Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.1-Linear/1.1.1.4-a+b-x-^m-c+d-x-^n-e+f-x-^p-g+h-x-^q

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [48]. This is test number [3].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. `IntegrateAlgebraic` under Mathematica 12.3.1 on windows 10. `https://github.com/stblake/algebraic_integration`. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (48)	0.00 (0)
Mathematica	100.00 (48)	0.00 (0)
Maple	100.00 (48)	0.00 (0)
Mupad	100.00 (48)	0.00 (0)
Fricas	95.83 (46)	4.17 (2)
Giac	85.42 (41)	14.58 (7)
Sympy	83.33 (40)	% 16.67 (8)
Maxima	79.17 (38)	20.83 (10)
IntegrateAlgebraic	75.00 (36)	25.00 (12)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

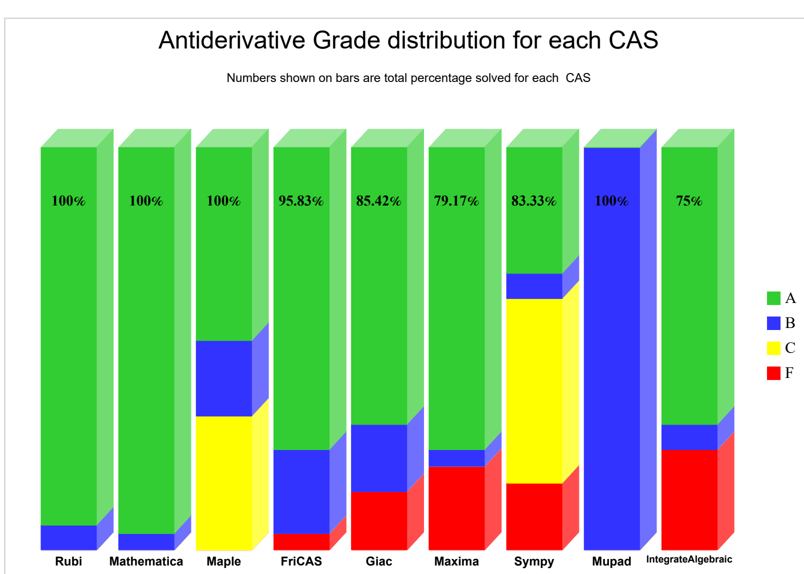
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

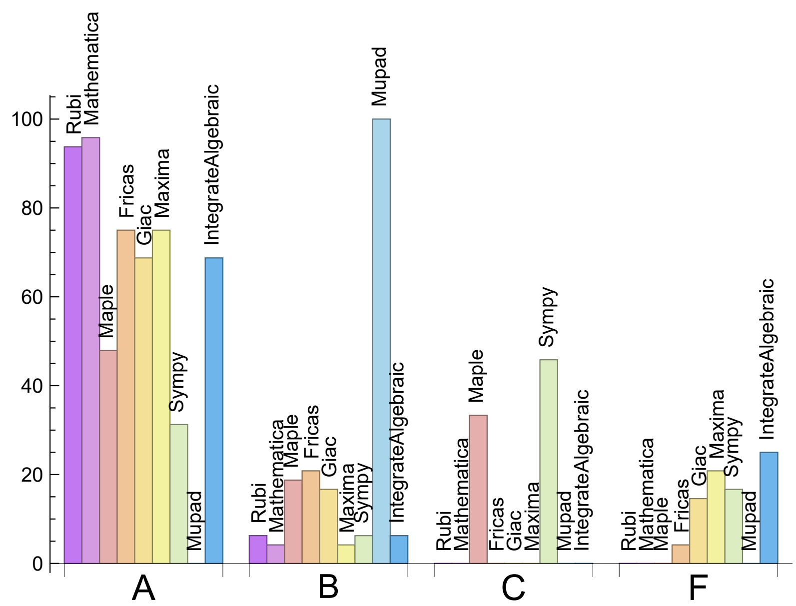
System	% A grade	% B grade	% C grade	% F grade
Mathematica	95.83	4.17	0.00	0.00
Rubi	93.75	6.25	0.00	0.00
Fricas	75.00	20.83	0.00	4.17
Maxima	75.00	4.17	0.00	20.83
IntegrateAlgebraic	68.75	6.25	0.00	25.00
Giac	68.75	16.67	0.00	14.58
Maple	47.92	18.75	33.33	0.00
Sympy	31.25	6.25	45.83	16.67
Mupad	N/A	100.00	0.00	0.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	0	0.00 %	0.00 %	0.00 %
Fricas	2	0.00 %	100.00 %	0.00 %
IntegrateAlgebraic	12	100.00 %	0.00 %	0.00 %
Giac	7	57.14 %	0.00 %	42.86 %
Maxima	10	40.00 %	0.00 %	60.00 %
Sympy	8	0.00 %	100.00 %	0.00 %
Mupad	0	0.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

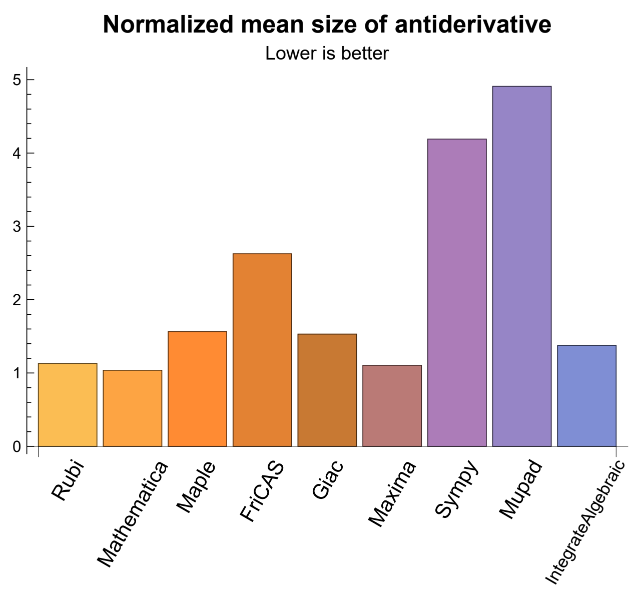
1.3 Performance

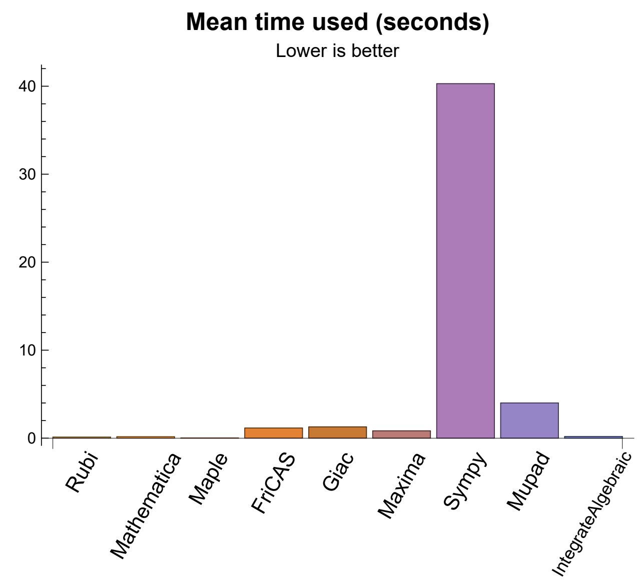
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	125.48	1.13	109.50	1.00
Mathematica	0.17	107.94	1.04	89.00	0.99
Maple	0.01	224.21	1.56	114.00	1.33
Maxima	0.83	102.61	1.11	85.00	1.05
Fricas	1.16	443.74	2.63	87.00	1.44
Sympy	40.29	451.55	4.19	217.00	2.90
Giac	1.29	170.88	1.53	112.00	1.36
Mupad	4.00	707.94	4.91	253.50	2.46
IntegrateAlgebraic	0.19	125.31	1.38	103.00	1.30

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.




Mean time used (seconds)
Lower is better
40
30
20
10
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad
IntegrateAlgebraic

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Mupad $\{\}$

IntegrateAlgebraic $\{\}$

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {43, 44, 45, 46}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

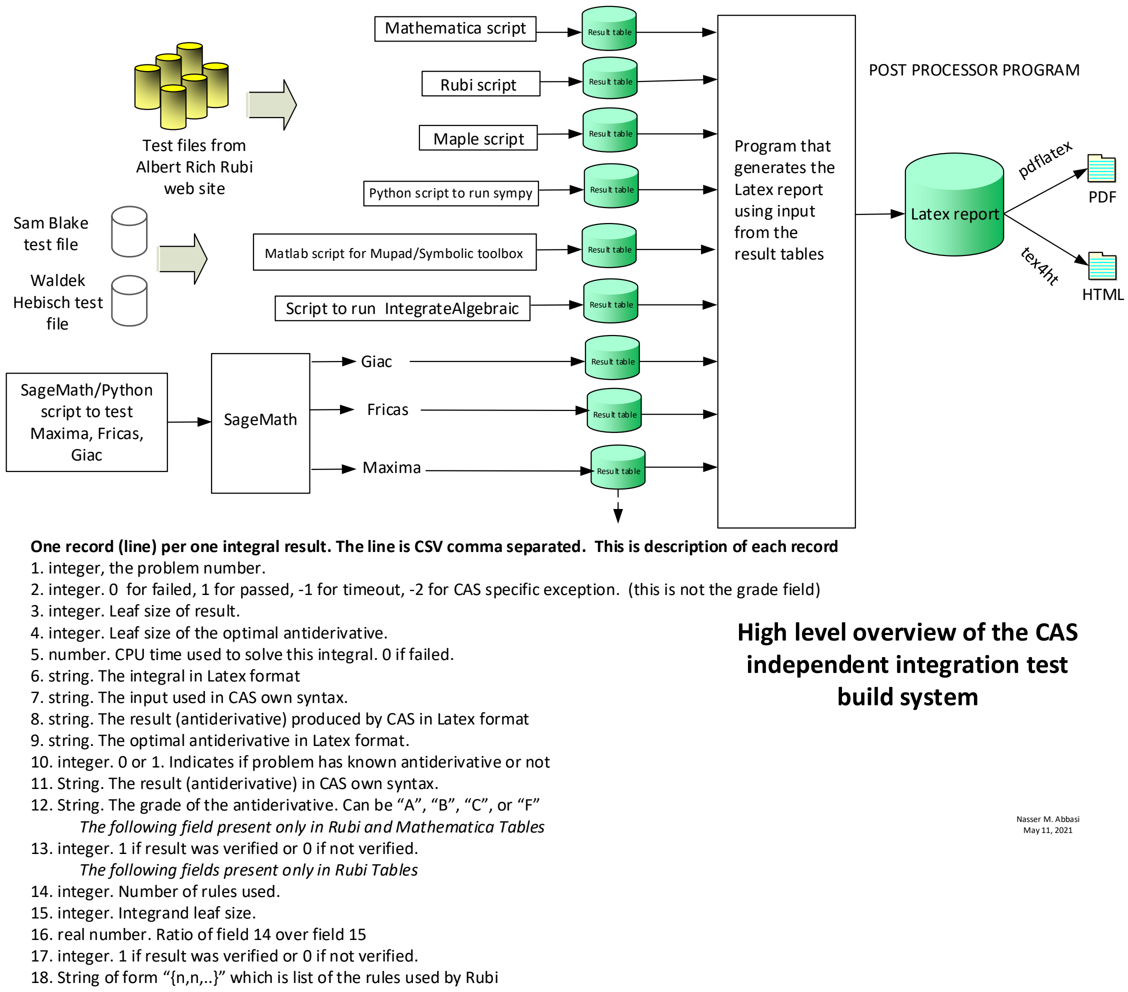
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 47, 48 }

B grade: { 44, 45, 46 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 46, 47, 48 }

B grade: { 44, 45 }

C grade: { }

F grade: { }

2.1.3 Maple

A grade: { 1, 4, 5, 6, 7, 8, 9, 10, 11, 13, 15, 16, 17, 18, 19, 20, 21, 27, 28, 29, 30, 31, 32 }

B grade: { 2, 3, 12, 14, 33, 34, 35, 36, 37 }

C grade: { 22, 23, 24, 25, 26, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48 }

F grade: { }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 38, 39, 40, 41, 42, 43, 45, 46, 47, 48 }

B grade: { 33, 44 }

C grade: { }

F grade: { 12, 13, 14, 19, 20, 21, 34, 35, 36, 37 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48 }

B grade: { 13, 14, 20, 21, 26, 33, 34, 35, 36, 37 }

C grade: { }

F grade: { 4, 5 }

2.1.6 Sympy

A grade: { 1, 2, 6, 7, 8, 9, 10, 11, 12, 15, 16, 17, 18, 19, 33 }

B grade: { 3, 13, 20 }

C grade: { 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48 }

F grade: { 4, 5, 14, 21, 34, 35, 36, 37 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 31, 32, 38, 39, 43, 44, 45, 46 }

B grade: { 5, 27, 28, 29, 30, 33, 47, 48 }

C grade: { }

F grade: { 34, 35, 36, 37, 40, 41, 42 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48 }

C grade: { }

F grade: { }

2.1.9 IntegrateAlgebraic

A grade: { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 39, 40, 41, 42, 45, 46, 47, 48 }

B grade: { 38, 43, 44 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 33, 34, 35, 36, 37 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA` and `IntegrateAlgebraic` to `I.A.`

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	112	109	108	142	148	150	115	0
N.S.	1	1.00	1.00	0.97	0.96	1.27	1.32	1.34	1.03	0.00
time (sec)	N/A	0.158	0.052	0.001	0.445	0.998	0.094	1.121	2.383	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	123	246	162	163	146	208	174	0
N.S.	1	1.00	0.98	1.95	1.29	1.29	1.16	1.65	1.38	0.00
time (sec)	N/A	0.211	0.088	0.005	0.443	0.695	0.580	1.189	2.537	0.001

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	85	196	104	117	507	112	105	0
N.S.	1	1.00	1.01	2.33	1.24	1.39	6.04	1.33	1.25	0.00
time (sec)	N/A	0.086	0.062	0.009	0.439	1.316	20.494	1.251	2.970	0.001

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	102	179	134	0	0	162	127	0
N.S.	1	1.00	0.94	1.66	1.24	0.00	0.00	1.50	1.18	0.00
time (sec)	N/A	0.110	0.082	0.010	0.443	0.000	0.000	1.137	4.168	0.001

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	F(-1)	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	164	164	310	0	0	363	317	0
N.S.	1	1.00	1.01	1.01	1.90	0.00	0.00	2.23	1.94	0.00
time (sec)	N/A	0.212	0.201	0.010	0.489	0.000	0.000	1.189	6.622	0.002

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	20	19	19	20	22	19	0
N.S.	1	1.00	1.00	0.87	0.83	0.83	0.87	0.96	0.83	0.00
time (sec)	N/A	0.011	0.006	0.007	0.430	1.709	0.153	1.251	0.076	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	33	34	34	53	32	31	29	0
N.S.	1	1.00	0.77	0.79	0.79	1.23	0.74	0.72	0.67	0.00
time (sec)	N/A	0.037	0.024	0.009	0.433	1.440	0.161	1.176	0.123	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	227	227	205	301	239	649	274	338	413	297
N.S.	1	1.00	0.90	1.33	1.05	2.86	1.21	1.49	1.82	1.31
time (sec)	N/A	0.257	0.275	0.010	0.991	1.294	37.787	1.405	0.161	0.201

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	145	176	152	405	167	201	263	202
N.S.	1	1.00	0.99	1.21	1.04	2.77	1.14	1.38	1.80	1.38
time (sec)	N/A	0.098	0.183	0.010	0.963	1.591	27.596	1.362	2.621	0.126

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	81	89	91	219	92	105	136	105
N.S.	1	1.00	1.05	1.16	1.18	2.84	1.19	1.36	1.77	1.36
time (sec)	N/A	0.024	0.159	0.008	0.979	0.901	25.991	1.328	0.091	0.060

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	55	46	60	111	54	57	45	57
N.S.	1	1.00	1.02	0.85	1.11	2.06	1.00	1.06	0.83	1.06
time (sec)	N/A	0.017	0.048	0.007	0.979	0.866	5.984	1.244	0.072	0.032

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	101	196	0	449	97	112	2368	111
N.S.	1	1.00	1.00	1.94	0.00	4.45	0.96	1.11	23.45	1.10
time (sec)	N/A	0.114	0.122	0.016	0.000	1.217	27.334	1.352	2.871	0.173

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	124	192	0	1018	1204	142	1827	151
N.S.	1	1.00	0.98	1.51	0.00	8.02	9.48	1.12	14.39	1.19
time (sec)	N/A	0.109	0.439	0.019	0.000	1.345	133.870	1.423	0.599	0.497

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	208	208	260	424	0	2216	0	300	4852	268
N.S.	1	1.00	1.25	2.04	0.00	10.65	0.00	1.44	23.33	1.29
time (sec)	N/A	0.274	0.602	0.020	0.000	3.164	0.000	1.394	4.543	1.081

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	226	226	204	301	238	641	274	338	413	343
N.S.	1	1.00	0.90	1.33	1.05	2.84	1.21	1.50	1.83	1.52
time (sec)	N/A	0.252	0.281	0.011	0.974	1.403	37.946	1.394	2.528	0.178

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	146	176	152	403	167	201	263	202
N.S.	1	1.00	1.01	1.21	1.05	2.78	1.15	1.39	1.81	1.39
time (sec)	N/A	0.094	0.179	0.010	0.980	1.071	26.173	1.368	0.089	0.128

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	87	89	90	217	92	105	136	91
N.S.	1	1.00	1.13	1.16	1.17	2.82	1.19	1.36	1.77	1.18
time (sec)	N/A	0.024	0.132	0.009	0.979	0.925	25.966	1.262	2.487	0.076

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	55	46	60	111	54	57	45	57
N.S.	1	1.00	1.02	0.85	1.11	2.06	1.00	1.06	0.83	1.06
time (sec)	N/A	0.016	0.050	0.007	0.974	0.782	6.083	1.201	0.069	0.035

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	100	103	0	450	100	112	2355	101
N.S.	1	1.00	0.99	1.02	0.00	4.46	0.99	1.11	23.32	1.00
time (sec)	N/A	0.119	0.227	0.016	0.000	1.473	24.251	1.219	2.819	0.170

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	128	128	122	137	0	1008	1149	142	1814	139
N.S.	1	1.00	0.95	1.07	0.00	7.88	8.98	1.11	14.17	1.09
time (sec)	N/A	0.116	0.196	0.017	0.000	1.355	142.242	1.323	2.952	0.482

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	205	259	221	0	2211	0	301	4839	248
N.S.	1	1.00	1.26	1.08	0.00	10.79	0.00	1.47	23.60	1.21
time (sec)	N/A	0.279	0.532	0.019	0.000	3.638	0.000	1.401	4.632	1.256

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	111	111	89	132	105	65	484	63	345	116
N.S.	1	1.00	0.80	1.19	0.95	0.59	4.36	0.57	3.11	1.05
time (sec)	N/A	0.045	0.089	0.036	0.974	1.318	35.797	1.245	7.778	0.096

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	81	111	83	57	393	53	269	100
N.S.	1	1.00	0.93	1.28	0.95	0.66	4.52	0.61	3.09	1.15
time (sec)	N/A	0.034	0.031	0.015	0.993	1.289	25.601	1.341	5.922	0.089

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	73	90	61	49	269	40	191	84
N.S.	1	1.00	1.16	1.43	0.97	0.78	4.27	0.63	3.03	1.33
time (sec)	N/A	0.023	0.030	0.016	0.962	1.312	20.771	1.275	4.527	0.078

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	61	70	41	43	133	28	118	51
N.S.	1	1.00	1.65	1.89	1.11	1.16	3.59	0.76	3.19	1.38
time (sec)	N/A	0.013	0.026	0.019	0.955	1.307	11.713	1.239	3.447	0.107

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	53	69	41	47	71	44	47	45
N.S.	1	1.00	1.83	2.38	1.41	1.62	2.45	1.52	1.62	1.55
time (sec)	N/A	0.016	0.025	0.019	0.953	0.808	25.620	1.233	2.985	0.099

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	29	25	42	27	107	88	24	37
N.S.	1	1.00	0.64	0.56	0.93	0.60	2.38	1.96	0.53	0.82
time (sec)	N/A	0.010	0.013	0.004	0.958	0.803	15.238	1.271	2.747	0.044

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	37	33	62	35	189	130	32	55
N.S.	1	1.00	0.51	0.45	0.85	0.48	2.59	1.78	0.44	0.75
time (sec)	N/A	0.019	0.016	0.005	0.962	1.326	17.839	1.242	2.731	0.045

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	45	41	84	43	274	175	40	72
N.S.	1	1.00	0.46	0.42	0.87	0.44	2.82	1.80	0.41	0.74
time (sec)	N/A	0.027	0.018	0.007	0.973	0.833	23.154	1.406	2.766	0.046

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	53	49	106	51	359	217	48	88
N.S.	1	1.00	0.44	0.40	0.88	0.42	2.97	1.79	0.40	0.73
time (sec)	N/A	0.038	0.019	0.006	0.966	0.906	33.865	1.315	2.829	0.049

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	48	44	21	40	117	43	65	49
N.S.	1	1.00	1.23	1.13	0.54	1.03	3.00	1.10	1.67	1.26
time (sec)	N/A	0.007	0.016	0.018	0.962	0.810	35.801	1.269	4.080	0.055

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	39	39	48	44	21	40	117	43	444	49
N.S.	1	1.00	1.23	1.13	0.54	1.03	3.00	1.10	11.38	1.26
time (sec)	N/A	0.011	0.006	0.005	0.976	1.076	72.778	1.291	5.273	0.064

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	167	167	149	726	474	877	8221	1665	819	0
N.S.	1	1.00	0.89	4.35	2.84	5.25	49.23	9.97	4.90	0.00
time (sec)	N/A	0.131	0.231	0.010	0.542	0.865	8.124	1.028	2.945	0.113

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	362	362	220	894	0	1659	0	0	1895	0
N.S.	1	1.00	0.61	2.47	0.00	4.58	0.00	0.00	5.23	0.00
time (sec)	N/A	0.362	0.478	0.010	0.000	0.927	0.000	0.000	4.487	0.122

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	507	507	279	2343	0	3441	0	0	3720	0
N.S.	1	1.00	0.55	4.62	0.00	6.79	0.00	0.00	7.34	0.00
time (sec)	N/A	0.586	0.702	0.017	0.000	1.180	0.000	0.000	6.752	0.123

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	363	360	227	906	0	1608	0	0	1890	0
N.S.	1	0.99	0.63	2.50	0.00	4.43	0.00	0.00	5.21	0.00
time (sec)	N/A	0.397	0.548	0.010	0.000	0.705	0.000	0.000	4.277	0.120

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F(-1)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	188	186	181	509	0	905	0	0	869	0
N.S.	1	0.99	0.96	2.71	0.00	4.81	0.00	0.00	4.62	0.00
time (sec)	N/A	0.098	0.128	0.010	0.000	0.842	0.000	0.000	3.407	0.079

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	79	79	57	139	87	78	313	101	244	179
N.S.	1	1.00	0.72	1.76	1.10	0.99	3.96	1.28	3.09	2.27
time (sec)	N/A	0.142	0.066	0.043	0.967	0.884	82.403	1.339	7.441	0.172

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	63	45	117	57	67	282	76	232	117
N.S.	1	1.00	0.71	1.86	0.90	1.06	4.48	1.21	3.68	1.86
time (sec)	N/A	0.061	0.035	0.020	0.970	0.940	49.786	1.315	6.988	0.126

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	96	57	81	245	0	122	95
N.S.	1	1.00	1.00	2.00	1.19	1.69	5.10	0.00	2.54	1.98
time (sec)	N/A	0.184	0.060	0.028	0.969	0.936	55.199	0.000	3.924	0.126

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	48	97	57	84	221	0	114	93
N.S.	1	1.00	1.00	2.02	1.19	1.75	4.60	0.00	2.38	1.94
time (sec)	N/A	0.183	0.060	0.023	0.971	0.872	49.853	0.000	3.741	0.166

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	56	108	98	65	218	0	312	112
N.S.	1	1.00	0.79	1.52	1.38	0.92	3.07	0.00	4.39	1.58
time (sec)	N/A	0.188	0.050	0.022	0.967	0.657	80.293	0.000	5.854	0.257

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	87	151	149	137	100	73	308	105	318	230
N.S.	1	1.74	1.71	1.57	1.15	0.84	3.54	1.21	3.66	2.64
time (sec)	N/A	0.146	0.349	0.026	0.434	0.907	78.825	1.303	12.354	0.171

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	B	C	B	A	C	A	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	52	135	126	120	90	61	277	80	312	112
N.S.	1	2.60	2.42	2.31	1.73	1.17	5.33	1.54	6.00	2.15
time (sec)	N/A	0.071	0.225	0.018	0.426	0.923	48.292	1.253	12.400	0.143

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	B	C	A	A	C	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	55	135	128	95	56	73	240	71	118	91
N.S.	1	2.45	2.33	1.73	1.02	1.33	4.36	1.29	2.15	1.65
time (sec)	N/A	0.184	0.418	0.023	0.962	0.744	47.403	1.295	3.975	0.148

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	C	A	A	C	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	55	135	89	96	56	82	216	83	118	89
N.S.	1	2.45	1.62	1.75	1.02	1.49	3.93	1.51	2.15	1.62
time (sec)	N/A	0.182	0.185	0.023	0.965	1.025	45.925	1.347	3.859	0.116

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	129	82	103	61	69	212	145	316	107
N.S.	1	1.55	0.99	1.24	0.73	0.83	2.55	1.75	3.81	1.29
time (sec)	N/A	0.189	0.144	0.023	0.972	0.881	74.795	1.350	9.891	0.115

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	C	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	171	94	123	86	90	219	197	304	168
N.S.	1	1.47	0.81	1.06	0.74	0.78	1.89	1.70	2.62	1.45
time (sec)	N/A	0.221	0.133	0.023	0.983	0.945	129.782	1.348	9.436	0.157

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [24] had the largest ratio of [.2500]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	21	0.048
2	A	2	1	1.00	23	0.043
3	A	2	1	1.00	25	0.040
4	A	2	1	1.00	27	0.037
5	A	2	1	1.00	29	0.034
6	A	2	1	1.00	17	0.059
7	A	3	2	1.00	22	0.091
8	A	6	5	1.00	25	0.200
9	A	5	5	1.00	25	0.200
10	A	4	4	1.00	23	0.174
11	A	4	4	1.00	18	0.222
12	A	6	4	1.00	25	0.160
13	A	6	4	1.00	25	0.160
14	A	7	5	1.00	25	0.200
15	A	6	5	1.00	25	0.200
16	A	5	5	1.00	25	0.200
17	A	4	4	1.00	23	0.174
18	A	4	4	1.00	18	0.222
19	A	6	5	1.00	25	0.200
20	A	6	5	1.00	25	0.200
21	A	7	6	1.00	25	0.240
22	A	8	6	1.00	26	0.231
23	A	7	6	1.00	26	0.231
24	A	6	6	1.00	24	0.250
25	A	4	4	1.00	23	0.174
26	A	5	5	1.00	26	0.192
27	A	3	3	1.00	26	0.115
28	A	4	4	1.00	26	0.154
29	A	5	4	1.00	26	0.154
30	A	6	4	1.00	26	0.154
31	A	4	4	1.00	24	0.167
32	A	5	5	1.00	36	0.139
33	A	2	1	1.00	23	0.043
34	A	3	3	1.00	29	0.103
35	A	4	3	1.00	29	0.103
36	A	3	3	0.99	29	0.103
37	A	3	3	0.99	24	0.125
38	A	4	4	1.00	31	0.129
39	A	4	4	1.00	30	0.133

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
40	A	7	7	1.00	33	0.212
41	A	7	7	1.00	33	0.212
42	A	6	6	1.00	33	0.182
43	A	5	5	1.74	30	0.167
44	B	5	5	2.60	29	0.172
45	B	8	8	2.45	32	0.250
46	B	8	8	2.45	32	0.250
47	A	6	6	1.55	32	0.188
48	A	7	7	1.47	32	0.219

Chapter 3

Listing of integrals

Local contents

3.1 $\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx$

Optimal. Leaf size=112

$$\frac{1}{4}x^4(adfh+b(cfh+deh+dfg))+\frac{1}{3}x^3(a(cfh+deh+dfg)+b(ceh+cfg+deg))+\frac{1}{2}x^2(a(ceh+cfg+deg)+bceg)+acegx$$

Rubi [A] time = 0.16, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {142}

$$\frac{1}{4}x^4(adfh+b(cfh+deh+dfg))+\frac{1}{3}x^3(a(cfh+deh+dfg)+b(ceh+cfg+deg))+\frac{1}{2}x^2(a(ceh+cfg+deg)+bceg)+acegx+\frac{1}{5}bdfhx^5$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] a*c*e*g*x + ((b*c*e*g + a*(d*e*g + c*f*g + c*e*h))*x^2)/2 + ((b*(d*e*g + c*
f*g + c*e*h) + a*(d*f*g + d*e*h + c*f*h))*x^3)/3 + ((a*d*f*h + b*(d*f*g + d
*e*h + c*f*h))*x^4)/4 + (b*d*f*h*x^5)/5
```

Rule 142

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx = \int \left(aceg+(bceg+a(deg+cfg+ceh))x+(b(deg+cfg+ceh)+a(d\right.$$

$$= acegx+\frac{1}{2}(bceg+a(deg+cfg+ceh))x^2+\frac{1}{3}(b(deg+cfg+ceh)+a($$

Mathematica [A] time = 0.05, size = 112, normalized size = 1.00

$$\frac{1}{4}x^4(adfh+bcfh+bdeh+bdfg)+\frac{1}{3}x^3(acfh+adeh+adfg+bceh+bcfg+bdeg)+\frac{1}{2}x^2(aceh+acfg+adeg+bceg)+acegx+\frac{1}{5}bdfhx^5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] a*c*e*g*x + ((b*c*e*g + a*d*e*g + a*c*f*g + a*c*e*h)*x^2)/2 + ((b*d*e*g + b
*c*f*g + a*d*f*g + b*c*e*h + a*d*e*h + a*c*f*h)*x^3)/3 + ((b*d*f*g + b*d*e*
h + b*c*f*h + a*d*f*h)*x^4)/4 + (b*d*f*h*x^5)/5
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+bx)(c+dx)(e+fx)(g+hx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)*(c + d*x)*(e + f*x)*(g + h*x), x]
```

fricas [A] time = 1.00, size = 142, normalized size = 1.27

$$\frac{1}{5}x^5hfdb+\frac{1}{4}x^4gfdb+\frac{1}{4}x^4hedb+\frac{1}{4}x^4hfcb+\frac{1}{4}x^4hfda+\frac{1}{3}x^3gedb+\frac{1}{3}x^3gfcb+\frac{1}{3}x^3hecb+\frac{1}{3}x^3gfda+\frac{1}{3}x^3heda+\frac{1}{3}x^3hfca+\frac{1}{2}x^2gecb+\frac{1}{2}x^2geda+\frac{1}{2}x^2gfca+\frac{1}{2}x^2heca+xgeca$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] 1/5*x^5*h*f*d*b + 1/4*x^4*g*f*d*b + 1/4*x^4*h*e*d*b + 1/4*x^4*h*f*c*b + 1/4
*x^4*h*f*d*a + 1/3*x^3*g*e*d*b + 1/3*x^3*g*f*c*b + 1/3*x^3*h*e*c*b + 1/3*x^
3*g*f*d*a + 1/3*x^3*h*e*d*a + 1/3*x^3*h*f*c*a + 1/2*x^2*g*e*c*b + 1/2*x^2*g
*e*d*a + 1/2*x^2*g*f*c*a + 1/2*x^2*h*e*c*a + x*g*e*c*a
```

giac [A] time = 1.12, size = 150, normalized size = 1.34

$$\frac{1}{5}bdfhx^5 + \frac{1}{4}bdfgx^4 + \frac{1}{4}bcfhx^4 + \frac{1}{4}adfhx^4 + \frac{1}{4}bdhx^4e + \frac{1}{3}bcfgx^3 + \frac{1}{3}adfgx^3 + \frac{1}{3}acfhx^3 + \frac{1}{3}bdgx^3e + \frac{1}{3}bchx^3e + \frac{1}{3}adhx^3e + \frac{1}{2}acfgx^2 + \frac{1}{2}bcgx^2e + \frac{1}{2}adgx^2e + \frac{1}{2}achx^2e + acgxe$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] 1/5*b*d*f*h*x^5 + 1/4*b*d*f*g*x^4 + 1/4*b*c*f*h*x^4 + 1/4*a*d*f*h*x^4 + 1/4
*b*d*h*x^4*e + 1/3*b*c*f*g*x^3 + 1/3*a*d*f*g*x^3 + 1/3*a*c*f*h*x^3 + 1/3*b*
d*g*x^3*e + 1/3*b*c*h*x^3*e + 1/3*a*d*h*x^3*e + 1/2*a*c*f*g*x^2 + 1/2*b*c*g
*x^2*e + 1/2*a*d*g*x^2*e + 1/2*a*c*h*x^2*e + a*c*g*x*e
```

maple [A] time = 0.00, size = 109, normalized size = 0.97

$$\frac{bdfh\,x^5}{5} + acegx + \frac{\left(bdfg + \left(bde + (ad+bc)\,f\right)h\right)x^4}{4} + \frac{\left(\left(bde + (ad+bc)\,f\right)g + \left(acf + (ad+bc)\,e\right)h\right)x^3}{3} + \frac{\left(aceh + \left(acf + (ad+bc)\,e\right)g\right)x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x)
```

```
[Out] 1/5*b*d*f*h*x^5+1/4*(((a*d+b*c)*f+b*d*e)*h+b*d*f*g)*x^4+1/3*((a*c*f+(a*d+b*
c)*e)*h+((a*d+b*c)*f+b*d*e)*g)*x^3+1/2*(a*c*e*h+(a*c*f+(a*d+b*c)*e)*g)*x^2+
a*c*e*g*x
```

maxima [A] time = 0.45, size = 108, normalized size = 0.96

$$\frac{1}{5}bdfhx^5 + acegx + \frac{1}{4}\left(bdfg + \left(bde + (bc+ad)f\right)h\right)x^4 + \frac{1}{3}\left(\left(bde + (bc+ad)f\right)g + \left(acf + (bc+ad)e\right)h\right)x^3 + \frac{1}{2}\left(aceh + \left(acf + (bc+ad)e\right)g\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] 1/5*b*d*f*h*x^5 + a*c*e*g*x + 1/4*(b*d*f*g + (b*d*e + (b*c + a*d)*f)*h)*x^4
 + 1/3*((b*d*e + (b*c + a*d)*f)*g + (a*c*f + (b*c + a*d)*e)*h)*x^3 + 1/2*(a
*c*e*h + (a*c*f + (b*c + a*d)*e)*g)*x^2
```

mupad [B] time = 2.38, size = 115, normalized size = 1.03

$$\frac{b\,d\,f\,h\,x^5}{5} + \left(\frac{a\,d\,f\,h}{4} + \frac{b\,c\,f\,h}{4} + \frac{b\,d\,e\,h}{4} + \frac{b\,d\,f\,g}{4}\right)x^4 + \left(\frac{a\,c\,f\,h}{3} + \frac{a\,d\,e\,h}{3} + \frac{a\,d\,f\,g}{3} + \frac{b\,c\,e\,h}{3} + \frac{b\,c\,f\,g}{3} + \frac{b\,d\,e\,g}{3}\right)x^3 + \left(\frac{a\,c\,e\,h}{2} + \frac{a\,c\,f\,g}{2} + \frac{a\,d\,e\,g}{2} + \frac{b\,c\,e\,g}{2}\right)x^2 + a\,c\,e\,g\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(g + h*x)*(a + b*x)*(c + d*x),x)
```

```
[Out] x^3*((a*c*f*h)/3 + (a*d*e*h)/3 + (a*d*f*g)/3 + (b*c*e*h)/3 + (b*c*f*g)/3 +
(b*d*e*g)/3) + x^2*((a*c*e*h)/2 + (a*c*f*g)/2 + (a*d*e*g)/2 + (b*c*e*g)/2)
+ x^4*((a*d*f*h)/4 + (b*c*f*h)/4 + (b*d*e*h)/4 + (b*d*f*g)/4) + a*c*e*g*x +
 (b*d*f*h*x^5)/5
```

sympy [A] time = 0.09, size = 148, normalized size = 1.32

$$acegx + \frac{bdfhx^5}{5} + x^4\left(\frac{adfh}{4} + \frac{bcfh}{4} + \frac{bdeh}{4} + \frac{bdfg}{4}\right) + x^3\left(\frac{acfh}{3} + \frac{adeh}{3} + \frac{adfg}{3} + \frac{bceh}{3} + \frac{bcfg}{3} + \frac{bdeg}{3}\right) + x^2\left(\frac{aceh}{2} + \frac{acfg}{2} + \frac{adeg}{2} + \frac{bceg}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)*(h*x+g),x)
```

```
[Out] a*c*e*g*x + b*d*f*h*x**5/5 + x**4*(a*d*f*h/4 + b*c*f*h/4 + b*d*e*h/4 + b*d*
f*g/4) + x**3*(a*c*f*h/3 + a*d*e*h/3 + a*d*f*g/3 + b*c*e*h/3 + b*c*f*g/3 +
b*d*e*g/3) + x**2*(a*c*e*h/2 + a*c*f*g/2 + a*d*e*g/2 + b*c*e*g/2)
```

3.2 $\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx$

Optimal. Leaf size=126

$$-\frac{(bg-ah)(dg-ch)(fg-eh)\log(g+hx)}{h^4}+\frac{x(b(dg-ch)(fg-eh)-ah(-cfh-deh+dfg))}{h^3}+\frac{x^2(adfh-b(-cfh-$$

Rubi [A] time = 0.21, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {142}

$$\frac{x^2(adfh-b(-cfh-deh+dfg))}{2h^2}+\frac{x(b(dg-ch)(fg-eh)-ah(-cfh-deh+dfg))}{h^3}-\frac{(bg-ah)(dg-ch)(fg-eh)\log(g+hx)}{h^4}+\frac{bdfx^3}{3h}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*(c + d*x)*(e + f*x))/(g + h*x),x]
```

```
[Out] ((b*(d*g - c*h)*(f*g - e*h) - a*h*(d*f*g - d*e*h - c*f*h))*x)/h^3 + ((a*d*f
*h - b*(d*f*g - d*e*h - c*f*h))*x^2)/(2*h^2) + (b*d*f*x^3)/(3*h) - ((b*g -
a*h)*(d*g - c*h)*(f*g - e*h)*Log[g + h*x])/h^4
```

Rule 142

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx &= \int \left(\frac{b(dg-ch)(fg-eh)-ah(dfg-deh-cfh)}{h^3}+\frac{(adfh-b(dfg-deh-cfh))x}{h^2}\right.\\ &= \frac{(b(dg-ch)(fg-eh)-ah(dfg-deh-cfh))x}{h^3}+\frac{(adfh-b(dfg-deh-cfh))x^2}{2h^2}\end{aligned}$$

Mathematica [A] time = 0.09, size = 123, normalized size = 0.98

$$\frac{hx\left(3ah(2cfh+d(2eh-2fg+fhx))+b\left(3ch(2eh-2fg+fhx)+3deh(hx-2g)+df\left(6g^2-3ghx+2h^2x^2\right)\right)\right)-6(bg-ah)(dg-ch)(fg-eh)\log(g+hx)}{6h^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*(c + d*x)*(e + f*x))/(g + h*x),x]
```

```
[Out] (h*x*(3*a*h*(2*c*f*h + d*(-2*f*g + 2*e*h + f*h*x)) + b*(3*d*e*h*(-2*g + h*x
) + 3*c*h*(-2*f*g + 2*e*h + f*h*x) + d*f*(6*g^2 - 3*g*h*x + 2*h^2*x^2))) -
6*(b*g - a*h)*(d*g - c*h)*(f*g - e*h)*Log[g + h*x])/(6*h^4)
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)(c+dx)(e+fx)}{g+hx}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x)*(c + d*x)*(e + f*x))/(g + h*x),x]
```

```
[Out] IntegrateAlgebraic[((a + b*x)*(c + d*x)*(e + f*x))/(g + h*x), x]
```

fricas [A] time = 0.70, size = 163, normalized size = 1.29

$$\frac{2\,bdfh^3x^3 - 3\left(bdfgh^2 - \left(bde + (bc + ad)f\right)h^3\right)x^2 + 6\left(bdfg^2h - \left(bde + (bc + ad)f\right)gh^2 + \left(acf + (bc + ad)e\right)h^3\right)x - 6\left(bdfg^3 - aceh^3 - \left(bde + (bc + ad)f\right)g^2h + \left(acf + (bc + ad)e\right)gh^2\right)\log\left(hx + g\right)}{6\,h^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] 1/6*(2*b*d*f*h^3*x^3 - 3*(b*d*f*g*h^2 - (b*d*e + (b*c + a*d)*f)*h^3)*x^2 +
6*(b*d*f*g^2*h - (b*d*e + (b*c + a*d)*f)*g*h^2 + (a*c*f + (b*c + a*d)*e)*h^
3)*x - 6*(b*d*f*g^3 - a*c*e*h^3 - (b*d*e + (b*c + a*d)*f)*g^2*h + (a*c*f +
(b*c + a*d)*e)*g*h^2)*log(h*x + g))/h^4
```

giac [A] time = 1.19, size = 208, normalized size = 1.65

$$\frac{2\,bdfh^2x^3 - 3\,bdfghx^2 + 3\,bcfh^2x^2 + 3\,adfh^2x^2 + 3\,bdh^2x^2e + 6\,bdfg^2x - 6\,bcfghx - 6\,adfghx + 6\,acfh^2x - 6\,bdghxe + 6\,bch^2xe + 6\,adh^2xe}{6\,h^3} - \frac{\left(bdfg^3 - bcfg^2h - adfg^2h + acfgh^2 - bdg^2he + bcgh^2e + adgh^2e - ach^3e\right)\log\left(\left|hx + g\right|\right)}{h^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] 1/6*(2*b*d*f*h^2*x^3 - 3*b*d*f*g*h*x^2 + 3*b*c*f*h^2*x^2 + 3*a*d*f*h^2*x^2
+ 3*b*d*h^2*x^2*e + 6*b*d*f*g^2*x - 6*b*c*f*g*h*x - 6*a*d*f*g*h*x + 6*a*c*f
*h^2*x - 6*b*d*g*h*x*e + 6*b*c*h^2*x*e + 6*a*d*h^2*x*e)/h^3 - (b*d*f*g^3 -
b*c*f*g^2*h - a*d*f*g^2*h + a*c*f*g*h^2 - b*d*g^2*h*e + b*c*g*h^2*e + a*d*g
*h^2*e - a*c*h^3*e)*log(abs(h*x + g))/h^4
```

maple [B] time = 0.00, size = 246, normalized size = 1.95

$$\frac{bdf\,x^3}{3h} + \frac{adf\,x^2}{2h} + \frac{bcf\,x^2}{2h} + \frac{bde\,x^2}{2h} - \frac{bdfg\,x^2}{2h^2} + \frac{ace\ln\left(hx + g\right)}{h} - \frac{acfg\ln\left(hx + g\right)}{h^2} + \frac{acfx}{h} - \frac{adeg\ln\left(hx + g\right)}{h^2} + \frac{adex}{h} + \frac{adf\,g^2\ln\left(hx + g\right)}{h^3} - \frac{adfgx}{h^2} - \frac{bceg\ln\left(hx + g\right)}{h^2} + \frac{bcex}{h} + \frac{bcf\,g^2\ln\left(hx + g\right)}{h^3} - \frac{bcfgx}{h^2} + \frac{bdeg^2\ln\left(hx + g\right)}{h^3} - \frac{bdegx}{h^2} - \frac{bdf\,g^3\ln\left(hx + g\right)}{h^4} + \frac{bdf\,g^2x}{h^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x)
```

```
[Out] 1/3*b*d*f*x^3/h+1/2/h*x^2*a*d*f+1/2/h*x^2*b*c*f+1/2/h*x^2*b*d*e-1/2/h^2*x^2
*b*d*f*g+1/h*x*a*c*f+1/h*x*a*d*e-1/h^2*x*a*d*f*g+1/h*x*b*c*e-1/h^2*x*b*c*f*
g-1/h^2*x*b*d*e*g+1/h^3*x*b*d*f*g^2+1/h*ln(h*x+g)*a*c*e-1/h^2*ln(h*x+g)*a*c
*f*g-1/h^2*ln(h*x+g)*a*d*e*g+1/h^3*ln(h*x+g)*a*d*f*g^2-1/h^2*ln(h*x+g)*b*c*
e*g+1/h^3*ln(h*x+g)*b*c*f*g^2+1/h^3*ln(h*x+g)*b*d*e*g^2-1/h^4*ln(h*x+g)*b*d
*f*g^3
```

maxima [A] time = 0.44, size = 162, normalized size = 1.29

$$\frac{2\,bdfh^2x^3 - 3\left(bdfgh - \left(bde + (bc + ad)f\right)h^2\right)x^2 + 6\left(bdfg^2 - \left(bde + (bc + ad)f\right)gh + \left(acf + (bc + ad)e\right)h^2\right)x}{6\,h^3} - \frac{\left(bdfg^3 - aceh^3 - \left(bde + (bc + ad)f\right)g^2h + \left(acf + (bc + ad)e\right)gh^2\right)\log\left(hx + g\right)}{h^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*d*f*h^2*x^3 - 3*(b*d*f*g*h - (b*d*e + (b*c + a*d)*f)*h^2)*x^2 + 6*
(b*d*f*g^2 - (b*d*e + (b*c + a*d)*f)*g*h + (a*c*f + (b*c + a*d)*e)*h^2)*x)/
h^3 - (b*d*f*g^3 - a*c*e*h^3 - (b*d*e + (b*c + a*d)*f)*g^2*h + (a*c*f + (b*
c + a*d)*e)*g*h^2)*log(h*x + g)/h^4
```

mupad [B] time = 2.54, size = 174, normalized size = 1.38

$$x\left(\frac{a\,c\,f + a\,d\,e + b\,c\,e}{h} - \frac{g\left(\frac{a\,d\,f + b\,c\,f + b\,d\,e}{h} - \frac{b\,d\,f\,g}{h^2}\right)}{h}\right) + x^2\left(\frac{a\,d\,f + b\,c\,f + b\,d\,e}{2\,h} - \frac{b\,d\,f\,g}{2\,h^2}\right) + \frac{\ln\left(g + h\,x\right)\,\left(a\,c\,e\,h^3 - b\,d\,f\,g^3 - a\,c\,f\,g\,h^2 - a\,d\,e\,g\,h^2 - b\,c\,e\,g\,h^2 + a\,d\,f\,g^2\,h + b\,c\,f\,g^2\,h + b\,d\,e\,g^2\,h\right)}{h^4} + \frac{b\,d\,f\,x^3}{3\,h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)*(c + d*x))/(g + h*x),x)
```

```
[Out] x*((a*c*f + a*d*e + b*c*e)/h - (g*((a*d*f + b*c*f + b*d*e)/h - (b*d*f*g)/h^
2))/h) + x^2*((a*d*f + b*c*f + b*d*e)/(2*h) - (b*d*f*g)/(2*h^2)) + (log(g +
 h*x)*(a*c*e*h^3 - b*d*f*g^3 - a*c*f*g*h^2 - a*d*e*g*h^2 - b*c*e*g*h^2 + a*
d*f*g^2*h + b*c*f*g^2*h + b*d*e*g^2*h))/h^4 + (b*d*f*x^3)/(3*h)
```

sympy [A] time = 0.58, size = 146, normalized size = 1.16

$$\frac{bdfx^3}{3h} + x^2\left(\frac{adf}{2h} + \frac{bcf}{2h} + \frac{bde}{2h} - \frac{bdfg}{2h^2}\right) + x\left(\frac{acf}{h} + \frac{ade}{h} - \frac{adfg}{h^2} + \frac{bce}{h} - \frac{bcfg}{h^2} - \frac{bdeg}{h^2} + \frac{bdfg^2}{h^3}\right) + \frac{(ah-bg)(ch-dg)(eh-fg)\log(g+hx)}{h^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)*(f*x+e)/(h*x+g),x)
```

```
[Out] b*d*f*x**3/(3*h) + x**2*(a*d*f/(2*h) + b*c*f/(2*h) + b*d*e/(2*h) - b*d*f*g/
(2*h**2)) + x*(a*c*f/h + a*d*e/h - a*d*f*g/h**2 + b*c*e/h - b*c*f*g/h**2 -
b*d*e*g/h**2 + b*d*f*g**2/h**3) + (a*h - b*g)*(c*h - d*g)*(e*h - f*g)*log(g
 + h*x)/h**4
```

3.3 $\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)}\,dx$

Optimal. Leaf size=84

$$\frac{(be-af)(de-cf)\log(e+fx)}{f^2(fg-eh)}-\frac{(bg-ah)(dg-ch)\log(g+hx)}{h^2(fg-eh)}+\frac{bdx}{fh}$$

Rubi [A] time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {142}

$$\frac{(be-af)(de-cf)\log(e+fx)}{f^2(fg-eh)}-\frac{(bg-ah)(dg-ch)\log(g+hx)}{h^2(fg-eh)}+\frac{bdx}{fh}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

```
[Out] (b*d*x)/(f*h) + ((b*e - a*f)*(d*e - c*f)*Log[e + f*x])/(f^2*(f*g - e*h)) -
((b*g - a*h)*(d*g - c*h)*Log[g + h*x])/(h^2*(f*g - e*h))
```

Rule 142

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)}\,dx &= \int \left(\frac{bd}{fh}+\frac{(-be+af)(-de+cf)}{f(fg-eh)(e+fx)}+\frac{(-bg+ah)(-dg+ch)}{h(-fg+eh)(g+hx)}\right)dx\\ &= \frac{bdx}{fh}+\frac{(be-af)(de-cf)\log(e+fx)}{f^2(fg-eh)}-\frac{(bg-ah)(dg-ch)\log(g+hx)}{h^2(fg-eh)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 85, normalized size = 1.01

$$\frac{f(bdhx(fg-eh)-f(bg-ah)(dg-ch)\log(g+hx))+h^2(be-af)(de-cf)\log(e+fx)}{f^2h^2(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

```
[Out] ((b*e - a*f)*(d*e - c*f)*h^2*Log[e + f*x] + f*(b*d*h*(f*g - e*h)*x - f*(b*g
 - a*h)*(d*g - c*h)*Log[g + h*x]))/(f^2*h^2*(f*g - e*h))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx)(c+dx)}{(e+fx)(g+hx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x)*(c + d*x))/((e + f*x)*(g + h*x)),x]
```

```
[Out] IntegrateAlgebraic[((a + b*x)*(c + d*x))/((e + f*x)*(g + h*x)), x]
```

fricas [A] time = 1.32, size = 117, normalized size = 1.39

$$\frac{\left(bde^2 + acf^2 - (bc + ad)ef\right)h^2 \log\left(fx + e\right) + \left(bdf^2gh - bdefh^2\right)x - \left(bdf^2g^2 + acf^2h^2 - (bc + ad)f^2gh\right)\log\left(hx + g\right)}{f^3gh^2 - ef^2h^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] ((b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)*h^2*log(f*x + e) + (b*d*f^2*g*h - b*
d*e*f*h^2)*x - (b*d*f^2*g^2 + a*c*f^2*h^2 - (b*c + a*d)*f^2*g*h)*log(h*x +
g))/(f^3*g*h^2 - e*f^2*h^3)
```

giac [A] time = 1.25, size = 112, normalized size = 1.33

$$\frac{bdx}{fh} + \frac{\left(acf^2 - bcfe - adfe + bde^2\right)\log\left(\left|fx + e\right|\right)}{f^3g - f^2he} - \frac{\left(bdg^2 - bcgh - adgh + ach^2\right)\log\left(\left|hx + g\right|\right)}{fgh^2 - h^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] b*d*x/(f*h) + (a*c*f^2 - b*c*f*e - a*d*f*e + b*d*e^2)*log(abs(f*x + e))/(f^
3*g - f^2*h*e) - (b*d*g^2 - b*c*g*h - a*d*g*h + a*c*h^2)*log(abs(h*x + g))/
(f*g*h^2 - h^3*e)
```

maple [B] time = 0.01, size = 196, normalized size = 2.33

$$-\frac{ac\ln\left(fx+e\right)}{eh-fg} + \frac{ac\ln\left(hx+g\right)}{eh-fg} + \frac{ade\ln\left(fx+e\right)}{\left(eh-fg\right)f} - \frac{adg\ln\left(hx+g\right)}{\left(eh-fg\right)h} + \frac{bce\ln\left(fx+e\right)}{\left(eh-fg\right)f} - \frac{bcg\ln\left(hx+g\right)}{\left(eh-fg\right)h} - \frac{bde^2\ln\left(fx+e\right)}{\left(eh-fg\right)f^2} + \frac{bdg^2\ln\left(hx+g\right)}{\left(eh-fg\right)h^2} + \frac{bdx}{fh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] b*d*x/f/h+1/(e*h-f*g)*ln(h*x+g)*a*c-1/h/(e*h-f*g)*ln(h*x+g)*a*d*g-1/h/(e*h-
f*g)*ln(h*x+g)*b*c*g+1/h^2/(e*h-f*g)*ln(h*x+g)*b*d*g^2-1/(e*h-f*g)*ln(f*x+e
)*a*c+1/f/(e*h-f*g)*ln(f*x+e)*a*d*e+1/f/(e*h-f*g)*ln(f*x+e)*b*c*e-1/f^2/(e*
h-f*g)*ln(f*x+e)*b*d*e^2
```

maxima [A] time = 0.44, size = 104, normalized size = 1.24

$$\frac{bdx}{fh} + \frac{\left(bde^2 + acf^2 - (bc + ad)ef\right)\log\left(fx + e\right)}{f^3g - ef^2h} - \frac{\left(bdg^2 + ach^2 - (bc + ad)gh\right)\log\left(hx + g\right)}{fgh^2 - eh^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] b*d*x/(f*h) + (b*d*e^2 + a*c*f^2 - (b*c + a*d)*e*f)*log(f*x + e)/(f^3*g - e
*f^2*h) - (b*d*g^2 + a*c*h^2 - (b*c + a*d)*g*h)*log(h*x + g)/(f*g*h^2 - e*h
^3)
```

mupad [B] time = 2.97, size = 105, normalized size = 1.25

$$\frac{\ln\left(e + f\,x\right)\,\left(a\,c\,f^2 - f\,\left(a\,d\,e + b\,c\,e\right) + b\,d\,e^2\right)}{f^3\,g - e\,f^2\,h} + \frac{\ln\left(g + h\,x\right)\,\left(a\,c\,h^2 - h\,\left(a\,d\,g + b\,c\,g\right) + b\,d\,g^2\right)}{e\,h^3 - f\,g\,h^2} + \frac{b\,d\,x}{f\,h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x)*(c + d*x))/((e + f*x)*(g + h*x)),x)
```

```
[Out] (log(e + f*x)*(a*c*f^2 - f*(a*d*e + b*c*e) + b*d*e^2))/(f^3*g - e*f^2*h) +
(log(g + h*x)*(a*c*h^2 - h*(a*d*g + b*c*g) + b*d*g^2))/(e*h^3 - f*g*h^2) +
(b*d*x)/(f*h)
```

sympy [B] time = 20.49, size = 507, normalized size = 6.04

$$\frac{bdx}{fh}+\frac{(ah-bg)\,(ch-dg)\log\left(x+\frac{acefh^2+acf^2gh-2adefgh-2bcefgh+bde^2gh+bdefg^2-\frac{e^2fh(ah-bg)(ch-dg)}{eh-fg}+\frac{2ef^2g(ah-bg)(ch-dg)}{eh-fg}-\frac{f^3g^2(ah-bg)(ch-dg)}{h(eh-fg)}}{2acf^2h^2-adefh^2-adf^2gh-bcefh^2-bcf^2gh+bde^2h^2+bdf^2g^2}\right)}{h^2\,(eh-fg)}-\frac{(af-be)\,(cf-de)\log\left(x+\frac{acefh^2+acf^2gh-2adefgh-2bcefgh+bde^2gh+bdefg^2+\frac{e^2h^3(af-be)(cf-de)}{f(eh-fg)}-\frac{2egh^2(af-be)(cf-de)}{eh-fg}+\frac{fg^2h(af-be)(cf-de)}{eh-fg}}{2acf^2h^2-adefh^2-adf^2gh-bcefh^2-bcf^2gh+bde^2h^2+bdf^2g^2}\right)}{f^2\,(eh-fg)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] b*d*x/(f*h) + (a*h - b*g)*(c*h - d*g)*log(x + (a*c*e*f*h**2 + a*c*f**2*g*h
- 2*a*d*e*f*g*h - 2*b*c*e*f*g*h + b*d*e**2*g*h + b*d*e*f*g**2 - e**2*f*h*(a
*h - b*g)*(c*h - d*g)/(e*h - f*g) + 2*e*f**2*g*(a*h - b*g)*(c*h - d*g)/(e*h
 - f*g) - f**3*g**2*(a*h - b*g)*(c*h - d*g)/(h*(e*h - f*g)))/(2*a*c*f**2*h*
*2 - a*d*e*f*h**2 - a*d*f**2*g*h - b*c*e*f*h**2 - b*c*f**2*g*h + b*d*e**2*h
**2 + b*d*f**2*g**2))/(h**2*(e*h - f*g)) - (a*f - b*e)*(c*f - d*e)*log(x +
(a*c*e*f*h**2 + a*c*f**2*g*h - 2*a*d*e*f*g*h - 2*b*c*e*f*g*h + b*d*e**2*g*h
 + b*d*e*f*g**2 + e**2*h**3*(a*f - b*e)*(c*f - d*e)/(f*(e*h - f*g)) - 2*e*g
*h**2*(a*f - b*e)*(c*f - d*e)/(e*h - f*g) + f*g**2*h*(a*f - b*e)*(c*f - d*e
)/(e*h - f*g))/(2*a*c*f**2*h**2 - a*d*e*f*h**2 - a*d*f**2*g*h - b*c*e*f*h**
2 - b*c*f**2*g*h + b*d*e**2*h**2 + b*d*f**2*g**2))/(f**2*(e*h - f*g))
```

3.4 $\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx$

Optimal. Leaf size=108

$$-\frac{(bc-ad)\log(c+dx)}{(de-cf)(dg-ch)}+\frac{(be-af)\log(e+fx)}{(de-cf)(fg-eh)}-\frac{(bg-ah)\log(g+hx)}{(dg-ch)(fg-eh)}$$

Rubi [A] time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.037, Rules used = {148}

$$-\frac{(bc-ad)\log(c+dx)}{(de-cf)(dg-ch)}+\frac{(be-af)\log(e+fx)}{(de-cf)(fg-eh)}-\frac{(bg-ah)\log(g+hx)}{(dg-ch)(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] -(((b*c - a*d)*Log[c + d*x])/((d*e - c*f)*(d*g - c*h))) + ((b*e - a*f)*Log[
e + f*x])/((d*e - c*f)*(f*g - e*h)) - ((b*g - a*h)*Log[g + h*x])/((d*g - c*
h)*(f*g - e*h))
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx &= \int \left(\frac{d(-bc+ad)}{(de-cf)(dg-ch)(c+dx)}+\frac{f(-be+af)}{(de-cf)(-fg+eh)(e+fx)}+\frac{h(-bg+}{(dg-ch)(fg-}\right.\\ &= -\frac{(bc-ad)\log(c+dx)}{(de-cf)(dg-ch)}+\frac{(be-af)\log(e+fx)}{(de-cf)(fg-eh)}-\frac{(bg-ah)\log(g+hx)}{(dg-ch)(fg-eh)}\end{aligned}$$

Mathematica [A] time = 0.08, size = 102, normalized size = 0.94

$$\frac{(bc-ad)\log(c+dx)(fg-eh)-(be-af)(dg-ch)\log(e+fx)+(bg-ah)(de-cf)\log(g+hx)}{(de-cf)(dg-ch)(eh-fg)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] ((b*c - a*d)*(f*g - e*h)*Log[c + d*x] - (b*e - a*f)*(d*g - c*h)*Log[e + f*x
] + (d*e - c*f)*(b*g - a*h)*Log[g + h*x])/((d*e - c*f)*(d*g - c*h)*(-(f*g)
+ e*h))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a+bx}{(c+dx)(e+fx)(g+hx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)/((c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] IntegrateAlgebraic[(a + b*x)/((c + d*x)*(e + f*x)*(g + h*x)), x]
```

fricas [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [**A**] time = 1.14, size = 162, normalized size = 1.50

$$\frac{\left(bcd-ad^2\right)\log\left(|dx+c|\right)}{cd^2fg-c^2dfh-d^3ge+cd^2he}+\frac{\left(af^2-bfe\right)\log\left(\left|fx+e\right|\right)}{cf^3g-df^2ge-cf^2he+dfhe^2}-\frac{\left(bgh-ah^2\right)\log\left(\left|hx+g\right|\right)}{dfg^2h-cfgh^2-dgh^2e+ch^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
```

```
[Out] (b*c*d - a*d^2)*log(abs(d*x + c))/(c*d^2*f*g - c^2*d*f*h - d^3*g*e + c*d^2*
h*e) + (a*f^2 - b*f*e)*log(abs(f*x + e))/(c*f^3*g - d*f^2*g*e - c*f^2*h*e +
 d*f*h*e^2) - (b*g*h - a*h^2)*log(abs(h*x + g))/(d*f*g^2*h - c*f*g*h^2 - d*
g*h^2*e + c*h^3*e)
```

maple [**A**] time = 0.01, size = 179, normalized size = 1.66

$$\frac{ad\ln\left(dx+c\right)}{\left(cf-de\right)\left(ch-dg\right)}-\frac{af\ln\left(fx+e\right)}{\left(cf-de\right)\left(eh-fg\right)}+\frac{ah\ln\left(hx+g\right)}{\left(ch-dg\right)\left(eh-fg\right)}-\frac{bc\ln\left(dx+c\right)}{\left(cf-de\right)\left(ch-dg\right)}+\frac{be\ln\left(fx+e\right)}{\left(cf-de\right)\left(eh-fg\right)}-\frac{bg\ln\left(hx+g\right)}{\left(ch-dg\right)\left(eh-fg\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] 1/(c*f-d*e)/(c*h-d*g)*ln(d*x+c)*a*d-1/(c*f-d*e)/(c*h-d*g)*ln(d*x+c)*b*c+1/(
c*h-d*g)/(e*h-f*g)*ln(h*x+g)*a*h-1/(c*h-d*g)/(e*h-f*g)*ln(h*x+g)*b*g-1/(c*f
-d*e)/(e*h-f*g)*ln(f*x+e)*a*f+1/(c*f-d*e)/(e*h-f*g)*ln(f*x+e)*b*e
```

maxima [**A**] time = 0.44, size = 134, normalized size = 1.24

$$-\frac{(bc-ad)\log\left(dx+c\right)}{\left(d^2e-cdf\right)g-\left(cde-c^2f\right)h}+\frac{\left(be-af\right)\log\left(fx+e\right)}{\left(def-cf^2\right)g-\left(de^2-cef\right)h}-\frac{\left(bg-ah\right)\log\left(hx+g\right)}{dfg^2+ceh^2-\left(de+cf\right)gh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
```

```
[Out] -(b*c - a*d)*log(d*x + c)/((d^2*e - c*d*f)*g - (c*d*e - c^2*f)*h) + (b*e -
a*f)*log(f*x + e)/((d*e*f - c*f^2)*g - (d*e^2 - c*e*f)*h) - (b*g - a*h)*log
(h*x + g)/(d*f*g^2 + c*e*h^2 - (d*e + c*f)*g*h)
```

mupad [**B**] time = 4.17, size = 127, normalized size = 1.18

$$\frac{\ln\left(e+f\,x\right)\,\left(a\,f-b\,e\right)}{c\,f^2\,g+d\,e^2\,h-c\,e\,f\,h-d\,e\,f\,g}+\frac{\ln\left(g+h\,x\right)\,\left(a\,h-b\,g\right)}{c\,e\,h^2+d\,f\,g^2-c\,f\,g\,h-d\,e\,g\,h}+\frac{\ln\left(c+d\,x\right)\,\left(a\,d-b\,c\right)}{d^2\,e\,g+c^2\,f\,h-c\,d\,e\,h-c\,d\,f\,g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x)/((e + f*x)*(g + h*x)*(c + d*x)),x)
```

```
[Out] (log(e + f*x)*(a*f - b*e))/(c*f^2*g + d*e^2*h - c*e*f*h - d*e*f*g) + (log(g
 + h*x)*(a*h - b*g))/(c*e*h^2 + d*f*g^2 - c*f*g*h - d*e*g*h) + (log(c + d*x
)*(a*d - b*c))/(d^2*e*g + c^2*f*h - c*d*e*h - c*d*f*g)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] Timed out
```

3.5 $\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx$

Optimal. Leaf size=163

$$\frac{b^2\log(a+bx)}{(bc-ad)(be-af)(bg-ah)}-\frac{d^2\log(c+dx)}{(bc-ad)(de-cf)(dg-ch)}+\frac{f^2\log(e+fx)}{(be-af)(de-cf)(fg-eh)}-\frac{h^2\log(g+hx)}{(bg-ah)(dg-ch)(fg}$$

Rubi [A] time = 0.21, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.034, Rules used = {180}

$$\frac{b^2\log(a+bx)}{(bc-ad)(be-af)(bg-ah)}-\frac{d^2\log(c+dx)}{(bc-ad)(de-cf)(dg-ch)}+\frac{f^2\log(e+fx)}{(be-af)(de-cf)(fg-eh)}-\frac{h^2\log(g+hx)}{(bg-ah)(dg-ch)(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*x)*(c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] (b^2*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)) - (d^2*Log[c + d*x
])/((b*c - a*d)*(d*e - c*f)*(d*g - c*h)) + (f^2*Log[e + f*x])/((b*e - a*f)*
(d*e - c*f)*(f*g - e*h)) - (h^2*Log[g + h*x])/((b*g - a*h)*(d*g - c*h)*(f*g
 - e*h))
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx &= \int \left(\frac{b^3}{(bc-ad)(be-af)(bg-ah)(a+bx)}-\frac{d^3}{(bc-ad)(-de+cf)(-dg+}\right.\\ &= \frac{b^2\log(a+bx)}{(bc-ad)(be-af)(bg-ah)}-\frac{d^2\log(c+dx)}{(bc-ad)(de-cf)(dg-ch)}+\frac{f}{(be-af}\end{aligned}$$

Mathematica [A] time = 0.20, size = 164, normalized size = 1.01

$$\frac{b^2\log(a+bx)}{(bc-ad)(be-af)(bg-ah)}-\frac{d^2\log(c+dx)}{(bc-ad)(cf-de)(ch-dg)}-\frac{f^2\log(e+fx)}{(be-af)(de-cf)(eh-fg)}-\frac{h^2\log(g+hx)}{(bg-ah)(dg-ch)(fg-eh)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x)*(c + d*x)*(e + f*x)*(g + h*x)),x]
```

```
[Out] (b^2*Log[a + b*x])/((b*c - a*d)*(b*e - a*f)*(b*g - a*h)) - (d^2*Log[c + d*x
])/((b*c - a*d)*(-(d*e) + c*f)*(-(d*g) + c*h)) - (f^2*Log[e + f*x])/((b*e -
 a*f)*(d*e - c*f)*(-(f*g) + e*h)) - (h^2*Log[g + h*x])/((b*g - a*h)*(d*g -
c*h)*(f*g - e*h))
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a+bx)(c+dx)(e+fx)(g+hx)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)*(e + f*x)*(g + h*x)),x]
[Out] IntegrateAlgebraic[1/((a + b*x)*(c + d*x)*(e + f*x)*(g + h*x)), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="fricas")
[Out] Timed out
```

giac [B] time = 1.19, size = 363, normalized size = 2.23

$$-\frac{b^{3} \log \left(|bx+a|\right)}{ab^{3}cfg-a^{2}b^{2}dfg-a^{2}b^{2}cfh+a^{3}bdfh-b^{4}cge+ab^{3}dge+ab^{3}che-a^{2}b^{2}dhe}+\frac{d^{3} \log \left(|dx+c|\right)}{bc^{2}d^{2}fg-acd^{3}fg-bc^{3}dfh+ac^{2}d^{2}fh-bcd^{3}ge+ad^{4}ge+bc^{2}d^{2}he-acd^{3}he}+\frac{f^{3} \log \left(|fx+e|\right)}{acf^{4}g-bcf^{3}ge-adf^{3}ge-acf^{3}he+bdf^{2}ge^{2}+bcf^{2}he^{2}+adf^{2}he^{2}-bdfhe^{3}}-\frac{h^{3} \log \left(|hx+g|\right)}{bdfg^{3}h-bcfg^{2}h^{2}-adfg^{2}h^{2}+acfgh^{3}-bdg^{2}h^{2}e+bcgh^{3}e+adgh^{3}e-ach^{4}e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="giac")
[Out] -b^3*log(abs(b*x + a))/(a*b^3*c*f*g - a^2*b^2*d*f*g - a^2*b^2*c*f*h + a^3*b
*d*f*h - b^4*c*g*e + a*b^3*d*g*e + a*b^3*c*h*e - a^2*b^2*d*h*e) + d^3*log(a
bs(d*x + c))/(b*c^2*d^2*f*g - a*c*d^3*f*g - b*c^3*d*f*h + a*c^2*d^2*f*h - b
*c*d^3*g*e + a*d^4*g*e + b*c^2*d^2*h*e - a*c*d^3*h*e) + f^3*log(abs(f*x + e
))/(a*c*f^4*g - b*c*f^3*g*e - a*d*f^3*g*e - a*c*f^3*h*e + b*d*f^2*g*e^2 + b
*c*f^2*h*e^2 + a*d*f^2*h*e^2 - b*d*f*h*e^3) - h^3*log(abs(h*x + g))/(b*d*f*
g^3*h - b*c*f*g^2*h^2 - a*d*f*g^2*h^2 + a*c*f*g*h^3 - b*d*g^2*h^2*e + b*c*g
*h^3*e + a*d*g*h^3*e - a*c*h^4*e)
```

maple [A] time = 0.01, size = 164, normalized size = 1.01

$$-\frac{b^{2} \ln \left(bx+a\right)}{\left(ad-bc\right) \left(af-be\right) \left(ah-bg\right)}+\frac{d^{2} \ln \left(dx+c\right)}{\left(ad-bc\right) \left(cf-de\right) \left(ch-dg\right)}-\frac{f^{2} \ln \left(fx+e\right)}{\left(cf-de\right) \left(eh-fg\right) \left(af-be\right)}+\frac{h^{2} \ln \left(hx+g\right)}{\left(ch-dg\right) \left(ah-bg\right) \left(eh-fg\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
[Out] d^2/(a*d-b*c)/(c*f-d*e)/(c*h-d*g)*ln(d*x+c)+h^2/(c*h-d*g)/(a*h-b*g)/(e*h-f*
g)*ln(h*x+g)-b^2/(a*d-b*c)/(a*f-b*e)/(a*h-b*g)*ln(b*x+a)-f^2/(c*f-d*e)/(e*h
-f*g)/(a*f-b*e)*ln(f*x+e)
```

maxima [A] time = 0.49, size = 310, normalized size = 1.90

$$\frac{b^{2} \log \left(bx+a\right)}{\left(\left(b^{3}c-ab^{2}d\right)e-\left(ab^{2}c-a^{2}bd\right)f\right)g-\left(\left(ab^{2}c-a^{2}bd\right)e-\left(a^{2}bc-a^{3}d\right)f\right)h}-\frac{d^{2} \log \left(dx+c\right)}{\left(\left(bcd^{2}-ad^{3}\right)e-\left(bc^{2}d-acd^{2}\right)f\right)g-\left(\left(bc^{2}d-acd^{2}\right)e-\left(bc^{3}-ac^{2}d\right)f\right)h}+\frac{f^{2} \log \left(fx+e\right)}{\left(bde^{2}f+acf^{3}-\left(bc+ad\right)ef^{2}\right)g-\left(bde^{3}+acef^{2}-\left(bc+ad\right)e^{2}f\right)h}-\frac{h^{2} \log \left(hx+g\right)}{bdfg^{3}-aceh^{3}-\left(bde+\left(bc+ad\right)f\right)g^{2}h+\left(acf+\left(bc+ad\right)e\right)gh^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x, algorithm="maxima")
[Out] b^2*log(b*x + a)/(((b^3*c - a*b^2*d)*e - (a*b^2*c - a^2*b*d)*f)*g - ((a*b^2
*c - a^2*b*d)*e - (a^2*b*c - a^3*d)*f)*h) - d^2*log(d*x + c)/(((b*c*d^2 - a
*d^3)*e - (b*c^2*d - a*c*d^2)*f)*g - ((b*c^2*d - a*c*d^2)*e - (b*c^3 - a*c^
2*d)*f)*h) + f^2*log(f*x + e)/((b*d*e^2*f + a*c*f^3 - (b*c + a*d)*e*f^2)*g
- (b*d*e^3 + a*c*e*f^2 - (b*c + a*d)*e^2*f)*h) - h^2*log(h*x + g)/(b*d*f*g^
3 - a*c*e*h^3 - (b*d*e + (b*c + a*d)*f)*g^2*h + (a*c*f + (b*c + a*d)*e)*g*h
^2)
```

mupad [B] time = 6.62, size = 317, normalized size = 1.94

$$\frac{b^{2} \ln \left(a+bx\right)}{b^{3}ceg-a^{3}dfh-ab^{2}ceh-ab^{2}cfg-ab^{2}deg+a^{2}bcfh+a^{2}bdeh+a^{2}bdfg}+\frac{d^{2} \ln \left(c+dx\right)}{ad^{3}eg-bc^{3}fh-acd^{2}eh-acd^{2}fg-bcd^{2}eg+ac^{2}dfh+bc^{2}deh+bc^{2}dfg}+\frac{f^{2} \ln \left(e+fx\right)}{acf^{3}g-bde^{3}h-acef^{2}h-adef^{2}g-bcef^{2}g+ade^{2}fh+bce^{2}fh+bde^{2}fg}+\frac{h^{2} \ln \left(g+hx\right)}{aceh^{3}-bdfg^{3}-acfgh^{2}-adegh^{2}-bcegh^{2}+adfg^{2}h+bcfg^{2}h+bdeg^{2}h}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e + f*x)*(g + h*x)*(a + b*x)*(c + d*x)),x)
```

```
[Out] (b^2*log(a + b*x))/(b^3*c*e*g - a^3*d*f*h - a*b^2*c*e*h - a*b^2*c*f*g - a*b
^2*d*e*g + a^2*b*c*f*h + a^2*b*d*e*h + a^2*b*d*f*g) + (d^2*log(c + d*x))/(a
*d^3*e*g - b*c^3*f*h - a*c*d^2*e*h - a*c*d^2*f*g - b*c*d^2*e*g + a*c^2*d*f*
h + b*c^2*d*e*h + b*c^2*d*f*g) + (f^2*log(e + f*x))/(a*c*f^3*g - b*d*e^3*h
- a*c*e*f^2*h - a*d*e*f^2*g - b*c*e*f^2*g + a*d*e^2*f*h + b*c*e^2*f*h + b*d
*e^2*f*g) + (h^2*log(g + h*x))/(a*c*e*h^3 - b*d*f*g^3 - a*c*f*g*h^2 - a*d*e
*g*h^2 - b*c*e*g*h^2 + a*d*f*g^2*h + b*c*f*g^2*h + b*d*e*g^2*h)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x+a)/(d*x+c)/(f*x+e)/(h*x+g),x)
```

```
[Out] Timed out
```

3.6 $\int \frac{x}{(1+x)(2+x)(3+x)}\,dx$

Optimal. Leaf size=23

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.059, Rules used = {148}

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Int[x/((1 + x)*(2 + x)*(3 + x)),x]
```

```
[Out] -Log[1 + x]/2 + 2*Log[2 + x] - (3*Log[3 + x])/2
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(1+x)(2+x)(3+x)}\,dx &= \int \left(-\frac{1}{2(1+x)}+\frac{2}{2+x}-\frac{3}{2(3+x)}\right)dx\\ &= -\frac{1}{2}\log(1+x)+2\log(2+x)-\frac{3}{2}\log(3+x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$-\frac{1}{2}\log(x+1)+2\log(x+2)-\frac{3}{2}\log(x+3)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((1 + x)*(2 + x)*(3 + x)),x]
```

```
[Out] -1/2*Log[1 + x] + 2*Log[2 + x] - (3*Log[3 + x])/2
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(1+x)(2+x)(3+x)}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/((1 + x)*(2 + x)*(3 + x)),x]
```

```
[Out] IntegrateAlgebraic[x/((1 + x)*(2 + x)*(3 + x)), x]
```

fricas [A] time = 1.71, size = 19, normalized size = 0.83

$$-\frac{3}{2}\log\left(x+3\right)+2\log\left(x+2\right)-\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="fricas")
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

giac [A] time = 1.25, size = 22, normalized size = 0.96

$$-\frac{3}{2}\log\left(|x+3|\right)+2\log\left(|x+2|\right)-\frac{1}{2}\log\left(|x+1|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="giac")
[Out] -3/2*log(abs(x + 3)) + 2*log(abs(x + 2)) - 1/2*log(abs(x + 1))
```

maple [A] time = 0.01, size = 20, normalized size = 0.87

$$-\frac{\ln\left(x+1\right)}{2}+2\ln\left(x+2\right)-\frac{3\ln\left(x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(x+1)/(x+2)/(x+3),x)
[Out] -1/2*ln(x+1)+2*ln(x+2)-3/2*ln(x+3)
```

maxima [A] time = 0.43, size = 19, normalized size = 0.83

$$-\frac{3}{2}\log\left(x+3\right)+2\log\left(x+2\right)-\frac{1}{2}\log\left(x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x, algorithm="maxima")
[Out] -3/2*log(x + 3) + 2*log(x + 2) - 1/2*log(x + 1)
```

mupad [B] time = 0.08, size = 19, normalized size = 0.83

$$2\ln\left(x+2\right)-\frac{\ln\left(x+1\right)}{2}-\frac{3\ln\left(x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((x + 1)*(x + 2)*(x + 3)),x)
[Out] 2*log(x + 2) - log(x + 1)/2 - (3*log(x + 3))/2
```

sympy [A] time = 0.15, size = 20, normalized size = 0.87

$$-\frac{\log\left(x+1\right)}{2}+2\log\left(x+2\right)-\frac{3\log\left(x+3\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(1+x)/(2+x)/(3+x),x)
[Out] -log(x + 1)/2 + 2*log(x + 2) - 3*log(x + 3)/2
```

3.7 $\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3} \, dx$

Optimal. Leaf size=43

$$\frac{201}{15125(5x+3)} - \frac{12}{1375(5x+3)^2} + \frac{20\log(6-x)}{3993} + \frac{1493\log(5x+3)}{499125}$$

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1593, 148}

$$\frac{201}{15125(5x+3)} - \frac{12}{1375(5x+3)^2} + \frac{20\log(6-x)}{3993} + \frac{1493\log(5x+3)}{499125}$$

Antiderivative was successfully verified.

```
[In] Int[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
```

```
[Out] -12/(1375*(3 + 5*x)^2) + 201/(15125*(3 + 5*x)) + (20*Log[6 - x])/3993 + (14
93*Log[3 + 5*x])/499125
```

Rule 148

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(
c + d*x)^n*(e + f*x)^p*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
m}, x] && (IntegersQ[m, n, p] || (IGtQ[n, 0] && IGtQ[p, 0]))
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3} \, dx &= \int \frac{(-1+x)x^2}{(-6+x)(3+5x)^3} \, dx \\
&= \int \left(\frac{20}{3993(-6+x)} + \frac{24}{275(3+5x)^3} - \frac{201}{3025(3+5x)^2} + \frac{1493}{99825(3+5x)}\right) dx \\
&= -\frac{12}{1375(3+5x)^2} + \frac{201}{15125(3+5x)} + \frac{20\log(6-x)}{3993} + \frac{1493\log(3+5x)}{499125}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 0.77

$$\frac{\frac{99(335x+157)}{(5x+3)^2} + 2500\log(x-6) + 1493\log(5x+3)}{499125}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
```

```
[Out] ((99*(157 + 335*x))/(3 + 5*x)^2 + 2500*Log[-6 + x] + 1493*Log[3 + 5*x])/499
125
```

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-x^2+x^3}{(-6+x)(3+5x)^3}\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3),x]
```

```
[Out] IntegrateAlgebraic[(-x^2 + x^3)/((-6 + x)*(3 + 5*x)^3), x]
```

fricas [A] time = 1.44, size = 53, normalized size = 1.23

$$\frac{1493\left(25\,x^2+30\,x+9\right)\log\left(5\,x+3\right)+2500\left(25\,x^2+30\,x+9\right)\log\left(x-6\right)+33165\,x+15543}{499125\left(25\,x^2+30\,x+9\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="fricas")
```

```
[Out] 1/499125*(1493*(25*x^2 + 30*x + 9)*log(5*x + 3) + 2500*(25*x^2 + 30*x + 9)*
log(x - 6) + 33165*x + 15543)/(25*x^2 + 30*x + 9)
```

giac [A] time = 1.18, size = 31, normalized size = 0.72

$$\frac{3\,(335\,x+157)}{15125\,(5\,x+3)^2}+\frac{1493}{499125}\,\log\left(|5\,x+3|\right)+\frac{20}{3993}\,\log\left(|x-6|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="giac")
```

```
[Out] 3/15125*(335*x + 157)/(5*x + 3)^2 + 1493/499125*log(abs(5*x + 3)) + 20/3993
*log(abs(x - 6))
```

maple [A] time = 0.01, size = 34, normalized size = 0.79

$$\frac{1493\ln\left(5x+3\right)}{499125}+\frac{20\ln\left(x-6\right)}{3993}-\frac{12}{1375\left(5x+3\right)^2}+\frac{201}{15125\left(5x+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3-x^2)/(x-6)/(5*x+3)^3,x)
```

```
[Out] -12/1375/(5*x+3)^2+201/15125/(5*x+3)+1493/499125*ln(5*x+3)+20/3993*ln(x-6)
```

maxima [A] time = 0.43, size = 34, normalized size = 0.79

$$\frac{3\,(335\,x+157)}{15125\left(25\,x^2+30\,x+9\right)}+\frac{1493}{499125}\,\log\left(5\,x+3\right)+\frac{20}{3993}\,\log\left(x-6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3-x^2)/(-6+x)/(3+5*x)^3,x, algorithm="maxima")
```

```
[Out] 3/15125*(335*x + 157)/(25*x^2 + 30*x + 9) + 1493/499125*log(5*x + 3) + 20/3
993*log(x - 6)
```

mupad [B] time = 0.12, size = 29, normalized size = 0.67

$$\frac{20\,\ln\left(x-6\right)}{3993}+\frac{1493\,\ln\left(x+\frac{3}{5}\right)}{499125}+\frac{\frac{201\,x}{75625}+\frac{471}{378125}}{x^2+\frac{6\,x}{5}+\frac{9}{25}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2 - x^3)/((5*x + 3)^3*(x - 6)),x)
```

```
[Out] (20*log(x - 6))/3993 + (1493*log(x + 3/5))/499125 + ((201*x)/75625 + 471/37
8125)/((6*x)/5 + x^2 + 9/25)
```

sympy [A] time = 0.16, size = 32, normalized size = 0.74

$$\frac{1005x + 471}{378125x^2 + 453750x + 136125} + \frac{20\log(x - 6)}{3993} + \frac{1493\log\left(x + \frac{3}{5}\right)}{499125}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3-x**2)/(-6+x)/(3+5*x)**3,x)
```

```
[Out] (1005*x + 471)/(378125*x**2 + 453750*x + 136125) + 20*log(x - 6)/3993 + 149
3*log(x + 3/5)/499125
```

3.8 $\int \frac{(a+bx)^3\sqrt{c+dx}\,(e+fx)}{x}\,dx$

Optimal. Leaf size=227

$$2a^3e\sqrt{c+dx}-2a^3\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\frac{2(c+dx)^{3/2}\left(2\left(20a^3d^3f+3a^2bd^2(45de-16cf)-9ab^2cd(7de-4c\right.\right.}{}$$

Rubi [A] time = 0.26, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {153, 147, 50, 63, 208}

$$\frac{2(c+dx)^{3/2}\left(2\left(3a^2bd^2(45de-16cf)+20a^3d^3f-9ab^2cd(7de-4cf)+4b^3c^2(3de-2cf)\right)+3bdx\left(21abd^2e-4(bc-ad)(2adf-2bcf+3bde)\right)\right)}{315d^4}+2a^3e\sqrt{c+dx}-2a^3\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\frac{2(a+bx)^2(c+dx)^{3/2}(2adf-2bcf+3bde)}{21d^2}+\frac{2f(a+bx)^3(c+dx)^{3/2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^3*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] 2*a^3*e*Sqrt[c + d*x] + (2*(3*b*d*e - 2*b*c*f + 2*a*d*f)*(a + b*x)^2*(c + d
*x)^(3/2))/(21*d^2) + (2*f*(a + b*x)^3*(c + d*x)^(3/2))/(9*d) + (2*(c + d*x
)^(3/2)*(2*(20*a^3*d^3*f + 3*a^2*b*d^2*(45*d*e - 16*c*f) - 9*a*b^2*c*d*(7*d
*e - 4*c*f) + 4*b^3*c^2*(3*d*e - 2*c*f)) + 3*b*d*(21*a*b*d^2*e - 4*(b*c - a
*d)*(3*b*d*e - 2*b*c*f + 2*a*d*f))*x))/(315*d^4) - 2*a^3*Sqrt[c]*e*ArcTanh[
Sqrt[c + d*x]/Sqrt[c]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
```

```
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+bx)^3\sqrt{c+dx}\,(e+fx)}{x}\,dx &= \frac{2f(a+bx)^3(c+dx)^{3/2}}{9d}+\frac{2\int \frac{(a+bx)^2\sqrt{c+dx}\left(\frac{9ade}{2}+\frac{3}{2}(3bde-2bcf+2adf)x\right)}{x}\,dx}{9d}\\
&= \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2}+\frac{2f(a+bx)^3(c+dx)^{3/2}}{9d}+\frac{4\int \frac{(a+\cdots}{\cdots}}{\cdots}\\
&= \frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2}+\frac{2f(a+bx)^3(c+dx)^{3/2}}{9d}+\frac{2(c+\cdots}{\cdots}\\
&= 2a^3e\sqrt{c+dx}+\frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2}+\frac{2f(a+bx)^3(c+\cdots}{9d}\\
&= 2a^3e\sqrt{c+dx}+\frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2}+\frac{2f(a+bx)^3(c+\cdots}{9d}\\
&= 2a^3e\sqrt{c+dx}+\frac{2(3bde-2bcf+2adf)(a+bx)^2(c+dx)^{3/2}}{21d^2}+\frac{2f(a+bx)^3(c+\cdots}{9d}
\end{aligned}
$$

Mathematica [A] time = 0.28, size = 205, normalized size = 0.90

$$\frac{2\left(3de\left(105a^3d^3\sqrt{c+dx}-105a^3\sqrt{c}\,d^3\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+35b(c+dx)^{3/2}\left(3a^2d^2-3abcd+b^2c^2\right)-21b^2(c+dx)^{5/2}(2bc-3ad)+15b^3(c+dx)^{7/2}\right)-f(c+dx)^{3/2}\left(135b^2(c+dx)^2(bc-ad)-189b(c+dx)(bc-ad)^2+105(bc-ad)^3-35b^3(c+dx)^3\right)\right)}{315d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^3*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*(-(f*(c + d*x)^(3/2)*(105*(b*c - a*d)^3 - 189*b*(b*c - a*d)^2*(c + d*x)
+ 135*b^2*(b*c - a*d)*(c + d*x)^2 - 35*b^3*(c + d*x)^3)) + 3*d*e*(105*a^3*d
^3*Sqrt[c + d*x] + 35*b*(b^2*c^2 - 3*a*b*c*d + 3*a^2*d^2)*(c + d*x)^(3/2) -
 21*b^2*(2*b*c - 3*a*d)*(c + d*x)^(5/2) + 15*b^3*(c + d*x)^(7/2) - 105*a^3*
Sqrt[c]*d^3*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])))/(315*d^4)
```

IntegrateAlgebraic [A] time = 0.20, size = 297, normalized size = 1.31

$$\frac{2\sqrt{c+dx}\left(105a^3d^3f(c+dx)+315a^3d^4e+315a^2bd^3e(c+dx)+189a^2bd^2f(c+dx)^2-315a^2bcd^2f(c+dx)+315ab^2c^2df(c+dx)+189ab^2d^2e(c+dx)^2-315ab^2cd^2e(c+dx)+135ab^2df(c+dx)^3-378ab^2cdf(c+dx)^2-105b^3c^3f(c+dx)+105b^3c^2de(c+dx)+189b^3c^2f(c+dx)^2+45b^3de(c+dx)^3-126b^3cde(c+dx)^2+35b^3f(c+dx)^4-135b^3cf(c+dx)^3\right)}{315d^4}-2a^3\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x)^3*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*Sqrt[c + d*x]*(315*a^3*d^4*e + 105*b^3*c^2*d*e*(c + d*x) - 315*a*b^2*c*d
^2*e*(c + d*x) + 315*a^2*b*d^3*e*(c + d*x) - 105*b^3*c^3*f*(c + d*x) + 315*
a*b^2*c^2*d*f*(c + d*x) - 315*a^2*b*c*d^2*f*(c + d*x) + 105*a^3*d^3*f*(c +
d*x) - 126*b^3*c*d*e*(c + d*x)^2 + 189*a*b^2*d^2*e*(c + d*x)^2 + 189*b^3*c^
2*f*(c + d*x)^2 - 378*a*b^2*c*d*f*(c + d*x)^2 + 189*a^2*b*d^2*f*(c + d*x)^2
```

```
 + 45*b^3*d*e*(c + d*x)^3 - 135*b^3*c*f*(c + d*x)^3 + 135*a*b^2*d*f*(c + d*
x)^3 + 35*b^3*f*(c + d*x)^4))/(315*d^4) - 2*a^3*Sqrt[c]*e*ArcTanh[Sqrt[c +
d*x]/Sqrt[c]]
```

fricas [A] time = 1.29, size = 649, normalized size = 2.86

$$\left[\frac{315\,a^{3}\sqrt{c}\,d^{4}e\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2\left(35\,b^{3}d^{4}fx^{4}+5\left(9\,b^{3}d^{4}e+\left(b^{3}cd^{3}+27\,ab^{2}d^{4}\right)f\right)x^{3}+3\left(3\left(b^{3}cd^{3}+21\,ab^{2}d^{4}\right)e-\left(2\,b^{3}c^{2}d^{2}-9\,ab^{2}cd^{3}-63\,a^{2}bd^{4}\right)f\right)x^{2}+3\left(8\,b^{3}c^{3}d-42\,ab^{2}c^{2}d^{2}+105\,a^{2}bcd^{3}+105\,a^{3}d^{4}\right)e-\left(16\,b^{3}c^{4}-72\,ab^{2}c^{3}d+126\,a^{2}bc^{2}d^{2}-105\,a^{3}cd^{3}\right)f-\left(3\left(4\,b^{3}c^{2}d^{2}-21\,ab^{2}cd^{3}-105\,a^{2}bd^{4}\right)e-\left(8\,b^{3}c^{3}d-36\,ab^{2}c^{2}d^{2}+63\,a^{2}bcd^{3}+105\,a^{3}d^{4}\right)f\right)x\right)\sqrt{dx+c}}{315\,d^{4}},\ \frac{2\left(315\,a^{3}\sqrt{-c}\,d^{4}e\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)+\left(35\,b^{3}d^{4}fx^{4}+5\left(9\,b^{3}d^{4}e+\left(b^{3}cd^{3}+27\,ab^{2}d^{4}\right)f\right)x^{3}+3\left(3\left(b^{3}cd^{3}+21\,ab^{2}d^{4}\right)e-\left(2\,b^{3}c^{2}d^{2}-9\,ab^{2}cd^{3}-63\,a^{2}bd^{4}\right)f\right)x^{2}+3\left(8\,b^{3}c^{3}d-42\,ab^{2}c^{2}d^{2}+105\,a^{2}bcd^{3}+105\,a^{3}d^{4}\right)e-\left(16\,b^{3}c^{4}-72\,ab^{2}c^{3}d+126\,a^{2}bc^{2}d^{2}-105\,a^{3}cd^{3}\right)f-\left(3\left(4\,b^{3}c^{2}d^{2}-21\,ab^{2}cd^{3}-105\,a^{2}bd^{4}\right)e-\left(8\,b^{3}c^{3}d-36\,ab^{2}c^{2}d^{2}+63\,a^{2}bcd^{3}+105\,a^{3}d^{4}\right)f\right)x\right)\sqrt{dx+c}\right)}{315\,d^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/315*(315*a^3*sqrt(c)*d^4*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x)
+ 2*(35*b^3*d^4*f*x^4 + 5*(9*b^3*d^4*e + (b^3*c*d^3 + 27*a*b^2*d^4)*f)*x^3
+ 3*(3*(b^3*c*d^3 + 21*a*b^2*d^4)*e - (2*b^3*c^2*d^2 - 9*a*b^2*c*d^3 - 63*a
^2*b*d^4)*f)*x^2 + 3*(8*b^3*c^3*d - 42*a*b^2*c^2*d^2 + 105*a^2*b*c*d^3 + 10
5*a^3*d^4)*e - (16*b^3*c^4 - 72*a*b^2*c^3*d + 126*a^2*b*c^2*d^2 - 105*a^3*c
*d^3)*f - (3*(4*b^3*c^2*d^2 - 21*a*b^2*c*d^3 - 105*a^2*b*d^4)*e - (8*b^3*c^
3*d - 36*a*b^2*c^2*d^2 + 63*a^2*b*c*d^3 + 105*a^3*d^4)*f)*x)*sqrt(d*x + c))
/d^4, 2/315*(315*a^3*sqrt(-c)*d^4*e*arctan(sqrt(d*x + c)*sqrt(-c)/c) + (35*
b^3*d^4*f*x^4 + 5*(9*b^3*d^4*e + (b^3*c*d^3 + 27*a*b^2*d^4)*f)*x^3 + 3*(3*(
b^3*c*d^3 + 21*a*b^2*d^4)*e - (2*b^3*c^2*d^2 - 9*a*b^2*c*d^3 - 63*a^2*b*d^4
)*f)*x^2 + 3*(8*b^3*c^3*d - 42*a*b^2*c^2*d^2 + 105*a^2*b*c*d^3 + 105*a^3*d^
4)*e - (16*b^3*c^4 - 72*a*b^2*c^3*d + 126*a^2*b*c^2*d^2 - 105*a^3*c*d^3)*f
- (3*(4*b^3*c^2*d^2 - 21*a*b^2*c*d^3 - 105*a^2*b*d^4)*e - (8*b^3*c^3*d - 36
*a*b^2*c^2*d^2 + 63*a^2*b*c*d^3 + 105*a^3*d^4)*f)*x)*sqrt(d*x + c))/d^4]
```

giac [A] time = 1.41, size = 338, normalized size = 1.49

$$\frac{2\,a^{3}c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{\sqrt{-c}}+\frac{2\left(35\,(dx+c)^{\frac{9}{2}}b^{3}d^{32}f-135\,(dx+c)^{\frac{7}{2}}b^{3}cd^{32}f+189\,(dx+c)^{\frac{5}{2}}b^{3}c^{2}d^{32}f-105\,(dx+c)^{\frac{3}{2}}b^{3}c^{3}d^{32}f+135\,(dx+c)^{\frac{7}{2}}ab^{2}d^{33}f-378\,(dx+c)^{\frac{5}{2}}ab^{2}cd^{33}f+315\,(dx+c)^{\frac{3}{2}}ab^{2}c^{2}d^{33}f+189\,(dx+c)^{\frac{5}{2}}a^{2}bd^{34}f-315\,(dx+c)^{\frac{3}{2}}a^{2}bcd^{34}f+105\,(dx+c)^{\frac{3}{2}}a^{3}d^{35}f+45\,(dx+c)^{\frac{7}{2}}b^{3}d^{33}e-126\,(dx+c)^{\frac{5}{2}}b^{3}cd^{33}e+105\,(dx+c)^{\frac{3}{2}}b^{3}c^{2}d^{33}e+189\,(dx+c)^{\frac{5}{2}}ab^{2}d^{34}e-315\,(dx+c)^{\frac{3}{2}}ab^{2}cd^{34}e+315\,(dx+c)^{\frac{3}{2}}a^{2}bd^{35}e+315\,\sqrt{dx+c}\,a^{3}d^{36}e\right)}{315\,d^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^3*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/sqrt(-c) + 2/315*(35*(d*x + c)^(9/
2)*b^3*d^32*f - 135*(d*x + c)^(7/2)*b^3*c*d^32*f + 189*(d*x + c)^(5/2)*b^3*
c^2*d^32*f - 105*(d*x + c)^(3/2)*b^3*c^3*d^32*f + 135*(d*x + c)^(7/2)*a*b^2
*d^33*f - 378*(d*x + c)^(5/2)*a*b^2*c*d^33*f + 315*(d*x + c)^(3/2)*a*b^2*c^
2*d^33*f + 189*(d*x + c)^(5/2)*a^2*b*d^34*f - 315*(d*x + c)^(3/2)*a^2*b*c*d
^34*f + 105*(d*x + c)^(3/2)*a^3*d^35*f + 45*(d*x + c)^(7/2)*b^3*d^33*e - 12
6*(d*x + c)^(5/2)*b^3*c*d^33*e + 105*(d*x + c)^(3/2)*b^3*c^2*d^33*e + 189*(
d*x + c)^(5/2)*a*b^2*d^34*e - 315*(d*x + c)^(3/2)*a*b^2*c*d^34*e + 315*(d*x
 + c)^(3/2)*a^2*b*d^35*e + 315*sqrt(d*x + c)*a^3*d^36*e)/d^36
```

maple [A] time = 0.01, size = 301, normalized size = 1.33

$$\frac{-2a^{3}\sqrt{c}\,d^{4}e\,\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)+2\sqrt{dx+c}\,a^{3}d^{4}e+\frac{2(dx+c)^{\frac{3}{2}}a^{3}d^{3}f}{3}-2\,(dx+c)^{\frac{3}{2}}a^{2}bcd^{2}f+2\,(dx+c)^{\frac{3}{2}}a^{2}bd^{3}e+2\,(dx+c)^{\frac{3}{2}}ab^{2}c^{2}df-2\,(dx+c)^{\frac{3}{2}}ab^{2}cd^{2}e-\frac{2(dx+c)^{\frac{3}{2}}b^{3}c^{3}f}{3}+\frac{2(dx+c)^{\frac{3}{2}}b^{3}c^{2}de}{3}+\frac{6(dx+c)^{\frac{5}{2}}a^{2}bd^{2}f}{5}-\frac{12(dx+c)^{\frac{5}{2}}ab^{2}cdf}{5}+\frac{6(dx+c)^{\frac{5}{2}}ab^{2}d^{2}e}{5}+\frac{6(dx+c)^{\frac{5}{2}}b^{3}c^{2}f}{5}-\frac{4(dx+c)^{\frac{5}{2}}b^{3}cde}{5}+\frac{6(dx+c)^{\frac{7}{2}}ab^{2}df}{7}-\frac{6(dx+c)^{\frac{7}{2}}b^{3}cf}{7}+\frac{2(dx+c)^{\frac{7}{2}}b^{3}de}{7}+\frac{2(dx+c)^{\frac{9}{2}}b^{3}f}{9}}{d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x)
```

```
[Out] 2/d^4*(1/9*f*b^3*(d*x+c)^(9/2)+3/7*(d*x+c)^(7/2)*a*b^2*d*f-3/7*(d*x+c)^(7/2
)*b^3*c*f+1/7*(d*x+c)^(7/2)*b^3*d*e+3/5*(d*x+c)^(5/2)*a^2*b*d^2*f-6/5*(d*x+
c)^(5/2)*a*b^2*c*d*f+3/5*(d*x+c)^(5/2)*a*b^2*d^2*e+3/5*(d*x+c)^(5/2)*b^3*c^
2*f-2/5*(d*x+c)^(5/2)*b^3*c*d*e+1/3*(d*x+c)^(3/2)*a^3*d^3*f-(d*x+c)^(3/2)*a
^2*b*c*d^2*f+(d*x+c)^(3/2)*a^2*b*d^3*e+(d*x+c)^(3/2)*a*b^2*c^2*d*f-(d*x+c)^
(3/2)*a*b^2*c*d^2*e-1/3*(d*x+c)^(3/2)*b^3*c^3*f+1/3*(d*x+c)^(3/2)*b^3*c^2*d
*e+a^3*d^4*e*(d*x+c)^(1/2)-a^3*c^(1/2)*d^4*e*arctanh((d*x+c)^(1/2)/c^(1/2))
)
```

maxima [A] time = 0.99, size = 239, normalized size = 1.05

$$a^{3}\sqrt{c}e\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)+\frac{2\left(315\sqrt{dx+c}\,a^{3}d^{4}e+35\,(dx+c)^{\frac{9}{2}}b^{3}f+45\left(b^{3}de-3\left(b^{3}c-ab^{2}d\right)f\right)(dx+c)^{\frac{7}{2}}-63\left(\left(2\,b^{3}cd-3\,ab^{2}d^{2}\right)e-3\left(b^{3}c^{2}-2\,ab^{2}cd+a^{2}bd^{2}\right)f\right)(dx+c)^{\frac{5}{2}}+105\left(\left(b^{3}c^{2}d-3\,ab^{2}cd^{2}+3\,a^{2}bd^{3}\right)e-\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)f\right)(dx+c)^{\frac{3}{2}}\right)}{315\,d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^3*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a^3*sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2/
315*(315*sqrt(d*x + c)*a^3*d^4*e + 35*(d*x + c)^(9/2)*b^3*f + 45*(b^3*d*e -
 3*(b^3*c - a*b^2*d)*f)*(d*x + c)^(7/2) - 63*((2*b^3*c*d - 3*a*b^2*d^2)*e -
 3*(b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*f)*(d*x + c)^(5/2) + 105*((b^3*c^2*d
 - 3*a*b^2*c*d^2 + 3*a^2*b*d^3)*e - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^
2 - a^3*d^3)*f)*(d*x + c)^(3/2))/d^4
```

mupad [B] time = 0.16, size = 413, normalized size = 1.82

$$\left(c\left(c\left(c\left(\frac{2b^3de-8b^3cf+6ab^2df}{d^4}+\frac{2b^3cf}{d^4}\right)+\frac{6b\,(ad-bc)\,(adf-2bcf+bde)}{d^4}\right)+\frac{2\,(ad-bc)^2\,(adf-4bcf+3bde)}{d^4}\right)-\frac{2\,(ad-bc)^3\,(cf-de)}{d^4}\right)\sqrt{c+dx}+\left(\frac{c\left(c\left(\frac{2b^3de-8b^3cf+6ab^2df}{d^4}+\frac{2b^3cf}{d^4}\right)+\frac{6b\,(ad-bc)\,(adf-2bcf+bde)}{d^4}\right)}{3}+\frac{2\,(ad-bc)^2\,(adf-4bcf+3bde)}{3d^4}\right)(c+dx)^{3/2}+\left(\frac{2b^3de-8b^3cf+6ab^2df}{7d^4}+\frac{2b^3cf}{7d^4}\right)(c+dx)^{7/2}+\left(\frac{c\left(\frac{2b^3de-8b^3cf+6ab^2df}{d^4}+\frac{2b^3cf}{d^4}\right)}{5}+\frac{6b\,(ad-bc)\,(adf-2bcf+bde)}{5d^4}\right)(c+dx)^{5/2}+\frac{2b^3f\,(c+dx)^{9/2}}{9d^4}+a^3\sqrt{c}\,e\,\mathrm{atan}\left(\frac{\sqrt{c+dx}\,1i}{\sqrt{c}}\right)2i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^3*(c + d*x)^(1/2))/x,x)
```

```
[Out] (c*(c*(c*((2*b^3*d*e - 8*b^3*c*f + 6*a*b^2*d*f)/d^4 + (2*b^3*c*f)/d^4) + (6
*b*(a*d - b*c)*(a*d*f - 2*b*c*f + b*d*e))/d^4) + (2*(a*d - b*c)^2*(a*d*f -
4*b*c*f + 3*b*d*e))/d^4) - (2*(a*d - b*c)^3*(c*f - d*e))/d^4)*(c + d*x)^(1/
2) + ((c*(c*((2*b^3*d*e - 8*b^3*c*f + 6*a*b^2*d*f)/d^4 + (2*b^3*c*f)/d^4) +
 (6*b*(a*d - b*c)*(a*d*f - 2*b*c*f + b*d*e))/d^4))/3 + (2*(a*d - b*c)^2*(a*
d*f - 4*b*c*f + 3*b*d*e))/(3*d^4))*(c + d*x)^(3/2) + ((2*b^3*d*e - 8*b^3*c*
f + 6*a*b^2*d*f)/(7*d^4) + (2*b^3*c*f)/(7*d^4))*(c + d*x)^(7/2) + ((c*((2*b
^3*d*e - 8*b^3*c*f + 6*a*b^2*d*f)/d^4 + (2*b^3*c*f)/d^4))/5 + (6*b*(a*d - b
*c)*(a*d*f - 2*b*c*f + b*d*e))/(5*d^4))*(c + d*x)^(5/2) + a^3*c^(1/2)*e*ata
n(((c + d*x)^(1/2)*1i)/c^(1/2))*2i + (2*b^3*f*(c + d*x)^(9/2))/(9*d^4)
```

sympy [A] time = 37.79, size = 274, normalized size = 1.21

$$\frac{2a^{3}ce\,\mathrm{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2a^{3}e\sqrt{c+dx}+\frac{2b^{3}f\,(c+dx)^{\frac{9}{2}}}{9d^{4}}+\frac{2\,(c+dx)^{\frac{7}{2}}\left(3ab^{2}df-3b^{3}cf+b^{3}de\right)}{7d^{4}}+\frac{2\,(c+dx)^{\frac{5}{2}}\left(3a^{2}bd^{2}f-6ab^{2}cdf+3ab^{2}d^{2}e+3b^{3}c^{2}f-2b^{3}cde\right)}{5d^{4}}+\frac{2\,(c+dx)^{\frac{3}{2}}\left(a^{3}d^{3}f-3a^{2}bcd^{2}f+3a^{2}bd^{3}e+3ab^{2}c^{2}df-3ab^{2}cd^{2}e-b^{3}c^{3}f+b^{3}c^{2}de\right)}{3d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**3*(f*x+e)*(d*x+c)**(1/2)/x,x)
```

```
[Out] 2*a**3*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*a**3*e*sqrt(c + d*x) +
 2*b**3*f*(c + d*x)**(9/2)/(9*d**4) + 2*(c + d*x)**(7/2)*(3*a*b**2*d*f - 3*
b**3*c*f + b**3*d*e)/(7*d**4) + 2*(c + d*x)**(5/2)*(3*a**2*b*d**2*f - 6*a*b
**2*c*d*f + 3*a*b**2*d**2*e + 3*b**3*c**2*f - 2*b**3*c*d*e)/(5*d**4) + 2*(c
 + d*x)**(3/2)*(a**3*d**3*f - 3*a**2*b*c*d**2*f + 3*a**2*b*d**3*e + 3*a*b**
2*c**2*d*f - 3*a*b**2*c*d**2*e - b**3*c**3*f + b**3*c**2*d*e)/(3*d**4)
```

3.9 $\int \frac{(a+bx)^2\sqrt{c+dx}\,(e+fx)}{x}\,dx$

Optimal. Leaf size=146

$$\frac{2(c+dx)^{3/2}\left(2\left(10a^2d^2f+7abd(5de-2cf)+b^2(-c)(7de-4cf)\right)+3bdx(4adf-4bcf+7bde)\right)}{105d^3}+2a^2e\sqrt{c+dx}$$

Rubi [A] time = 0.10, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {153, 147, 50, 63, 208}

$$\frac{2(c+dx)^{3/2}\left(2\left(10a^2d^2f+7abd(5de-2cf)+b^2(-c)(7de-4cf)\right)+3bdx(4adf-4bcf+7bde)\right)}{105d^3}+2a^2e\sqrt{c+dx}-2a^2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\frac{2f(a+bx)^2(c+dx)^{3/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)^2*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] 2*a^2*e*Sqrt[c + d*x] + (2*f*(a + b*x)^2*(c + d*x)^(3/2))/(7*d) + (2*(c + d
*x)^(3/2)*(2*(10*a^2*d^2*f - b^2*c*(7*d*e - 4*c*f) + 7*a*b*d*(5*d*e - 2*c*f
)) + 3*b*d*(7*b*d*e - 4*b*c*f + 4*a*d*f)*x))/(105*d^3) - 2*a^2*Sqrt[c]*e*Ar
cTanh[Sqrt[c + d*x]/Sqrt[c]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)^2\sqrt{c+dx}\,(e+fx)}{x}\,dx = \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2\int \frac{(a+bx)\sqrt{c+dx}\left(\frac{7ade}{2}+\frac{1}{2}(7bde-4bcf+4adf)x\right)}{x}dx}{7d}$$

$$= \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2\left(10a^2d^2f-b^2c(7de-4cf)+7abd(5d\right.\right.}{105d^3}$$

$$= 2a^2e\sqrt{c+dx} + \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2\left(10a^2d^2f-b^2c(7de-\right.\right.}{}$$

$$= 2a^2e\sqrt{c+dx} + \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2\left(10a^2d^2f-b^2c(7de-\right.\right.}{}$$

$$= 2a^2e\sqrt{c+dx} + \frac{2f(a+bx)^2(c+dx)^{3/2}}{7d} + \frac{2(c+dx)^{3/2}\left(2\left(10a^2d^2f-b^2c(7de-\right.\right.}{}$$

Mathematica [A] time = 0.18, size = 145, normalized size = 0.99

$$\frac{2\left(7de\left(\sqrt{c+dx}\left(15a^2d^2+10abd(c+dx)+b^2\left(-2c^2+cdx+3d^2x^2\right)\right)-15a^2\sqrt{c}\,d^2\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)\right)+f(c+dx)^{3/2}\left(-42b(c+dx)(bc-ad)+35(bc-ad)^2+15b^2(c+dx)^2\right)\right)}{105d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)^2*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*(f*(c + d*x)^(3/2)*(35*(b*c - a*d)^2 - 42*b*(b*c - a*d)*(c + d*x) + 15*b
^2*(c + d*x)^2) + 7*d*e*(Sqrt[c + d*x]*(15*a^2*d^2 + 10*a*b*d*(c + d*x) + b
^2*(-2*c^2 + c*d*x + 3*d^2*x^2)) - 15*a^2*Sqrt[c]*d^2*ArcTanh[Sqrt[c + d*x]
/Sqrt[c]])))/(105*d^3)
```

IntegrateAlgebraic [A] time = 0.13, size = 202, normalized size = 1.38

$$\frac{2\left(105a^2d^3e\sqrt{c+dx}+35a^2d^2f(c+dx)^{3/2}+70abd^2e(c+dx)^{3/2}+42abdf(c+dx)^{5/2}-70abcdf(c+dx)^{3/2}+35b^2c^2f(c+dx)^{3/2}+21b^2de(c+dx)^{5/2}-35b^2cde(c+dx)^{3/2}+15b^2f(c+dx)^{7/2}-42b^2cf(c+dx)^{5/2}\right)}{105d^3}-2a^2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x)^2*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*(105*a^2*d^3*e*Sqrt[c + d*x] - 35*b^2*c*d*e*(c + d*x)^(3/2) + 70*a*b*d^2
*e*(c + d*x)^(3/2) + 35*b^2*c^2*f*(c + d*x)^(3/2) - 70*a*b*c*d*f*(c + d*x)^
(3/2) + 35*a^2*d^2*f*(c + d*x)^(3/2) + 21*b^2*d*e*(c + d*x)^(5/2) - 42*b^2*
c*f*(c + d*x)^(5/2) + 42*a*b*d*f*(c + d*x)^(5/2) + 15*b^2*f*(c + d*x)^(7/2)
))/(105*d^3) - 2*a^2*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]
```

fricas [A] time = 1.59, size = 405, normalized size = 2.77

$$\left[\frac{105a^2\sqrt{c}\,d^3e\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2\left(15b^2d^3fx^3+3\left(7b^2d^3e+\left(b^2cd^2+14abd^3\right)f\right)x^2-7\left(2b^2c^2d-10abcd^2-15a^2d^3\right)e+\left(8b^2c^3-28abc^2d+35a^2cd^2\right)f+\left(7\left(b^2cd^2+10abd^3\right)e-\left(4b^2c^2d-14abcd^2-35a^2d^3\right)f\right)x\right)\sqrt{dx+c}}{105d^3},\ \frac{2\left(105a^2\sqrt{-c}\,d^3e\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)+\left(15b^2d^3fx^3+3\left(7b^2d^3e+\left(b^2cd^2+14abd^3\right)f\right)x^2-7\left(2b^2c^2d-10abcd^2-15a^2d^3\right)e+\left(8b^2c^3-28abc^2d+35a^2cd^2\right)f+\left(7\left(b^2cd^2+10abd^3\right)e-\left(4b^2c^2d-14abcd^2-35a^2d^3\right)f\right)x\right)\sqrt{dx+c}\right)}{105d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/105*(105*a^2*sqrt(c)*d^3*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x)
+ 2*(15*b^2*d^3*f*x^3 + 3*(7*b^2*d^3*e + (b^2*c*d^2 + 14*a*b*d^3)*f)*x^2 -
7*(2*b^2*c^2*d - 10*a*b*c*d^2 - 15*a^2*d^3)*e + (8*b^2*c^3 - 28*a*b*c^2*d +
 35*a^2*c*d^2)*f + (7*(b^2*c*d^2 + 10*a*b*d^3)*e - (4*b^2*c^2*d - 14*a*b*c*
d^2 - 35*a^2*d^3)*f)*x)*sqrt(d*x + c))/d^3, 2/105*(105*a^2*sqrt(-c)*d^3*e*a
rctan(sqrt(d*x + c)*sqrt(-c)/c) + (15*b^2*d^3*f*x^3 + 3*(7*b^2*d^3*e + (b^2
*c*d^2 + 14*a*b*d^3)*f)*x^2 - 7*(2*b^2*c^2*d - 10*a*b*c*d^2 - 15*a^2*d^3)*e
 + (8*b^2*c^3 - 28*a*b*c^2*d + 35*a^2*c*d^2)*f + (7*(b^2*c*d^2 + 10*a*b*d^3
)*e - (4*b^2*c^2*d - 14*a*b*c*d^2 - 35*a^2*d^3)*f)*x)*sqrt(d*x + c))/d^3]
```

giac [A] time = 1.36, size = 201, normalized size = 1.38

$$\frac{2\,a^{2} c \arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right) e}{\sqrt{-c}} + \frac{2\,\left(15\,(dx+c)^{\frac{7}{2}} b^{2} d^{18} f - 42\,(dx+c)^{\frac{5}{2}} b^{2} c d^{18} f + 35\,(dx+c)^{\frac{3}{2}} b^{2} c^{2} d^{18} f + 42\,(dx+c)^{\frac{5}{2}} a b d^{19} f - 70\,(dx+c)^{\frac{3}{2}} a b c d^{19} f + 35\,(dx+c)^{\frac{3}{2}} a^{2} d^{20} f + 21\,(dx+c)^{\frac{5}{2}} b^{2} d^{19} e - 35\,(dx+c)^{\frac{3}{2}} b^{2} c d^{19} e + 70\,(dx+c)^{\frac{3}{2}} a b d^{20} e + 105\,\sqrt{dx+c}\, a^{2} d^{21} e\right)}{105\,d^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a^2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/sqrt(-c) + 2/105*(15*(d*x + c)^(7/
2)*b^2*d^18*f - 42*(d*x + c)^(5/2)*b^2*c*d^18*f + 35*(d*x + c)^(3/2)*b^2*c^
2*d^18*f + 42*(d*x + c)^(5/2)*a*b*d^19*f - 70*(d*x + c)^(3/2)*a*b*c*d^19*f
+ 35*(d*x + c)^(3/2)*a^2*d^20*f + 21*(d*x + c)^(5/2)*b^2*d^19*e - 35*(d*x +
 c)^(3/2)*b^2*c*d^19*e + 70*(d*x + c)^(3/2)*a*b*d^20*e + 105*sqrt(d*x + c)*
a^2*d^21*e)/d^21
```

maple [A] time = 0.01, size = 176, normalized size = 1.21

$$\frac{-2a^{2}\sqrt{c}\,d^{3}e\,\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right) + 2\sqrt{dx+c}\,a^{2}d^{3}e + \frac{2(dx+c)^{\frac{3}{2}}a^{2}d^{2}f}{3} - \frac{4(dx+c)^{\frac{3}{2}}abcdf}{3} + \frac{4(dx+c)^{\frac{3}{2}}abd^{2}e}{3} + \frac{2(dx+c)^{\frac{3}{2}}b^{2}c^{2}f}{3} - \frac{2(dx+c)^{\frac{3}{2}}b^{2}cde}{3} + \frac{4(dx+c)^{\frac{5}{2}}abdf}{5} - \frac{4(dx+c)^{\frac{5}{2}}b^{2}cf}{5} + \frac{2(dx+c)^{\frac{5}{2}}b^{2}de}{5} + \frac{2(dx+c)^{\frac{7}{2}}b^{2}f}{7}}{d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x)
```

```
[Out] 2/d^3*(1/7*b^2*f*(d*x+c)^(7/2)+2/5*(d*x+c)^(5/2)*a*b*d*f-2/5*(d*x+c)^(5/2)*
b^2*c*f+1/5*(d*x+c)^(5/2)*b^2*d*e+1/3*(d*x+c)^(3/2)*a^2*d^2*f-2/3*(d*x+c)^(
3/2)*a*b*c*d*f+2/3*(d*x+c)^(3/2)*a*b*d^2*e+1/3*(d*x+c)^(3/2)*b^2*c^2*f-1/3*
(d*x+c)^(3/2)*b^2*c*d*e+a^2*d^3*e*(d*x+c)^(1/2)-a^2*c^(1/2)*d^3*e*arctanh((
d*x+c)^(1/2)/c^(1/2)))
```

maxima [A] time = 0.96, size = 152, normalized size = 1.04

$$a^{2}\sqrt{c}\,e\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right) + \frac{2\left(105\,\sqrt{dx+c}\,a^{2}d^{3}e + 15\,(dx+c)^{\frac{7}{2}}b^{2}f + 21\,\left(b^{2}de - 2\,\left(b^{2}c - abd\right)f\right)(dx+c)^{\frac{5}{2}} - 35\,\left(\left(b^{2}cd - 2\,abd^{2}\right)e - \left(b^{2}c^{2} - 2\,abcd + a^{2}d^{2}\right)f\right)(dx+c)^{\frac{3}{2}}\right)}{105\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^2*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a^2*sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2/
105*(105*sqrt(d*x + c)*a^2*d^3*e + 15*(d*x + c)^(7/2)*b^2*f + 21*(b^2*d*e -
 2*(b^2*c - a*b*d)*f)*(d*x + c)^(5/2) - 35*((b^2*c*d - 2*a*b*d^2)*e - (b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*f)*(d*x + c)^(3/2))/d^3
```

mupad [B] time = 2.62, size = 263, normalized size = 1.80

$$\left(\frac{2\,b^{2}\,d\,e-6\,b^{2}\,c\,f+4\,a\,b\,d\,f}{5\,d^{3}}+\frac{2\,b^{2}\,c\,f}{5\,d^{3}}\right)(c+d\,x)^{5/2}+\left(c\left(c\left(\frac{2\,b^{2}\,d\,e-6\,b^{2}\,c\,f+4\,a\,b\,d\,f}{d^{3}}+\frac{2\,b^{2}\,c\,f}{d^{3}}\right)+\frac{2\,(a\,d-b\,c)\,\left(a\,d\,f-3\,b\,c\,f+2\,b\,d\,e\right)}{d^{3}}\right)-\frac{2\,(a\,d-b\,c)^{2}\,\left(c\,f-d\,e\right)}{d^{3}}\right)\sqrt{c+d\,x}+\left(\frac{c\,\left(\frac{2\,b^{2}\,d\,e-6\,b^{2}\,c\,f+4\,a\,b\,d\,f}{d^{3}}+\frac{2\,b^{2}\,c\,f}{d^{3}}\right)}{3}+\frac{2\,(a\,d-b\,c)\,\left(a\,d\,f-3\,b\,c\,f+2\,b\,d\,e\right)}{3\,d^{3}}\right)(c+d\,x)^{3/2}+\frac{2\,b^{2}\,f\,(c+d\,x)^{7/2}}{7\,d^{3}}+a^{2}\,\sqrt{c}\,e\,\mathrm{atan}\left(\frac{\sqrt{c+d\,x}\,1\mathrm{i}}{\sqrt{c}}\right)2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^2*(c + d*x)^(1/2))/x,x)
```

```
[Out] ((2*b^2*d*e - 6*b^2*c*f + 4*a*b*d*f)/(5*d^3) + (2*b^2*c*f)/(5*d^3))*(c + d*
x)^(5/2) + (c*(c*((2*b^2*d*e - 6*b^2*c*f + 4*a*b*d*f)/d^3 + (2*b^2*c*f)/d^3
) + (2*(a*d - b*c)*(a*d*f - 3*b*c*f + 2*b*d*e))/d^3) - (2*(a*d - b*c)^2*(c*
f - d*e))/d^3)*(c + d*x)^(1/2) + ((c*((2*b^2*d*e - 6*b^2*c*f + 4*a*b*d*f)/d
^3 + (2*b^2*c*f)/d^3))/3 + (2*(a*d - b*c)*(a*d*f - 3*b*c*f + 2*b*d*e))/(3*d
^3))*(c + d*x)^(3/2) + a^2*c^(1/2)*e*atan(((c + d*x)^(1/2)*1i)/c^(1/2))*2i
+ (2*b^2*f*(c + d*x)^(7/2))/(7*d^3)
```

sympy [**A**] time = 27.60, size = 167, normalized size = 1.14

$$\frac{2a^2ce\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2a^2e\sqrt{c+dx}+\frac{2b^2f\,(c+dx)^{\frac{7}{2}}}{7d^3}+\frac{2\,(c+dx)^{\frac{5}{2}}\left(2abdf-2b^2cf+b^2de\right)}{5d^3}+\frac{2\,(c+dx)^{\frac{3}{2}}\left(a^2d^2f-2abcdf+2abd^2e+b^2c^2f-b^2cde\right)}{3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**2*(f*x+e)*(d*x+c)**(1/2)/x,x)
```

```
[Out] 2*a**2*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*a**2*e*sqrt(c + d*x) +
 2*b**2*f*(c + d*x)**(7/2)/(7*d**3) + 2*(c + d*x)**(5/2)*(2*a*b*d*f - 2*b**
2*c*f + b**2*d*e)/(5*d**3) + 2*(c + d*x)**(3/2)*(a**2*d**2*f - 2*a*b*c*d*f
+ 2*a*b*d**2*e + b**2*c**2*f - b**2*c*d*e)/(3*d**3)
```

3.10 $\int \frac{(a+bx)\sqrt{c+dx}\,(e+fx)}{x}\,dx$

Optimal. Leaf size=77

$$-\frac{2(c+dx)^{3/2}(-5d(af+be)+2bcf-3bdfx)}{15d^2}+2ae\sqrt{c+dx}-2a\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {147, 50, 63, 208}

$$-\frac{2(c+dx)^{3/2}(-5d(af+be)+2bcf-3bdfx)}{15d^2}+2ae\sqrt{c+dx}-2a\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*x)*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] 2*a*e*Sqrt[c + d*x] - (2*(c + d*x)^(3/2)*(2*b*c*f - 5*d*(b*e + a*f) - 3*b*d
*f*x))/(15*d^2) - 2*a*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(a+bx)\sqrt{c+dx}\,(e+fx)}{x}\,dx = -\frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2} + (ae)\int \frac{\sqrt{c+dx}}{x}\,dx$$

$$= 2ae\sqrt{c+dx} - \frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2} + (ace)\int \frac{1}{x\sqrt{c+dx}}\,d$$

$$= 2ae\sqrt{c+dx} - \frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2} + \frac{(2ace)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}}\right.}{d}$$

$$= 2ae\sqrt{c+dx} - \frac{2(c+dx)^{3/2}(2bcf-5d(be+af)-3bdfx)}{15d^2} - 2a\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{}}\right.$$

Mathematica [A] time = 0.16, size = 81, normalized size = 1.05

$$\frac{2\sqrt{c+dx}\,(5ad(cf+3de+dfx)-b(c+dx)(2cf-5de-3dfx))}{15d^2} - 2a\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x)*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*Sqrt[c + d*x]*(-(b*(c + d*x)*(-5*d*e + 2*c*f - 3*d*f*x)) + 5*a*d*(3*d*e
+ c*f + d*f*x)))/(15*d^2) - 2*a*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]
```

IntegrateAlgebraic [A] time = 0.06, size = 105, normalized size = 1.36

$$\frac{2\left(15ad^2e\sqrt{c+dx}+5adf(c+dx)^{3/2}+5bde(c+dx)^{3/2}+3bf(c+dx)^{5/2}-5bcf(c+dx)^{3/2}\right)}{15d^2} - 2a\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((a + b*x)*Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*(15*a*d^2*e*Sqrt[c + d*x] + 5*b*d*e*(c + d*x)^(3/2) - 5*b*c*f*(c + d*x)^
(3/2) + 5*a*d*f*(c + d*x)^(3/2) + 3*b*f*(c + d*x)^(5/2)))/(15*d^2) - 2*a*Sq
rt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]
```

fricas [A] time = 0.90, size = 219, normalized size = 2.84

$$\left[\frac{15a\sqrt{c}d^2e\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2\left(3bd^2fx^2+5\left(bcd+3ad^2\right)e-\left(2bc^2-5acd\right)f+\left(5bd^2e+\left(bcd+5ad^2\right)f\right)x\right)\sqrt{dx+c}}{15d^2}, \frac{2\left(15a\sqrt{-c}d^2e\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)+\left(3bd^2fx^2+5\left(bcd+3ad^2\right)e-\left(2bc^2-5acd\right)f+\left(5bd^2e+\left(bcd+5ad^2\right)f\right)x\right)\sqrt{dx+c}\right)}{15d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*a*sqrt(c)*d^2*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*
(3*b*d^2*f*x^2 + 5*(b*c*d + 3*a*d^2)*e - (2*b*c^2 - 5*a*c*d)*f + (5*b*d^2*e
 + (b*c*d + 5*a*d^2)*f)*x)*sqrt(d*x + c))/d^2, 2/15*(15*a*sqrt(-c)*d^2*e*ar
ctan(sqrt(d*x + c)*sqrt(-c)/c) + (3*b*d^2*f*x^2 + 5*(b*c*d + 3*a*d^2)*e - (
2*b*c^2 - 5*a*c*d)*f + (5*b*d^2*e + (b*c*d + 5*a*d^2)*f)*x)*sqrt(d*x + c))/
d^2]
```

giac [A] time = 1.33, size = 105, normalized size = 1.36

$$\frac{2ac\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{\sqrt{-c}} + \frac{2\left(3(dx+c)^{\frac{5}{2}}bd^8f-5(dx+c)^{\frac{3}{2}}bcd^8f+5(dx+c)^{\frac{3}{2}}ad^9f+5(dx+c)^{\frac{3}{2}}bd^9e+15\sqrt{dx+c}\,ad^{10}e\right)}{15d^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/sqrt(-c) + 2/15*(3*(d*x + c)^(5/2)*b
*d^8*f - 5*(d*x + c)^(3/2)*b*c*d^8*f + 5*(d*x + c)^(3/2)*a*d^9*f + 5*(d*x +
 c)^(3/2)*b*d^9*e + 15*sqrt(d*x + c)*a*d^10*e)/d^10
```

maple [A] time = 0.01, size = 89, normalized size = 1.16

$$\frac{-2a\sqrt{c}\,d^2 e \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)+2\sqrt{dx+c}\,a\,d^2 e+\frac{2(dx+c)^{\frac{3}{2}}adf}{3}-\frac{2(dx+c)^{\frac{3}{2}}bcf}{3}+\frac{2(dx+c)^{\frac{3}{2}}bde}{3}+\frac{2(dx+c)^{\frac{5}{2}}bf}{5}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x)
```

```
[Out] 2/d^2*(1/5*f*b*(d*x+c)^(5/2)+1/3*(d*x+c)^(3/2)*a*d*f-1/3*(d*x+c)^(3/2)*b*c*
f+1/3*(d*x+c)^(3/2)*b*d*e+a*d^2*e*(d*x+c)^(1/2)-a*c^(1/2)*d^2*e*arctanh((d*
x+c)^(1/2)/c^(1/2)))
```

maxima [A] time = 0.98, size = 91, normalized size = 1.18

$$a\sqrt{c}\,e\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)+\frac{2\left(15\sqrt{dx+c}\,ad^2e+3\,(dx+c)^{\frac{5}{2}}bf+5\left(bde-(bc-ad)f\right)(dx+c)^{\frac{3}{2}}\right)}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] a*sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2/15
*(15*sqrt(d*x + c)*a*d^2*e + 3*(d*x + c)^(5/2)*b*f + 5*(b*d*e - (b*c - a*d)
*f)*(d*x + c)^(3/2))/d^2
```

mupad [B] time = 0.09, size = 136, normalized size = 1.77

$$\left(c\left(\frac{2\,a\,d\,f-4\,b\,c\,f+2\,b\,d\,e}{d^2}+\frac{2\,b\,c\,f}{d^2}\right)-\frac{2\,(a\,d-b\,c)\,\left(c\,f-d\,e\right)}{d^2}\right)\sqrt{c+d\,x}+\left(\frac{2\,a\,d\,f-4\,b\,c\,f+2\,b\,d\,e}{3\,d^2}+\frac{2\,b\,c\,f}{3\,d^2}\right)(c+d\,x)^{3/2}+\frac{2\,b\,f\,(c+d\,x)^{5/2}}{5\,d^2}+a\,\sqrt{c}\,e\,\mathrm{atan}\left(\frac{\sqrt{c+d\,x}\,1\mathrm{i}}{\sqrt{c}}\right)\,2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)*(c + d*x)^(1/2))/x,x)
```

```
[Out] (c*((2*a*d*f - 4*b*c*f + 2*b*d*e)/d^2 + (2*b*c*f)/d^2) - (2*(a*d - b*c)*(c*
f - d*e))/d^2)*(c + d*x)^(1/2) + ((2*a*d*f - 4*b*c*f + 2*b*d*e)/(3*d^2) + (
2*b*c*f)/(3*d^2))*(c + d*x)^(3/2) + (2*b*f*(c + d*x)^(5/2))/(5*d^2) + a*c^(
1/2)*e*atan(((c + d*x)^(1/2)*1i)/c^(1/2))*2i
```

sympy [A] time = 25.99, size = 92, normalized size = 1.19

$$\frac{2ace\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2ae\sqrt{c+dx}+\frac{2bf\,(c+dx)^{\frac{5}{2}}}{5d^2}+\frac{2\,(c+dx)^{\frac{3}{2}}\left(adf-bcf+bde\right)}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(f*x+e)*(d*x+c)**(1/2)/x,x)
```

```
[Out] 2*a*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*a*e*sqrt(c + d*x) + 2*b*f
*(c + d*x)**(5/2)/(5*d**2) + 2*(c + d*x)**(3/2)*(a*d*f - b*c*f + b*d*e)/(3*
d**2)
```

3.11 $\int \frac{\sqrt{c+dx}\,(e+fx)}{x}\,dx$

Optimal. Leaf size=54

$$2e\sqrt{c+dx}-2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\frac{2f(c+dx)^{3/2}}{3d}$$

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {80, 50, 63, 208}

$$2e\sqrt{c+dx}-2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)+\frac{2f(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] 2*e*Sqrt[c + d*x] + (2*f*(c + d*x)^(3/2))/(3*d) - 2*Sqrt[c]*e*ArcTanh[Sqrt[
c + d*x]/Sqrt[c]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{c+dx}\,(e+fx)}{x}\,dx &= \frac{2f(c+dx)^{3/2}}{3d}+e\int \frac{\sqrt{c+dx}}{x}\,dx\\ &= 2e\sqrt{c+dx}+\frac{2f(c+dx)^{3/2}}{3d}+(ce)\int \frac{1}{x\sqrt{c+dx}}\,dx\\ &= 2e\sqrt{c+dx}+\frac{2f(c+dx)^{3/2}}{3d}+\frac{(2ce)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{d}\\ &= 2e\sqrt{c+dx}+\frac{2f(c+dx)^{3/2}}{3d}-2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 1.02

$$e\left(2\sqrt{c+dx}-2\sqrt{c}\,\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)\right)+\frac{2f(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*f*(c + d*x)^(3/2))/(3*d) + e*(2*Sqrt[c + d*x] - 2*Sqrt[c]*ArcTanh[Sqrt[c
 + d*x]/Sqrt[c]])
```

IntegrateAlgebraic [A] time = 0.03, size = 57, normalized size = 1.06

$$\frac{2\left(3de\sqrt{c+dx}+f(c+dx)^{3/2}\right)}{3d}-2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c + d*x]*(e + f*x))/x,x]
```

```
[Out] (2*(3*d*e*Sqrt[c + d*x] + f*(c + d*x)^(3/2)))/(3*d) - 2*Sqrt[c]*e*ArcTanh[S
qrt[c + d*x]/Sqrt[c]]
```

fricas [A] time = 0.87, size = 111, normalized size = 2.06

$$\left[\frac{3\,\sqrt{c}\,de\log\left(\frac{dx-2\,\sqrt{dx+c}\,\sqrt{c}+2\,c}{x}\right)+2\left(dfx+3\,de+cf\right)\sqrt{dx+c}}{3\,d},\ \frac{2\left(3\,\sqrt{-c}\,de\arctan\left(\frac{\sqrt{dx+c}\,\sqrt{-c}}{c}\right)+\left(dfx+3\,de+cf\right)\sqrt{dx+c}\right)}{3\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(c)*d*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*(d*f*x
 + 3*d*e + c*f)*sqrt(d*x + c))/d, 2/3*(3*sqrt(-c)*d*e*arctan(sqrt(d*x + c)*
sqrt(-c)/c) + (d*f*x + 3*d*e + c*f)*sqrt(d*x + c))/d]
```

giac [A] time = 1.24, size = 57, normalized size = 1.06

$$\frac{2\,c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{\sqrt{-c}}+\frac{2\left((dx+c)^{\frac{3}{2}}d^2f+3\,\sqrt{dx+c}\,d^3e\right)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/sqrt(-c) + 2/3*((d*x + c)^(3/2)*d^2*f
+ 3*sqrt(d*x + c)*d^3*e)/d^3
```

maple [A] time = 0.01, size = 46, normalized size = 0.85

$$\frac{-2\sqrt{c}\, de \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)+2\sqrt{dx+c}\; de+\frac{2(dx+c)^{\frac{3}{2}} f}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(d*x+c)^(1/2)/x,x)
```

```
[Out] 2/d*(1/3*f*(d*x+c)^(3/2)+d*e*(d*x+c)^(1/2)-c^(1/2)*d*e*arctanh((d*x+c)^(1/2
)/c^(1/2)))
```

maxima [A] time = 0.98, size = 60, normalized size = 1.11

$$\sqrt{c}\, e \log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)+\frac{2\left(3\sqrt{dx+c}\, de+(dx+c)^{\frac{3}{2}} f\right)}{3\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(c)*e*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c))) + 2/3*(3
*sqrt(d*x + c)*d*e + (d*x + c)^(3/2)*f)/d
```

mupad [B] time = 0.07, size = 45, normalized size = 0.83

$$2\, e\, \sqrt{c+d\, x}+\frac{2\, f\, {(c+d\, x)}^{3/2}}{3\, d}+\sqrt{c}\; e\, \mathrm{atan}\left(\frac{\sqrt{c+d\, x}\; 1{}\mathrm{i}}{\sqrt{c}}\right)\, 2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(c + d*x)^(1/2))/x,x)
```

```
[Out] 2*e*(c + d*x)^(1/2) + c^(1/2)*e*atan(((c + d*x)^(1/2)*1i)/c^(1/2))*2i + (2*
f*(c + d*x)^(3/2))/(3*d)
```

sympy [A] time = 5.98, size = 54, normalized size = 1.00

$$\frac{2ce \operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}}+2e\sqrt{c+dx}+\frac{2f\left(c+dx\right)^{\frac{3}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x,x)
```

```
[Out] 2*c*e*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + 2*e*sqrt(c + d*x) + 2*f*(c +
d*x)**(3/2)/(3*d)
```

3.12 $\int \frac{\sqrt{c+dx}\,(e+fx)}{x(a+bx)}\,dx$

Optimal. Leaf size=101

$$\frac{2\sqrt{bc-ad}\,(be-af)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a}+\frac{2f\sqrt{c+dx}}{b}$$

Rubi [A] time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {154, 156, 63, 208}

$$\frac{2\sqrt{bc-ad}\,(be-af)\tanh^{-1}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a}+\frac{2f\sqrt{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)),x]
```

```
[Out] (2*f*Sqrt[c + d*x])/b - (2*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/a + (2
*Sqrt[b*c - a*d]*(b*e - a*f)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d
]])/(a*b^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,(e+fx)}{x(a+bx)}\,dx = \frac{2f\sqrt{c+dx}}{b} + \frac{2\int \frac{\frac{bce}{2}+\frac{1}{2}(bde+bcf-adf)x}{x(a+bx)\sqrt{c+dx}}\,dx}{b}$$

$$= \frac{2f\sqrt{c+dx}}{b} + \frac{(ce)\int \frac{1}{x\sqrt{c+dx}}\,dx}{a} - \frac{((bc-ad)(be-af))\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{ab}$$

$$= \frac{2f\sqrt{c+dx}}{b} + \frac{(2ce)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{ad} - \frac{(2(bc-ad)(be-af))\,\text{Subst}\left(\int -\right.}{abd}$$

$$= \frac{2f\sqrt{c+dx}}{b} - \frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a} + \frac{2\sqrt{bc-ad}\,(be-af)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{ab^{3/2}}$$

Mathematica [**A**] time = 0.12, size = 101, normalized size = 1.00

$$\frac{2\sqrt{bc-ad}\,(be-af)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{ab^{3/2}} - \frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a} + \frac{2f\sqrt{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)),x]
```

```
[Out] (2*f*Sqrt[c + d*x])/b - (2*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/a + (2
*Sqrt[b*c - a*d]*(b*e - a*f)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d
]])/(a*b^(3/2))
```

IntegrateAlgebraic [**A**] time = 0.17, size = 111, normalized size = 1.10

$$-\frac{2\sqrt{ad-bc}\,(be-af)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{ab^{3/2}} - \frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a} + \frac{2f\sqrt{c+dx}}{b}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)),x]
```

```
[Out] (2*f*Sqrt[c + d*x])/b - (2*Sqrt[-(b*c) + a*d]*(b*e - a*f)*ArcTan[(Sqrt[b]*S
qrt[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(a*b^(3/2)) - (2*Sqrt[c]*e*A
rcTanh[Sqrt[c + d*x]/Sqrt[c]])/a
```

fricas [**A**] time = 1.22, size = 449, normalized size = 4.45

$$\left[\frac{b\sqrt{c}e\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2\sqrt{dx+c}\,af-(be-af)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2bc-ad-2\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)}{ab}, \frac{b\sqrt{c}e\log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c}+2c}{x}\right)+2\sqrt{dx+c}\,af+2(be-af)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)}{ab}, \frac{2b\sqrt{-c}e\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)+2\sqrt{dx+c}\,af-(be-af)\sqrt{\frac{bc-ad}{b}}\log\left(\frac{bdx+2bc-ad-2\sqrt{dx+c}\,b\sqrt{\frac{bc-ad}{b}}}{bx+a}\right)}{ab}, \frac{2\left(b\sqrt{-c}e\arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right)+\sqrt{dx+c}\,af+(be-af)\sqrt{-\frac{bc-ad}{b}}\arctan\left(-\frac{\sqrt{dx+c}\,b\sqrt{-\frac{bc-ad}{b}}}{bc-ad}\right)\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x, algorithm="fricas")
```

```
[Out] [(b*sqrt(c)*e*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(d*x + c
)*a*f - (b*e - a*f)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt(d
*x + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)))/(a*b), (b*sqrt(c)*e*log((d*x - 2
*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(d*x + c)*a*f + 2*(b*e - a*f)*sqrt
(-(b*c - a*d)/b)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)))
/(a*b), (2*b*sqrt(-c)*e*arctan(sqrt(d*x + c)*sqrt(-c)/c) + 2*sqrt(d*x + c)*
a*f - (b*e - a*f)*sqrt((b*c - a*d)/b)*log((b*d*x + 2*b*c - a*d - 2*sqrt(d*x
 + c)*b*sqrt((b*c - a*d)/b))/(b*x + a)))/(a*b), 2*(b*sqrt(-c)*e*arctan(sqrt
(d*x + c)*sqrt(-c)/c) + sqrt(d*x + c)*a*f + (b*e - a*f)*sqrt(-(b*c - a*d)/b
)*arctan(-sqrt(d*x + c)*b*sqrt(-(b*c - a*d)/b)/(b*c - a*d)))/(a*b)]
```

giac [A] time = 1.35, size = 112, normalized size = 1.11

$$\frac{2\,c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{a\sqrt{-c}}+\frac{2\sqrt{dx+c}\,f}{b}+\frac{2\left(abcf-a^2df-b^2ce+abde\right)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x, algorithm="giac")
```

```
[Out] 2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/(a*sqrt(-c)) + 2*sqrt(d*x + c)*f/b + 2
*(a*b*c*f - a^2*d*f - b^2*c*e + a*b*d*e)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c
 + a*b*d))/(sqrt(-b^2*c + a*b*d)*a*b)
```

maple [B] time = 0.02, size = 196, normalized size = 1.94

$$-\frac{2adf\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,b}-\frac{2bce\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,a}+\frac{2cf\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}}+\frac{2de\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}}-\frac{2\sqrt{c}\,e\,\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{a}+\frac{2\sqrt{dx+c}\,f}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x)
```

```
[Out] 2*f*(d*x+c)^(1/2)/b-2*a/b/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*
c)*b)^(1/2)*b)*d*f+2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)
^(1/2)*b)*c*f+2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2
)*b)*d*e-2/a*b/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)
*b)*c*e-2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/a
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 2.87, size = 2368, normalized size = 23.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(c + d*x)^(1/2))/(x*(a + b*x)),x)
```

```
[Out] (2*f*(c + d*x)^(1/2))/b - (c^(1/2)*e*atan(((c^(1/2)*e*((8*(c + d*x)^(1/2)*(
a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2*b
^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2*e*
f + 4*a^2*b^2*c*d^3*e*f))/b + (c^(1/2)*e*((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2
*d^2*f))/b + (8*c^(1/2)*e*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(c + d*x)^(1/2))/
(a*b)))/a)*1i)/a + (c^(1/2)*e*((8*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^
4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3
*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f)
)/b - (c^(1/2)*e*((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2*d^2*f))/b - (8*c^(1/2)*
e*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(c + d*x)^(1/2))/(a*b)))/a)*1i)/a)/((16*(
b^3*c^2*d^3*e^3 - a*b^2*c*d^4*e^3 - a^3*c*d^4*e*f^2 + b^3*c^3*d^2*e^2*f - 3
*a*b^2*c^2*d^3*e^2*f - a*b^2*c^3*d^2*e*f^2 + 2*a^2*b*c^2*d^3*e*f^2 + 2*a^2*
b*c*d^4*e^2*f))/b - (c^(1/2)*e*((8*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d
^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^
```

```
3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f
))/b + (c^(1/2)*e*((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2*d^2*f))/b + (8*c^(1/2)
*e*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(c + d*x)^(1/2))/(a*b)))/a))/a + (c^(1/2
)*e*((8*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2
- 2*a^3*b*d^4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3
*f^2 - 2*a*b^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f))/b - (c^(1/2)*e*((8*(a^3*
b^2*c*d^3*f - a^2*b^3*c^2*d^2*f))/b - (8*c^(1/2)*e*(a^3*b^3*d^3 - 2*a^2*b^4
*c*d^2)*(c + d*x)^(1/2))/(a*b)))/a))/a))*2i)/a - (atan(((((8*(c + d*x)^(1/2
)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^
2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2
*e*f + 4*a^2*b^2*c*d^3*e*f))/b + (((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2*d^2*f)
)/b + (8*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/
2)*(c + d*x)^(1/2))/(a*b^4))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2))/(a*b^3))
*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2)*1i)/(a*b^3) + (((8*(c + d*x)^(1/2)*(a
^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^4*e*f + a^2*b^
2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b^3*c^2*d^2*e*f
 + 4*a^2*b^2*c*d^3*e*f))/b - (((8*(a^3*b^2*c*d^3*f - a^2*b^3*c^2*d^2*f))/b
- (8*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2)*(
c + d*x)^(1/2))/(a*b^4))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2))/(a*b^3))*(a*
f - b*e)*(-b^3*(a*d - b*c))^(1/2)*1i)/(a*b^3))/((16*(b^3*c^2*d^3*e^3 - a*b^
2*c*d^4*e^3 - a^3*c*d^4*e*f^2 + b^3*c^3*d^2*e^2*f - 3*a*b^2*c^2*d^3*e^2*f -
 a*b^2*c^3*d^2*e*f^2 + 2*a^2*b*c^2*d^3*e*f^2 + 2*a^2*b*c*d^4*e^2*f))/b - ((
(8*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a
^3*b*d^4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2
- 2*a*b^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f))/b + (((8*(a^3*b^2*c*d^3*f - a
^2*b^3*c^2*d^2*f))/b + (8*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(a*f - b*e)*(-b^3
*(a*d - b*c))^(1/2)*(c + d*x)^(1/2))/(a*b^4))*(a*f - b*e)*(-b^3*(a*d - b*c)
)^(1/2))/(a*b^3))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2))/(a*b^3) + (((8*(c +
 d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 2*b^4*c^2*d^2*e^2 - 2*a^3*b*d^
4*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a^3*b*c*d^3*f^2 - 2*a*b
^3*c^2*d^2*e*f + 4*a^2*b^2*c*d^3*e*f))/b - (((8*(a^3*b^2*c*d^3*f - a^2*b^3*
c^2*d^2*f))/b - (8*(a^3*b^3*d^3 - 2*a^2*b^4*c*d^2)*(a*f - b*e)*(-b^3*(a*d -
 b*c))^(1/2)*(c + d*x)^(1/2))/(a*b^4))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2)
)/(a*b^3))*(a*f - b*e)*(-b^3*(a*d - b*c))^(1/2))/(a*b^3)))*(a*f - b*e)*(-b^
3*(a*d - b*c))^(1/2)*2i)/(a*b^3)
```

sympy [A] time = 27.33, size = 97, normalized size = 0.96

$$\frac{2f\sqrt{c+dx}}{b}+\frac{2ce\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{a\sqrt{-c}}-\frac{2\left(ad-bc\right)\left(af-be\right)\operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{\frac{ad-bc}{b}}}\right)}{ab^{2}\sqrt{\frac{ad-bc}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x/(b*x+a),x)
```

```
[Out] 2*f*sqrt(c + d*x)/b + 2*c*e*atan(sqrt(c + d*x)/sqrt(-c))/(a*sqrt(-c)) - 2*(
a*d - b*c)*(a*f - b*e)*atan(sqrt(c + d*x)/sqrt((a*d - b*c)/b))/(a*b**2*sqrt
((a*d - b*c)/b))
```

3.13 $\int \frac{\sqrt{c+dx}\,(e+fx)}{x(a+bx)^2}\,dx$

Optimal. Leaf size=127

$$\frac{\left(2b^2ce-ad(af+be)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx}\,(be-af)}{ab(a+bx)}$$

Rubi [A] time = 0.11, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {149, 156, 63, 208}

$$\frac{\left(2b^2ce-ad(af+be)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2}+\frac{\sqrt{c+dx}\,(be-af)}{ab(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^2),x]
```

```
[Out] ((b*e - a*f)*Sqrt[c + d*x])/(a*b*(a + b*x)) - (2*Sqrt[c]*e*ArcTanh[Sqrt[c +
 d*x]/Sqrt[c]])/a^2 + ((2*b^2*c*e - a*d*(b*e + a*f))*ArcTanh[(Sqrt[b]*Sqrt[
c + d*x])/Sqrt[b*c - a*d]])/(a^2*b^(3/2)*Sqrt[b*c - a*d])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{c+dx}\,(e+fx)}{x(a+bx)^2}\,dx = \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} - \frac{\int \frac{-bce-\frac{1}{2}d(be+af)x}{x(a+bx)\sqrt{c+dx}}\,dx}{ab}$$

$$= \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} + \frac{(ce)\int \frac{1}{x\sqrt{c+dx}}\,dx}{a^2} + \frac{\left(-b^2ce+\frac{1}{2}ad(be+af)\right)\int \frac{1}{(a+bx)\sqrt{c+dx}}\,dx}{a^2b}$$

$$= \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} + \frac{(2ce)\,\mathrm{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{a^2d} + \frac{\left(2\left(-b^2ce+\frac{1}{2}ad(be+af)\right)\right.}{}$$

$$= \frac{(be-af)\sqrt{c+dx}}{ab(a+bx)} - \frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2} + \frac{\left(2b^2ce-ad(be+af)\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{a^2b^{3/2}\sqrt{bc-ad}}$$

Mathematica [A] time = 0.44, size = 124, normalized size = 0.98

$$\frac{-\frac{\left(a^2df+abde-2b^2ce\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{b^{3/2}\sqrt{bc-ad}} + \frac{a\sqrt{c+dx}\,(be-af)}{b(a+bx)} - 2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^2),x]
```

```
[Out] ((a*(b*e - a*f)*Sqrt[c + d*x])/(b*(a + b*x)) - 2*Sqrt[c]*e*ArcTanh[Sqrt[c +
 d*x]/Sqrt[c]] - ((-2*b^2*c*e + a*b*d*e + a^2*d*f)*ArcTanh[(Sqrt[b]*Sqrt[c
+ d*x])/Sqrt[b*c - a*d]])/(b^(3/2)*Sqrt[b*c - a*d]))/a^2
```

IntegrateAlgebraic [A] time = 0.50, size = 151, normalized size = 1.19

$$\frac{\left(a^2(-d)f-abde+2b^2ce\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{a^2b^{3/2}\sqrt{ad-bc}} - \frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2} - \frac{d\sqrt{c+dx}\,(af-be)}{ab(ad+b(c+dx)-bc)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^2),x]
```

```
[Out] -((d*(-(b*e) + a*f)*Sqrt[c + d*x])/(a*b*(-(b*c) + a*d + b*(c + d*x)))) + ((
2*b^2*c*e - a*b*d*e - a^2*d*f)*ArcTan[(Sqrt[b]*Sqrt[-(b*c) + a*d]*Sqrt[c +
d*x])/(b*c - a*d)])/(a^2*b^(3/2)*Sqrt[-(b*c) + a*d]) - (2*Sqrt[c]*e*ArcTanh
[Sqrt[c + d*x]/Sqrt[c]])/a^2
```

fricas [B] time = 1.35, size = 1018, normalized size = 8.02

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^3*d*f - (2*a*b^2*c - a^2*b*d)*e + (a^2*b*d*f - (2*b^3*c - a*b^2*d)
*e)*x)*sqrt(b^2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)
*sqrt(d*x + c))/(b*x + a)) + 2*((b^4*c - a*b^3*d)*e*x + (a*b^3*c - a^2*b^2*
d)*e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*((a*b^3*c -
a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)*f)*sqrt(d*x + c))/(a^3*b^3*c - a^4*b^2
*d + (a^2*b^4*c - a^3*b^3*d)*x), ((a^3*d*f - (2*a*b^2*c - a^2*b*d)*e + (a^2
*b*d*f - (2*b^3*c - a*b^2*d)*e)*x)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c
```

```
+ a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + ((b^4*c - a*b^3*d)*e*x + (a*b^3*c -
 a^2*b^2*d)*e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + ((a*b
^3*c - a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)*f)*sqrt(d*x + c))/(a^3*b^3*c -
a^4*b^2*d + (a^2*b^4*c - a^3*b^3*d)*x), 1/2*(4*((b^4*c - a*b^3*d)*e*x + (a*
b^3*c - a^2*b^2*d)*e)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c) + (a^3*d*f
- (2*a*b^2*c - a^2*b*d)*e + (a^2*b*d*f - (2*b^3*c - a*b^2*d)*e)*x)*sqrt(b^2
*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))
/(b*x + a)) + 2*((a*b^3*c - a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)*f)*sqrt(d*
x + c))/(a^3*b^3*c - a^4*b^2*d + (a^2*b^4*c - a^3*b^3*d)*x), ((a^3*d*f - (2
*a*b^2*c - a^2*b*d)*e + (a^2*b*d*f - (2*b^3*c - a*b^2*d)*e)*x)*sqrt(-b^2*c
+ a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) + 2*((b^4
*c - a*b^3*d)*e*x + (a*b^3*c - a^2*b^2*d)*e)*sqrt(-c)*arctan(sqrt(d*x + c)*
sqrt(-c)/c) + ((a*b^3*c - a^2*b^2*d)*e - (a^2*b^2*c - a^3*b*d)*f)*sqrt(d*x
+ c))/(a^3*b^3*c - a^4*b^2*d + (a^2*b^4*c - a^3*b^3*d)*x)]
```

giac [A] time = 1.42, size = 142, normalized size = 1.12

$$\frac{2\,c\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)e}{a^2\sqrt{-c}}+\frac{\left(a^2df-2\,b^2ce+abde\right)\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{-b^2c+abd}}\right)}{\sqrt{-b^2c+abd}\,a^2b}-\frac{\sqrt{dx+c}\,adf-\sqrt{dx+c}\,bde}{((dx+c)b-bc+ad)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/(a^2*sqrt(-c)) + (a^2*d*f - 2*b^2*c*e
+ a*b*d*e)*arctan(sqrt(d*x + c)*b/sqrt(-b^2*c + a*b*d))/(sqrt(-b^2*c + a*b*
d)*a^2*b) - (sqrt(d*x + c)*a*d*f - sqrt(d*x + c)*b*d*e)/(((d*x + c)*b - b*c
 + a*d)*a*b)
```

maple [A] time = 0.02, size = 192, normalized size = 1.51

$$\frac{de\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,\,a}-\frac{2bce\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,\,a^2}+\frac{df\arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{(ad-bc)b}}\right)}{\sqrt{(ad-bc)\,b}\,\,b}+\frac{\sqrt{dx+c}\,de}{(bdx+ad)\,a}-\frac{\sqrt{dx+c}\,df}{(bdx+ad)\,b}-\frac{2\sqrt{c}\,e\,\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x)
```

```
[Out] -d/b*(d*x+c)^(1/2)/(b*d*x+a*d)*f+d/a*(d*x+c)^(1/2)/(b*d*x+a*d)*e+d/b/((a*d-
b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*f+d/a/((a*d-b*c)*
b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*e-2/a^2*b/((a*d-b*c)*b
)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*c*e-2*e*arctanh((d*x+c)
^(1/2)/c^(1/2))*c^(1/2)/a^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 0.60, size = 1827, normalized size = 14.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(c + d*x)^(1/2))/(x*(a + b*x)^2),x)
```

```
[Out] (atan((((((((2*(2*a^4*b^3*c*d^3*e - 2*a^5*b^2*c*d^3*f))/(a^3*b) + ((4*a^5*b^
3*d^3 - 8*a^4*b^4*c*d^2)*(-b^3*(a*d - b*c))^(1/2)*(c + d*x)^(1/2)*(a^2*d*f
- 2*b^2*c*e + a*b*d*e))/(a^2*b*(a^2*b^4*c - a^3*b^3*d)))*(-b^3*(a*d - b*c))
^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(a^2*b^4*c - a^3*b^3*d)) + (2*(c
 + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 8*b^4*c^2*d^2*e^2 + 2*a^3*b*
d^4*e*f - 4*a*b^3*c*d^3*e^2 - 4*a^2*b^2*c*d^3*e*f))/(a^2*b))*(-b^3*(a*d - b
*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e)*1i)/(2*(a^2*b^4*c - a^3*b^3*d))
- (((((2*(2*a^4*b^3*c*d^3*e - 2*a^5*b^2*c*d^3*f))/(a^3*b) - ((4*a^5*b^3*d^3
 - 8*a^4*b^4*c*d^2)*(-b^3*(a*d - b*c))^(1/2)*(c + d*x)^(1/2)*(a^2*d*f - 2*b
^2*c*e + a*b*d*e))/(a^2*b*(a^2*b^4*c - a^3*b^3*d)))*(-b^3*(a*d - b*c))^(1/2
)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(a^2*b^4*c - a^3*b^3*d)) - (2*(c + d*
x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 8*b^4*c^2*d^2*e^2 + 2*a^3*b*d^4*e
*f - 4*a*b^3*c*d^3*e^2 - 4*a^2*b^2*c*d^3*e*f))/(a^2*b))*(-b^3*(a*d - b*c))^
(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e)*1i)/(2*(a^2*b^4*c - a^3*b^3*d)))/((4*
(a*b^2*c*d^4*e^3 - 2*b^3*c^2*d^3*e^3 + a^3*c*d^4*e*f^2 - 2*a*b^2*c^2*d^3*e^
2*f + 2*a^2*b*c*d^4*e^2*f))/(a^3*b) + (((((2*(2*a^4*b^3*c*d^3*e - 2*a^5*b^2
*c*d^3*f))/(a^3*b) + ((4*a^5*b^3*d^3 - 8*a^4*b^4*c*d^2)*(-b^3*(a*d - b*c))^
(1/2)*(c + d*x)^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(a^2*b*(a^2*b^4*c -
a^3*b^3*d)))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(
a^2*b^4*c - a^3*b^3*d)) + (2*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2
 + 8*b^4*c^2*d^2*e^2 + 2*a^3*b*d^4*e*f - 4*a*b^3*c*d^3*e^2 - 4*a^2*b^2*c*d^
3*e*f))/(a^2*b))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/
(2*(a^2*b^4*c - a^3*b^3*d)) + (((((2*(2*a^4*b^3*c*d^3*e - 2*a^5*b^2*c*d^3*f
))/(a^3*b) - ((4*a^5*b^3*d^3 - 8*a^4*b^4*c*d^2)*(-b^3*(a*d - b*c))^(1/2)*(c
 + d*x)^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(a^2*b*(a^2*b^4*c - a^3*b^3*
d)))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(a^2*b^4*
c - a^3*b^3*d)) - (2*(c + d*x)^(1/2)*(a^4*d^4*f^2 + a^2*b^2*d^4*e^2 + 8*b^4
*c^2*d^2*e^2 + 2*a^3*b*d^4*e*f - 4*a*b^3*c*d^3*e^2 - 4*a^2*b^2*c*d^3*e*f))/
(a^2*b))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d*e))/(2*(a^2*
b^4*c - a^3*b^3*d))))*(-b^3*(a*d - b*c))^(1/2)*(a^2*d*f - 2*b^2*c*e + a*b*d
*e)*1i)/(a^2*b^4*c - a^3*b^3*d) - (2*c^(1/2)*e*atanh((4*c^(1/2)*d^4*e*f^2*(
c + d*x)^(1/2))/(4*c*d^4*e*f^2 + (4*b^2*c*d^4*e^3)/a^2 - (16*b^2*c^2*d^3*e^
2*f)/a^2 + (8*b*c*d^4*e^2*f)/a) + (8*c^(1/2)*d^4*e^2*f*(c + d*x)^(1/2))/(8*
c*d^4*e^2*f + (4*b*c*d^4*e^3)/a - (16*b*c^2*d^3*e^2*f)/a + (4*a*c*d^4*e*f^2
)/b) + (4*b*c^(1/2)*d^4*e^3*(c + d*x)^(1/2))/(4*b*c*d^4*e^3 + 8*a*c*d^4*e^2
*f - 16*b*c^2*d^3*e^2*f + (4*a^2*c*d^4*e*f^2)/b) - (16*b*c^(3/2)*d^3*e^2*f*
(c + d*x)^(1/2))/(4*b*c*d^4*e^3 + 8*a*c*d^4*e^2*f - 16*b*c^2*d^3*e^2*f + (4
*a^2*c*d^4*e*f^2)/b)))/a^2 - ((a*d*f - b*d*e)*(c + d*x)^(1/2))/(a*b*(a*d -
b*c + b*(c + d*x)))
```

sympy [B] time = 133.87, size = 1204, normalized size = 9.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x/(b*x+a)**2,x)
```

```
[Out] -2*a*d**2*f*sqrt(c + d*x)/(2*a**2*b*d**2 - 2*a*b**2*c*d + 2*a*b**2*d**2*x -
 2*b**3*c*d*x) + a*d**2*f*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-
1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*s
qrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*b) - a*d**2*f*sqrt(-1/(b*(a*
d - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1
/(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x
))/(2*b) - 2*b*c*d*e*sqrt(c + d*x)/(2*a**3*d**2 - 2*a**2*b*c*d + 2*a**2*b*d
**2*x - 2*a*b**2*c*d*x) - c*d*f*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*
sqrt(-1/(b*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*
c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + c*d*f*sqrt(-1/(b*(a*d
 - b*c)**3))*log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/
(b*(a*d - b*c)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x)
```

```
)/2 + 2*c*d*f*sqrt(c + d*x)/(2*a**2*d**2 - 2*a*b*c*d + 2*a*b*d**2*x - 2*b**
2*c*d*x) - d**2*e*sqrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*
d - b*c)**3)) + 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(
b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + d**2*e*sqrt(-1/(b*(a*d - b*c)**3))*
log(a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c
)**3)) + b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/2 + 2*d**2*
e*sqrt(c + d*x)/(2*a**2*d**2 - 2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c*d*x) + 2
*d*f*atan(sqrt(c + d*x)/sqrt(a*d/b - c))/(b**2*sqrt(a*d/b - c)) + b*c*d*e*s
qrt(-1/(b*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(-1/(b*(a*d - b*c)**3)) + 2*a
*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) - b**2*c**2*sqrt(-1/(b*(a*d - b*c)**3))
 + sqrt(c + d*x))/(2*a) - b*c*d*e*sqrt(-1/(b*(a*d - b*c)**3))*log(a**2*d**2*
sqrt(-1/(b*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(-1/(b*(a*d - b*c)**3)) + b**2*
c**2*sqrt(-1/(b*(a*d - b*c)**3)) + sqrt(c + d*x))/(2*a) - 2*c*e*atan(sqrt(c
 + d*x)/sqrt(a*d/b - c))/(a**2*sqrt(a*d/b - c)) + 2*c*e*atan(sqrt(c + d*x)/
sqrt(-c))/(a**2*sqrt(-c))
```

3.14 $\int \frac{\sqrt{c+dx}\,(e+fx)}{x(a+bx)^3}\,dx$

Optimal. Leaf size=208

$$-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^3}+\frac{\sqrt{c+dx}\,\left(a^2(-d)f-3abde+4b^2ce\right)}{4a^2b(a+bx)(bc-ad)}+\frac{\left(a^3d^2f+3a^2bd^2e-12ab^2cde+8b^3c^2e\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4a^3b^{3/2}(bc-ad)^{3/2}}$$

Rubi [A] time = 0.27, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {149, 151, 156, 63, 208}

$$\frac{\left(3a^2bd^2e+a^3d^2f-12ab^2cde+8b^3c^2e\right)\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)}{4a^3b^{3/2}(bc-ad)^{3/2}}+\frac{\sqrt{c+dx}\,\left(a^2(-d)f-3abde+4b^2ce\right)}{4a^2b(a+bx)(bc-ad)}-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^3}+\frac{\sqrt{c+dx}\,(be-af)}{2ab(a+bx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^3),x]
```

```
[Out] ((b*e - a*f)*Sqrt[c + d*x])/(2*a*b*(a + b*x)^2) + ((4*b^2*c*e - 3*a*b*d*e -
 a^2*d*f)*Sqrt[c + d*x])/(4*a^2*b*(b*c - a*d)*(a + b*x)) - (2*Sqrt[c]*e*Arc
Tanh[Sqrt[c + d*x]/Sqrt[c]])/a^3 + ((8*b^3*c^2*e - 12*a*b^2*c*d*e + 3*a^2*b
*d^2*e + a^3*d^2*f)*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]])/(4*a^
3*b^(3/2)*(b*c - a*d)^(3/2))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{c+dx}\,(e+fx)}{x(a+bx)^3}\,dx &= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} - \frac{\int \frac{-2bce-\frac{1}{2}d(3be+af)x}{x(a+bx)^2\sqrt{c+dx}}\,dx}{2ab} \\
&= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{\left(4b^2ce-3abde-a^2df\right)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} - \frac{\int \frac{-2bc(bc-ad)e-\frac{1}{4}d\left(4b^2ce-ad(3be+\cdots\right.}{x(a+bx)\sqrt{c+dx}}}{2a^2b(bc-ad)} \\
&= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{\left(4b^2ce-3abde-a^2df\right)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} + \frac{(ce)\int \frac{1}{x\sqrt{c+dx}}\,dx}{a^3} - \frac{\left(8b^3c^2e\cdots\right.}{\cdots} \\
&= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{\left(4b^2ce-3abde-a^2df\right)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} + \frac{(2ce)\,\text{Subst}\left(\int \frac{1}{-\frac{c}{d}+\frac{x^2}{d}}\,dx,x,\cdots\right.}{a^3d} \\
&= \frac{(be-af)\sqrt{c+dx}}{2ab(a+bx)^2} + \frac{\left(4b^2ce-3abde-a^2df\right)\sqrt{c+dx}}{4a^2b(bc-ad)(a+bx)} - \frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^3} + \frac{\left(8\cdots\right.}{\cdots}
\end{aligned}$$

Mathematica [A] time = 0.60, size = 260, normalized size = 1.25

$$\frac{-\frac{(c+dx)^{3/2}\left(a^2df-5abde+4b^2ce\right)}{2a(a+bx)(ad-bc)}+\frac{4e(bc-ad)\left(\sqrt{c+dx}-\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)\right)}{a^2}+\frac{\left(a^3d^2f+3a^2bd^2e-12ab^2cde+8b^3c^2e\right)\left(\sqrt{b}\sqrt{c+dx}-\sqrt{bc-ad}\tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{bc-ad}}\right)\right)}{2a^2b^{3/2}(ad-bc)}+\frac{(c+dx)^{3/2}(be-af)}{(a+bx)^2}}{2a(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^3),x]
```

```
[Out] (((b*e - a*f)*(c + d*x)^(3/2))/(a + b*x)^2 - ((4*b^2*c*e - 5*a*b*d*e + a^2*
d*f)*(c + d*x)^(3/2))/(2*a*(-(b*c) + a*d)*(a + b*x)) + (4*(b*c - a*d)*e*(Sq
rt[c + d*x] - Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]))/a^2 + ((8*b^3*c^2*e
- 12*a*b^2*c*d*e + 3*a^2*b*d^2*e + a^3*d^2*f)*(Sqrt[b]*Sqrt[c + d*x] - Sqrt
[b*c - a*d]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[b*c - a*d]]))/(2*a^2*b^(3/
2)*(-(b*c) + a*d)))/(2*a*(b*c - a*d))
```

IntegrateAlgebraic [A] time = 1.08, size = 268, normalized size = 1.29

$$-\frac{2\sqrt{c}\,e\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^3}-\frac{d\sqrt{c+dx}\,\left(a^3d^2f-a^2bdf(c+dx)-a^2bcdf-5a^2bd^2e-3ab^2de(c+dx)+9ab^2cde-4b^3c^2e+4b^3ce(c+dx)\right)}{4a^2b(ad-bc)(ad+b(c+dx)-bc)^2}+\frac{\left(a^3d^2f+3a^2bd^2e-12ab^2cde+8b^3c^2e\right)\tan^{-1}\left(\frac{\sqrt{b}\sqrt{c+dx}\sqrt{ad-bc}}{bc-ad}\right)}{4a^3b^{3/2}(bc-ad)\sqrt{ad-bc}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[c + d*x]*(e + f*x))/(x*(a + b*x)^3),x]
```

```
[Out] -1/4*(d*Sqrt[c + d*x]*(-4*b^3*c^2*e + 9*a*b^2*c*d*e - 5*a^2*b*d^2*e - a^2*b
*c*d*f + a^3*d^2*f + 4*b^3*c*e*(c + d*x) - 3*a*b^2*d*e*(c + d*x) - a^2*b*d*
f*(c + d*x)))/(a^2*b*(-(b*c) + a*d)*(-(b*c) + a*d + b*(c + d*x))^2) + ((8*b
^3*c^2*e - 12*a*b^2*c*d*e + 3*a^2*b*d^2*e + a^3*d^2*f)*ArcTan[(Sqrt[b]*Sqrt
[-(b*c) + a*d]*Sqrt[c + d*x])/(b*c - a*d)])/(4*a^3*b^(3/2)*(b*c - a*d)*Sqrt
[-(b*c) + a*d]) - (2*Sqrt[c]*e*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/a^3
```

fricas [B] time = 3.16, size = 2216, normalized size = 10.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/8*((a^5*d^2*f + (a^3*b^2*d^2*f + (8*b^5*c^2 - 12*a*b^4*c*d + 3*a^2*b^3*
d^2)*e)*x^2 + (8*a^2*b^3*c^2 - 12*a^3*b^2*c*d + 3*a^4*b*d^2)*e + 2*(a^4*b*d
^2*f + (8*a*b^4*c^2 - 12*a^2*b^3*c*d + 3*a^3*b^2*d^2)*e)*x)*sqrt(b^2*c - a*
b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c))/(b*x +
 a)) - 8*((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*e*x^2 + 2*(a*b^5*c^2 - 2*a^
2*b^4*c*d + a^3*b^3*d^2)*e*x + (a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*
e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 2*((6*a^2*b^4*c^2
 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e - (2*a^3*b^3*c^2 - 3*a^4*b^2*c*d + a^5
*b*d^2)*f + ((4*a*b^5*c^2 - 7*a^2*b^4*c*d + 3*a^3*b^3*d^2)*e - (a^3*b^3*c*d
 - a^4*b^2*d^2)*f)*x)*sqrt(d*x + c))/(a^5*b^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2
*d^2 + (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*x^2 + 2*(a^4*b^5*c^2 - 2
*a^5*b^4*c*d + a^6*b^3*d^2)*x), -1/4*((a^5*d^2*f + (a^3*b^2*d^2*f + (8*b^5*
c^2 - 12*a*b^4*c*d + 3*a^2*b^3*d^2)*e)*x^2 + (8*a^2*b^3*c^2 - 12*a^3*b^2*c*
d + 3*a^4*b*d^2)*e + 2*(a^4*b*d^2*f + (8*a*b^4*c^2 - 12*a^2*b^3*c*d + 3*a^3
*b^2*d^2)*e)*x)*sqrt(-b^2*c + a*b*d)*arctan(sqrt(-b^2*c + a*b*d)*sqrt(d*x +
 c)/(b*d*x + b*c)) - 4*((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*e*x^2 + 2*(a*
b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*e*x + (a^2*b^4*c^2 - 2*a^3*b^3*c*d +
 a^4*b^2*d^2)*e)*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - ((6
*a^2*b^4*c^2 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e - (2*a^3*b^3*c^2 - 3*a^4*b
^2*c*d + a^5*b*d^2)*f + ((4*a*b^5*c^2 - 7*a^2*b^4*c*d + 3*a^3*b^3*d^2)*e -
(a^3*b^3*c*d - a^4*b^2*d^2)*f)*x)*sqrt(d*x + c))/(a^5*b^4*c^2 - 2*a^6*b^3*c
*d + a^7*b^2*d^2 + (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*x^2 + 2*(a^4
*b^5*c^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x), 1/8*(16*((b^6*c^2 - 2*a*b^5*c*d
 + a^2*b^4*d^2)*e*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*e*x + (
a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*e)*sqrt(-c)*arctan(sqrt(d*x + c)
*sqrt(-c)/c) - (a^5*d^2*f + (a^3*b^2*d^2*f + (8*b^5*c^2 - 12*a*b^4*c*d + 3*
a^2*b^3*d^2)*e)*x^2 + (8*a^2*b^3*c^2 - 12*a^3*b^2*c*d + 3*a^4*b*d^2)*e + 2*
(a^4*b*d^2*f + (8*a*b^4*c^2 - 12*a^2*b^3*c*d + 3*a^3*b^2*d^2)*e)*x)*sqrt(b^
2*c - a*b*d)*log((b*d*x + 2*b*c - a*d - 2*sqrt(b^2*c - a*b*d)*sqrt(d*x + c)
)/(b*x + a)) + 2*((6*a^2*b^4*c^2 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e - (2*a
^3*b^3*c^2 - 3*a^4*b^2*c*d + a^5*b*d^2)*f + ((4*a*b^5*c^2 - 7*a^2*b^4*c*d +
 3*a^3*b^3*d^2)*e - (a^3*b^3*c*d - a^4*b^2*d^2)*f)*x)*sqrt(d*x + c))/(a^5*b
^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d^2 + (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b
^4*d^2)*x^2 + 2*(a^4*b^5*c^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x), -1/4*((a^5*
d^2*f + (a^3*b^2*d^2*f + (8*b^5*c^2 - 12*a*b^4*c*d + 3*a^2*b^3*d^2)*e)*x^2
+ (8*a^2*b^3*c^2 - 12*a^3*b^2*c*d + 3*a^4*b*d^2)*e + 2*(a^4*b*d^2*f + (8*a*
b^4*c^2 - 12*a^2*b^3*c*d + 3*a^3*b^2*d^2)*e)*x)*sqrt(-b^2*c + a*b*d)*arctan
(sqrt(-b^2*c + a*b*d)*sqrt(d*x + c)/(b*d*x + b*c)) - 8*((b^6*c^2 - 2*a*b^5*
c*d + a^2*b^4*d^2)*e*x^2 + 2*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*e*x
+ (a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*e)*sqrt(-c)*arctan(sqrt(d*x +
 c)*sqrt(-c)/c) - ((6*a^2*b^4*c^2 - 11*a^3*b^3*c*d + 5*a^4*b^2*d^2)*e - (2*
a^3*b^3*c^2 - 3*a^4*b^2*c*d + a^5*b*d^2)*f + ((4*a*b^5*c^2 - 7*a^2*b^4*c*d
+ 3*a^3*b^3*d^2)*e - (a^3*b^3*c*d - a^4*b^2*d^2)*f)*x)*sqrt(d*x + c))/(a^5*
b^4*c^2 - 2*a^6*b^3*c*d + a^7*b^2*d^2 + (a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*
b^4*d^2)*x^2 + 2*(a^4*b^5*c^2 - 2*a^5*b^4*c*d + a^6*b^3*d^2)*x)]
```

giac [A] time = 1.39, size = 300, normalized size = 1.44

$$-\frac{\left(a^{3} d^{2} f + 8 \, b^{3} c^{2} e - 12 \, a b^{2} c d e + 3 \, a^{2} b d^{2} e\right) \arctan\left(\frac{\sqrt{d x + c} b}{\sqrt{-b^{2} c + a b d}}\right)}{4 \, {\left(a^{3} b^{2} c - a^{4} b d\right)} \sqrt{-b^{2} c + a b d}} + \frac{2 \, c \arctan\left(\frac{\sqrt{d x + c}}{\sqrt{-c}}\right) e}{a^{3} \sqrt{-c}} - \frac{{\left(d x + c\right)}^{\frac{3}{2}} a^{2} b d^{2} f + \sqrt{d x + c} a^{2} b c d^{2} f - \sqrt{d x + c} a^{3} d^{3} f - 4 \, {\left(d x + c\right)}^{\frac{3}{2}} b^{3} c d e + 4 \, \sqrt{d x + c} b^{3} c^{2} d e + 3 \, {\left(d x + c\right)}^{\frac{3}{2}} a b^{2} d^{2} e - 9 \, \sqrt{d x + c} a b^{2} c d^{2} e + 5 \, \sqrt{d x + c} a^{2} b d^{3} e}{4 \, {\left(a^{2} b^{2} c - a^{3} b d\right)} {\left({\left(d x + c\right)} b - b c + a d\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/4*(a^3*d^2*f + 8*b^3*c^2*e - 12*a*b^2*c*d*e + 3*a^2*b*d^2*e)*arctan(sqrt
(d*x + c)*b/sqrt(-b^2*c + a*b*d))/((a^3*b^2*c - a^4*b*d)*sqrt(-b^2*c + a*b*
d)) + 2*c*arctan(sqrt(d*x + c)/sqrt(-c))*e/(a^3*sqrt(-c)) - 1/4*((d*x + c)^
(3/2)*a^2*b*d^2*f + sqrt(d*x + c)*a^2*b*c*d^2*f - sqrt(d*x + c)*a^3*d^3*f -
 4*(d*x + c)^(3/2)*b^3*c*d*e + 4*sqrt(d*x + c)*b^3*c^2*d*e + 3*(d*x + c)^(3
/2)*a*b^2*d^2*e - 9*sqrt(d*x + c)*a*b^2*c*d^2*e + 5*sqrt(d*x + c)*a^2*b*d^3
*e)/((a^2*b^2*c - a^3*b*d)*((d*x + c)*b - b*c + a*d)^2)
```

maple [B] time = 0.02, size = 424, normalized size = 2.04

$$\frac{3\left(dx+c\right)^{\frac{3}{2}} b\,d^{2} e}{4\left(bdx+ad\right)^{2}\left(ad-bc\right)a}+\frac{3d^{2} e \arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{\left(ad-bc\right)b}}\right)}{4\left(ad-bc\right)\sqrt{\left(ad-bc\right)b}\,a}-\frac{\left(dx+c\right)^{\frac{3}{2}} b^{2} c d e}{\left(bdx+ad\right)^{2}\left(ad-bc\right)a^{2}}-\frac{3bcde \arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{\left(ad-bc\right)b}}\right)}{\left(ad-bc\right)\sqrt{\left(ad-bc\right)b}\,a^{2}}+\frac{2b^{2} c^{2} e \arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{\left(ad-bc\right)b}}\right)}{\left(ad-bc\right)\sqrt{\left(ad-bc\right)b}\,a^{3}}+\frac{d^{2} f \arctan\left(\frac{\sqrt{dx+c}\,b}{\sqrt{\left(ad-bc\right)b}}\right)}{4\left(ad-bc\right)\sqrt{\left(ad-bc\right)b}\,b}+\frac{\left(dx+c\right)^{\frac{3}{2}} d^{2} f}{4\left(bdx+ad\right)^{2}\left(ad-bc\right)}+\frac{5\sqrt{dx+c}\,d^{2} e}{4\left(bdx+ad\right)^{2} a}-\frac{\sqrt{dx+c}\,bcde}{\left(bdx+ad\right)^{2} a^{2}}-\frac{\sqrt{dx+c}\,d^{2} f}{4\left(bdx+ad\right)^{2} b}-\frac{2\sqrt{c}\,e \,\mathrm{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x)
```

```
[Out] 1/4*d^2/(b*d*x+a*d)^2/(a*d-b*c)*(d*x+c)^(3/2)*f+3/4*d^2/a/(b*d*x+a*d)^2/(a*
d-b*c)*(d*x+c)^(3/2)*b*e-d/a^2/(b*d*x+a*d)^2/(a*d-b*c)*(d*x+c)^(3/2)*b^2*c*
e-1/4*d^2/(b*d*x+a*d)^2/b*(d*x+c)^(1/2)*f+5/4*d^2/a/(b*d*x+a*d)^2*(d*x+c)^(
1/2)*e-d/a^2/(b*d*x+a*d)^2*b*(d*x+c)^(1/2)*c*e+1/4*d^2/(a*d-b*c)/b/((a*d-b*
c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*f+3/4*d^2/a/(a*d-b*
c)/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*b)*e-3*d/a^
2/(a*d-b*c)*b/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b*c)*b)^(1/2)*
b)*c*e+2/a^3/(a*d-b*c)*b^2/((a*d-b*c)*b)^(1/2)*arctan((d*x+c)^(1/2)/((a*d-b
*c)*b)^(1/2)*b)*c^2*e-2*e*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/a^3
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)^(1/2)/x/(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.54, size = 4852, normalized size = 23.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(c + d*x)^(1/2))/(x*(a + b*x)^3),x)
```

```
[Out] (c^(1/2)*e*atan(((c^(1/2)*e*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b^2*d^6*
e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 320
*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*f - 24*a^4
*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) + (c^(1/2)*e
*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*d^3*e - 9*a^7*b^4*c^
2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d) + (c
^(1/2)*e*(c + d*x)^(1/2)*(64*a^9*b^3*d^5 - 256*a^8*b^4*c*d^4 - 128*a^6*b^6*
c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(8*a^3*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2
*c*d))))/a^3)*1i)/a^3 + (c^(1/2)*e*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b
^2*d^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4*e^
2 - 320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*f -
 24*a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) - (c^
(1/2)*e*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*d^3*e - 9*a^7
*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*
d) - (c^(1/2)*e*(c + d*x)^(1/2)*(64*a^9*b^3*d^5 - 256*a^8*b^4*c*d^4 - 128*a
^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(8*a^3*(a^6*b*d^2 + a^4*b^3*c^2 - 2*
```

```
a^5*b^2*c*d))))/a^3)*1i)/a^3)/(((a^5*c*d^6*e*f^2)/4 - 12*a^2*b^3*c^2*d^5*e^
3 - 8*b^5*c^4*d^3*e^3 + 18*a*b^4*c^3*d^4*e^3 + (9*a^3*b^2*c*d^6*e^3)/4 + 2*
a^2*b^3*c^3*d^4*e^2*f - 4*a^3*b^2*c^2*d^5*e^2*f + (3*a^4*b*c*d^6*e^2*f)/2)/
(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d) + (c^(1/2)*e*(((c + d*x)^(1/2)*(a
^6*d^6*f^2 + 9*a^4*b^2*d^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 25
6*a^2*b^4*c^2*d^4*e^2 - 320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a
^3*b^3*c^2*d^4*e*f - 24*a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2
*a^5*b^2*c*d)) + (c^(1/2)*e*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b
^5*c^3*d^3*e - 9*a^7*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^
3*c^2 - 2*a^7*b^2*c*d) + (c^(1/2)*e*(c + d*x)^(1/2)*(64*a^9*b^3*d^5 - 256*a
^8*b^4*c*d^4 - 128*a^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(8*a^3*(a^6*b*d^
2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d))))/a^3))/a^3 - (c^(1/2)*e*(((c + d*x)^(1/2
)*(a^6*d^6*f^2 + 9*a^4*b^2*d^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f
+ 256*a^2*b^4*c^2*d^4*e^2 - 320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 +
16*a^3*b^3*c^2*d^4*e*f - 24*a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2
 - 2*a^5*b^2*c*d)) - (c^(1/2)*e*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a
^6*b^5*c^3*d^3*e - 9*a^7*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^
6*b^3*c^2 - 2*a^7*b^2*c*d) - (c^(1/2)*e*(c + d*x)^(1/2)*(64*a^9*b^3*d^5 - 2
56*a^8*b^4*c*d^4 - 128*a^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(8*a^3*(a^6*
b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d))))/a^3))/a^3))*2i)/a^3 - (((c + d*x)^(
1/2)*(a^2*d^2*f - 5*a*b*d^2*e + 4*b^2*c*d*e))/(4*a^2*b) - ((c + d*x)^(3/2)*
(a^2*d^2*f + 3*a*b*d^2*e - 4*b^2*c*d*e))/(4*a^2*(a*d - b*c)))/(b^2*(c + d*x
)^2 - (2*b^2*c - 2*a*b*d)*(c + d*x) + a^2*d^2 + b^2*c^2 - 2*a*b*c*d) + (ata
n((((-b^3*(a*d - b*c)^3)^(1/2)*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b^2*d
^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4*e^2 -
320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*f - 24*
a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) - ((-b^3*
(a*d - b*c)^3)^(1/2)*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*
d^3*e - 9*a^7*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 -
 2*a^7*b^2*c*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x)^(1/2)*(8*b^3*c^2*e
+ a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e)*(64*a^9*b^3*d^5 - 256*a^8*b^4
*c*d^4 - 128*a^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(64*(a^6*b*d^2 + a^4*b
^3*c^2 - 2*a^5*b^2*c*d)*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^
5*b^4*c*d^2)))*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e))/
(8*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)))*(8*b^3
*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e)*1i)/(8*(a^3*b^6*c^3 -
a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)) + ((-b^3*(a*d - b*c)^3)^(
1/2)*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b^2*d^6*e^2 + 128*b^6*c^4*d^2*e
^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 320*a*b^5*c^3*d^3*e^2 - 72
*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*f - 24*a^4*b^2*c*d^5*e*f))/(8*(a^
6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) + ((-b^3*(a*d - b*c)^3)^(1/2)*((5*a
^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*d^3*e - 9*a^7*b^4*c^2*d^4*
e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d) + ((-b^3*(
a*d - b*c)^3)^(1/2)*(c + d*x)^(1/2)*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*
e - 12*a*b^2*c*d*e)*(64*a^9*b^3*d^5 - 256*a^8*b^4*c*d^4 - 128*a^6*b^6*c^3*d
^2 + 320*a^7*b^5*c^2*d^3))/(64*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)*(a
^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)))*(8*b^3*c^2*
e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e))/(8*(a^3*b^6*c^3 - a^6*b^3*
d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)))*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2
*b*d^2*e - 12*a*b^2*c*d*e)*1i)/(8*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^
2*d + 3*a^5*b^4*c*d^2)))/(((a^5*c*d^6*e*f^2)/4 - 12*a^2*b^3*c^2*d^5*e^3 - 8
*b^5*c^4*d^3*e^3 + 18*a*b^4*c^3*d^4*e^3 + (9*a^3*b^2*c*d^6*e^3)/4 + 2*a^2*b
^3*c^3*d^4*e^2*f - 4*a^3*b^2*c^2*d^5*e^2*f + (3*a^4*b*c*d^6*e^2*f)/2)/(a^8*
b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d) - ((-b^3*(a*d - b*c)^3)^(1/2)*(((c + d
*x)^(1/2)*(a^6*d^6*f^2 + 9*a^4*b^2*d^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*
d^6*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^
5*e^2 + 16*a^3*b^3*c^2*d^4*e*f - 24*a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4
*b^3*c^2 - 2*a^5*b^2*c*d)) - ((-b^3*(a*d - b*c)^3)^(1/2)*((5*a^8*b^3*c*d^5*
e - a^9*b^2*c*d^5*f + 4*a^6*b^5*c^3*d^3*e - 9*a^7*b^4*c^2*d^4*e + a^8*b^3*c
```

```
^2*d^4*f)/(a^8*b*d^2 + a^6*b^3*c^2 - 2*a^7*b^2*c*d) - ((-b^3*(a*d - b*c)^3)
^(1/2)*(c + d*x)^(1/2)*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*
c*d*e)*(64*a^9*b^3*d^5 - 256*a^8*b^4*c*d^4 - 128*a^6*b^6*c^3*d^2 + 320*a^7*
b^5*c^2*d^3))/(64*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)*(a^3*b^6*c^3 -
a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)))*(8*b^3*c^2*e + a^3*d^2*f
 + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e))/(8*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b
^5*c^2*d + 3*a^5*b^4*c*d^2)))*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12
*a*b^2*c*d*e))/(8*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*
c*d^2)) + ((-b^3*(a*d - b*c)^3)^(1/2)*(((c + d*x)^(1/2)*(a^6*d^6*f^2 + 9*a^
4*b^2*d^6*e^2 + 128*b^6*c^4*d^2*e^2 + 6*a^5*b*d^6*e*f + 256*a^2*b^4*c^2*d^4
*e^2 - 320*a*b^5*c^3*d^3*e^2 - 72*a^3*b^3*c*d^5*e^2 + 16*a^3*b^3*c^2*d^4*e*
f - 24*a^4*b^2*c*d^5*e*f))/(8*(a^6*b*d^2 + a^4*b^3*c^2 - 2*a^5*b^2*c*d)) +
((-b^3*(a*d - b*c)^3)^(1/2)*((5*a^8*b^3*c*d^5*e - a^9*b^2*c*d^5*f + 4*a^6*b
^5*c^3*d^3*e - 9*a^7*b^4*c^2*d^4*e + a^8*b^3*c^2*d^4*f)/(a^8*b*d^2 + a^6*b^
3*c^2 - 2*a^7*b^2*c*d) + ((-b^3*(a*d - b*c)^3)^(1/2)*(c + d*x)^(1/2)*(8*b^3
*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e)*(64*a^9*b^3*d^5 - 256*
a^8*b^4*c*d^4 - 128*a^6*b^6*c^3*d^2 + 320*a^7*b^5*c^2*d^3))/(64*(a^6*b*d^2
+ a^4*b^3*c^2 - 2*a^5*b^2*c*d)*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d
 + 3*a^5*b^4*c*d^2)))*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c
*d*e))/(8*(a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2)))
*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e))/(8*(a^3*b^6*c^
3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2))))*(-b^3*(a*d - b*c)^3
)^(1/2)*(8*b^3*c^2*e + a^3*d^2*f + 3*a^2*b*d^2*e - 12*a*b^2*c*d*e)*1i)/(4*(
a^3*b^6*c^3 - a^6*b^3*d^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(d*x+c)**(1/2)/x/(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.15 $\int \frac{\sqrt{a+bx}\,(c+dx)^3(e+fx)}{x}\,dx$

Optimal. Leaf size=226

$$-\frac{2(a+bx)^{3/2}\left(2\left(8a^3d^3f-12a^2bd^2(3cf+de)+3ab^2cd(16cf+21de)-5b^3c^2(4cf+27de)\right)-3bdx\left(4(bc-ad)(-\right.\right.}{315b^4}$$

Rubi [A] time = 0.25, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {153, 147, 50, 63, 208}

$$-\frac{2(a+bx)^{3/2}\left(2\left(-12a^2bd^2(3cf+de)+8a^3d^3f+3ab^2cd(16cf+21de)-5b^3c^2(4cf+27de)\right)-3bdx\left(4(bc-ad)(-2adf+2bcf+3bde)+21b^2cde\right)\right)}{315b^4}+\frac{2(a+bx)^{3/2}(c+dx)^2(-2adf+2bcf+3bde)}{21b^2}+2c^3e\sqrt{a+bx}-2\sqrt{a}\,c^3e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{2f(a+bx)^{3/2}(c+dx)^3}{9b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(c + d*x)^3*(e + f*x))/x,x]
```

```
[Out] 2*c^3*e*Sqrt[a + b*x] + (2*(3*b*d*e + 2*b*c*f - 2*a*d*f)*(a + b*x)^(3/2)*(c
 + d*x)^2)/(21*b^2) + (2*f*(a + b*x)^(3/2)*(c + d*x)^3)/(9*b) - (2*(a + b*x
)^(3/2)*(2*(8*a^3*d^3*f - 12*a^2*b*d^2*(d*e + 3*c*f) - 5*b^3*c^2*(27*d*e +
4*c*f) + 3*a*b^2*c*d*(21*d*e + 16*c*f)) - 3*b*d*(21*b^2*c*d*e + 4*(b*c - a*
d)*(3*b*d*e + 2*b*c*f - 2*a*d*f))*x))/(315*b^4) - 2*Sqrt[a]*c^3*e*ArcTanh[S
qrt[a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
```

```
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\,(c+dx)^3(e+fx)}{x}\,dx = \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} + \frac{2\int \frac{\sqrt{a+bx}\,(c+dx)^2\left(\frac{9bce}{2}+\frac{3}{2}(3bde+2bcf-2adf)x\right)}{x}\,dx}{9b}$$

$$= \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} + \frac{4\int}{}$$

$$= \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}(c+dx)^3}{9b} - \frac{2(a}{}$$

$$= 2c^3e\sqrt{a+bx} + \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}}{9b}$$

$$= 2c^3e\sqrt{a+bx} + \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}}{9b}$$

$$= 2c^3e\sqrt{a+bx} + \frac{2(3bde+2bcf-2adf)(a+bx)^{3/2}(c+dx)^2}{21b^2} + \frac{2f(a+bx)^{3/2}}{9b}$$

Mathematica [A] time = 0.28, size = 204, normalized size = 0.90

$$\frac{2\left(3be\left(35d(a+bx)^{3/2}\left(a^2d^2-3abcd+3b^2c^2\right)+105b^3c^3\sqrt{a+bx}-105\sqrt{a}\,b^3c^3\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+21d^2(a+bx)^{5/2}(3bc-2ad)+15d^3(a+bx)^{7/2}\right)+f(a+bx)^{3/2}\left(135d^2(a+bx)^2(bc-ad)+189d(a+bx)(bc-ad)^2+105(bc-ad)^3+35d^3(a+bx)^3\right)\right)}{315b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(c + d*x)^3*(e + f*x))/x,x]
```

```
[Out] (2*(f*(a + b*x)^(3/2)*(105*(b*c - a*d)^3 + 189*d*(b*c - a*d)^2*(a + b*x) +
135*d^2*(b*c - a*d)*(a + b*x)^2 + 35*d^3*(a + b*x)^3) + 3*b*e*(105*b^3*c^3*
Sqrt[a + b*x] + 35*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*(a + b*x)^(3/2) + 21
*d^2*(3*b*c - 2*a*d)*(a + b*x)^(5/2) + 15*d^3*(a + b*x)^(7/2) - 105*Sqrt[a]
*b^3*c^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])))/(315*b^4)
```

IntegrateAlgebraic [A] time = 0.18, size = 343, normalized size = 1.52

$$\frac{2\left(-105a^3d^3f(a+bx)^{3/2}+315a^2bcd^2f(a+bx)^{3/2}+105a^2bd^3e(a+bx)^{3/2}+189a^2d^3f(a+bx)^{5/2}+315b^4c^3e\sqrt{a+bx}+105b^3c^3f(a+bx)^{3/2}+315b^3c^2de(a+bx)^{3/2}+189b^2c^2df(a+bx)^{5/2}-315ab^2c^2df(a+bx)^{3/2}+189b^2cd^2e(a+bx)^{5/2}-315ab^2cd^2e(a+bx)^{3/2}+135bcd^2f(a+bx)^{7/2}-378abcd^2f(a+bx)^{5/2}+45bd^3e(a+bx)^{7/2}-126abd^3e(a+bx)^{5/2}+35d^3f(a+bx)^{9/2}-135ad^3f(a+bx)^{7/2}\right)}{315b^4}-2\sqrt{a}\,c^3e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x]*(c + d*x)^3*(e + f*x))/x,x]
```

```
[Out] (2*(315*b^4*c^3*e*Sqrt[a + b*x] + 315*b^3*c^2*d*e*(a + b*x)^(3/2) - 315*a*b
^2*c*d^2*e*(a + b*x)^(3/2) + 105*a^2*b*d^3*e*(a + b*x)^(3/2) + 105*b^3*c^3*
f*(a + b*x)^(3/2) - 315*a*b^2*c^2*d*f*(a + b*x)^(3/2) + 315*a^2*b*c*d^2*f*(
a + b*x)^(3/2) - 105*a^3*d^3*f*(a + b*x)^(3/2) + 189*b^2*c*d^2*e*(a + b*x)^
(5/2) - 126*a*b*d^3*e*(a + b*x)^(5/2) + 189*b^2*c^2*d*f*(a + b*x)^(5/2) - 3
```

```
78*a*b*c*d^2*f*(a + b*x)^(5/2) + 189*a^2*d^3*f*(a + b*x)^(5/2) + 45*b*d^3*e
*(a + b*x)^(7/2) + 135*b*c*d^2*f*(a + b*x)^(7/2) - 135*a*d^3*f*(a + b*x)^(7
/2) + 35*d^3*f*(a + b*x)^(9/2)))/(315*b^4) - 2*Sqrt[a]*c^3*e*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]]
```

fricas [A] time = 1.40, size = 641, normalized size = 2.84

$$\left[\frac{315\,\sqrt{a}b^{4}c^{3}e\log\left(\frac{bx-2\,\sqrt{bx+a}\sqrt{a}+2\,a}{x}\right)+2\left(35\,b^{4}d^{3}fx^{4}+5\left(9\,b^{4}d^{3}e+\left(27\,b^{4}cd^{2}+ab^{3}d^{3}\right)f\right)x^{3}+3\left(3\left(21\,b^{4}cd^{2}+ab^{3}d^{3}\right)e+\left(63\,b^{4}c^{2}d+9\,ab^{3}cd^{2}-2\,a^{2}b^{2}d^{3}\right)f\right)x^{2}+3\,105\,b^{4}c^{3}+105\,ab^{3}c^{2}d-42\,a^{2}b^{2}cd^{2}+8\,a^{3}bd^{3}\right)e+\left(105\,ab^{3}c^{3}-126\,a^{2}b^{2}c^{2}d+72\,a^{3}bcd^{2}-16\,a^{4}d^{3}\right)f+\left(3\left(105\,b^{4}c^{2}d+21\,ab^{3}cd^{2}-4\,a^{2}b^{2}d^{3}\right)e+\left(105\,b^{4}c^{3}+63\,ab^{3}c^{2}d-36\,a^{2}b^{2}cd^{2}+8\,a^{3}bd^{3}\right)f\right)x\right)\sqrt{bx+a}}{315\,b^{4}},\ \frac{2\left(315\,\sqrt{-a}b^{4}c^{3}e\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(35\,b^{4}d^{3}fx^{4}+5\left(9\,b^{4}d^{3}e+\left(27\,b^{4}cd^{2}+ab^{3}d^{3}\right)f\right)x^{3}+3\left(3\left(21\,b^{4}cd^{2}+ab^{3}d^{3}\right)e+\left(63\,b^{4}c^{2}d+9\,ab^{3}cd^{2}-2\,a^{2}b^{2}d^{3}\right)f\right)x^{2}+3\left(105\,b^{4}c^{3}+105\,ab^{3}c^{2}d-42\,a^{2}b^{2}cd^{2}+8\,a^{3}bd^{3}\right)e+\left(105\,ab^{3}c^{3}-126\,a^{2}b^{2}c^{2}d+72\,a^{3}bcd^{2}-16\,a^{4}d^{3}\right)f+\left(3\left(105\,b^{4}c^{2}d+21\,ab^{3}cd^{2}-4\,a^{2}b^{2}d^{3}\right)e+\left(105\,b^{4}c^{3}+63\,ab^{3}c^{2}d-36\,a^{2}b^{2}cd^{2}+8\,a^{3}bd^{3}\right)f\right)x\right)\sqrt{bx+a}\right)}{315\,b^{4}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/315*(315*sqrt(a)*b^4*c^3*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
+ 2*(35*b^4*d^3*f*x^4 + 5*(9*b^4*d^3*e + (27*b^4*c*d^2 + a*b^3*d^3)*f)*x^3
+ 3*(3*(21*b^4*c*d^2 + a*b^3*d^3)*e + (63*b^4*c^2*d + 9*a*b^3*c*d^2 - 2*a^2
*b^2*d^3)*f)*x^2 + 3*(105*b^4*c^3 + 105*a*b^3*c^2*d - 42*a^2*b^2*c*d^2 + 8*
a^3*b*d^3)*e + (105*a*b^3*c^3 - 126*a^2*b^2*c^2*d + 72*a^3*b*c*d^2 - 16*a^4
*d^3)*f + (3*(105*b^4*c^2*d + 21*a*b^3*c*d^2 - 4*a^2*b^2*d^3)*e + (105*b^4*
c^3 + 63*a*b^3*c^2*d - 36*a^2*b^2*c*d^2 + 8*a^3*b*d^3)*f)*x)*sqrt(b*x + a))
/b^4, 2/315*(315*sqrt(-a)*b^4*c^3*e*arctan(sqrt(b*x + a)*sqrt(-a)/a) + (35*
b^4*d^3*f*x^4 + 5*(9*b^4*d^3*e + (27*b^4*c*d^2 + a*b^3*d^3)*f)*x^3 + 3*(3*(
21*b^4*c*d^2 + a*b^3*d^3)*e + (63*b^4*c^2*d + 9*a*b^3*c*d^2 - 2*a^2*b^2*d^3
)*f)*x^2 + 3*(105*b^4*c^3 + 105*a*b^3*c^2*d - 42*a^2*b^2*c*d^2 + 8*a^3*b*d^
3)*e + (105*a*b^3*c^3 - 126*a^2*b^2*c^2*d + 72*a^3*b*c*d^2 - 16*a^4*d^3)*f
+ (3*(105*b^4*c^2*d + 21*a*b^3*c*d^2 - 4*a^2*b^2*d^3)*e + (105*b^4*c^3 + 63
*a*b^3*c^2*d - 36*a^2*b^2*c*d^2 + 8*a^3*b*d^3)*f)*x)*sqrt(b*x + a))/b^4]
```

giac [A] time = 1.39, size = 338, normalized size = 1.50

$$\frac{2\,ac^{3}\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}}+\frac{2\left(105\,(bx+a)^{\frac{3}{2}}b^{35}c^{3}f+189\,(bx+a)^{\frac{5}{2}}b^{34}c^{2}df-315\,(bx+a)^{\frac{3}{2}}ab^{34}c^{2}df+135\,(bx+a)^{\frac{7}{2}}b^{33}cd^{2}f-378\,(bx+a)^{\frac{5}{2}}ab^{33}cd^{2}f+315\,(bx+a)^{\frac{3}{2}}a^{2}b^{33}cd^{2}f+35\,(bx+a)^{\frac{9}{2}}b^{32}d^{3}f-135\,(bx+a)^{\frac{7}{2}}ab^{32}d^{3}f+189\,(bx+a)^{\frac{5}{2}}a^{2}b^{32}d^{3}f-105\,(bx+a)^{\frac{3}{2}}a^{3}b^{32}d^{3}f+315\,\sqrt{bx+a}\,b^{36}c^{3}e+315\,(bx+a)^{\frac{3}{2}}b^{35}c^{2}de+189\,(bx+a)^{\frac{5}{2}}b^{34}cd^{2}e-315\,(bx+a)^{\frac{3}{2}}ab^{34}cd^{2}e+45\,(bx+a)^{\frac{7}{2}}b^{33}d^{3}e-126\,(bx+a)^{\frac{5}{2}}ab^{33}d^{3}e+105\,(bx+a)^{\frac{3}{2}}a^{2}b^{33}d^{3}e\right)}{315\,b^{36}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c^3*arctan(sqrt(b*x + a)/sqrt(-a))*e/sqrt(-a) + 2/315*(105*(b*x + a)^(3
/2)*b^35*c^3*f + 189*(b*x + a)^(5/2)*b^34*c^2*d*f - 315*(b*x + a)^(3/2)*a*b
^34*c^2*d*f + 135*(b*x + a)^(7/2)*b^33*c*d^2*f - 378*(b*x + a)^(5/2)*a*b^33
*c*d^2*f + 315*(b*x + a)^(3/2)*a^2*b^33*c*d^2*f + 35*(b*x + a)^(9/2)*b^32*d
^3*f - 135*(b*x + a)^(7/2)*a*b^32*d^3*f + 189*(b*x + a)^(5/2)*a^2*b^32*d^3*
f - 105*(b*x + a)^(3/2)*a^3*b^32*d^3*f + 315*sqrt(b*x + a)*b^36*c^3*e + 315
*(b*x + a)^(3/2)*b^35*c^2*d*e + 189*(b*x + a)^(5/2)*b^34*c*d^2*e - 315*(b*x
 + a)^(3/2)*a*b^34*c*d^2*e + 45*(b*x + a)^(7/2)*b^33*d^3*e - 126*(b*x + a)^
(5/2)*a*b^33*d^3*e + 105*(b*x + a)^(3/2)*a^2*b^33*d^3*e)/b^36
```

maple [A] time = 0.01, size = 301, normalized size = 1.33

$$\frac{-2\sqrt{a}\,b^{4}c^{3}e\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\,b^{4}c^{3}e-\frac{2(bx+a)^{\frac{3}{2}}a^{3}d^{3}f}{3}+2\,(bx+a)^{\frac{3}{2}}a^{2}bcd^{2}f+\frac{2(bx+a)^{\frac{3}{2}}a^{2}bd^{3}e}{3}-2\,(bx+a)^{\frac{3}{2}}ab^{2}c^{2}df-2\,(bx+a)^{\frac{3}{2}}ab^{2}cd^{2}e+\frac{2(bx+a)^{\frac{3}{2}}b^{3}c^{3}f}{3}+2\,(bx+a)^{\frac{3}{2}}b^{3}c^{2}de+\frac{6(bx+a)^{\frac{5}{2}}a^{2}d^{3}f}{5}-\frac{12(bx+a)^{\frac{5}{2}}abcd^{2}f}{5}-\frac{4(bx+a)^{\frac{5}{2}}abd^{3}e}{5}+\frac{6(bx+a)^{\frac{5}{2}}b^{2}c^{2}df}{5}+\frac{6(bx+a)^{\frac{5}{2}}b^{2}cd^{2}e}{5}-\frac{6(bx+a)^{\frac{7}{2}}ad^{3}f}{7}+\frac{6(bx+a)^{\frac{7}{2}}bcd^{2}f}{7}+\frac{2(bx+a)^{\frac{7}{2}}bd^{3}e}{7}+\frac{2(bx+a)^{\frac{9}{2}}d^{3}f}{9}}{b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x)
```

```
[Out] 2/b^4*(1/9*f*d^3*(b*x+a)^(9/2)-3/7*(b*x+a)^(7/2)*a*d^3*f+3/7*(b*x+a)^(7/2)*
b*c*d^2*f+1/7*(b*x+a)^(7/2)*b*d^3*e+3/5*(b*x+a)^(5/2)*a^2*d^3*f-6/5*(b*x+a)
^(5/2)*a*b*c*d^2*f-2/5*(b*x+a)^(5/2)*a*b*d^3*e+3/5*(b*x+a)^(5/2)*b^2*c^2*d*
f+3/5*(b*x+a)^(5/2)*b^2*c*d^2*e-1/3*(b*x+a)^(3/2)*a^3*d^3*f+(b*x+a)^(3/2)*a
^2*b*c*d^2*f+1/3*(b*x+a)^(3/2)*a^2*b*d^3*e-(b*x+a)^(3/2)*a*b^2*c^2*d*f-(b*x
+a)^(3/2)*a*b^2*c*d^2*e+1/3*(b*x+a)^(3/2)*b^3*c^3*f+(b*x+a)^(3/2)*b^3*c^2*d
*e+b^4*c^3*e*(b*x+a)^(1/2)-a^(1/2)*b^4*c^3*e*arctanh((b*x+a)^(1/2)/a^(1/2))
)
```

maxima [A] time = 0.97, size = 238, normalized size = 1.05

$$\sqrt{a}\,c^{3}e\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2\left(315\,\sqrt{bx+a}\,b^{4}c^{3}e+35\,(bx+a)^{\frac{9}{2}}d^{3}f+45\left(bd^{3}e+3\left(bcd^{2}-ad^{3}\right)f\right)(bx+a)^{\frac{7}{2}}+63\left(\left(3\,b^{2}cd^{2}-2\,abd^{3}\right)e+3\left(b^{2}c^{2}d-2\,abcd^{2}+a^{2}d^{3}\right)f\right)(bx+a)^{\frac{5}{2}}+105\left(\left(3\,b^{3}c^{2}d-3\,ab^{2}cd^{2}+a^{2}bd^{3}\right)e+\left(b^{3}c^{3}-3\,ab^{2}c^{2}d+3\,a^{2}bcd^{2}-a^{3}d^{3}\right)f\right)(bx+a)^{\frac{3}{2}}\right)}{315\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*c^3*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/
315*(315*sqrt(b*x + a)*b^4*c^3*e + 35*(b*x + a)^(9/2)*d^3*f + 45*(b*d^3*e +
 3*(b*c*d^2 - a*d^3)*f)*(b*x + a)^(7/2) + 63*((3*b^2*c*d^2 - 2*a*b*d^3)*e +
 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*f)*(b*x + a)^(5/2) + 105*((3*b^3*c^2
*d - 3*a*b^2*c*d^2 + a^2*b*d^3)*e + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^
2 - a^3*d^3)*f)*(b*x + a)^(3/2))/b^4
```

mupad [B] time = 2.53, size = 413, normalized size = 1.83

$$\left(\frac{2bd^3e-8ad^3f+6bcd^2f}{7b^4}+\frac{2ad^3f}{7b^4}\right)(a+bx)^{7/2}+\left(\frac{a\left(\frac{2bd^3e-8ad^3f+6bcd^2f}{b^4}+\frac{2ad^3f}{b^4}\right)}{5}-\frac{6d\,(ad-bc)\left(bcf-2adf+bde\right)}{5b^4}\right)(a+bx)^{5/2}+\left(a\left(a\left(a\left(\frac{2bd^3e-8ad^3f+6bcd^2f}{b^4}+\frac{2ad^3f}{b^4}\right)-\frac{6d\,(ad-bc)\left(bcf-2adf+bde\right)}{b^4}\right)+\frac{2(ad-bc)^2\left(bcf-4adf+3bde\right)}{b^4}\right)+\frac{2(ad-bc)^3\left(af-be\right)}{b^4}\right)\sqrt{a+bx}+\left(\frac{a\left(a\left(\frac{2bd^3e-8ad^3f+6bcd^2f}{b^4}+\frac{2ad^3f}{b^4}\right)-\frac{6d(ad-bc)\left(bcf-2adf+bde\right)}{b^4}\right)}{3}+\frac{2(ad-bc)^2\left(bcf-4adf+3bde\right)}{3b^4}\right)(a+bx)^{3/2}+\frac{2d^3f\,(a+bx)^{9/2}}{9b^4}+\sqrt{a}\,c^3e\,\mathrm{atan}\left(\frac{\sqrt{a+bx}\,1\mathrm{i}}{\sqrt{a}}\right)2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2)*(c + d*x)^3)/x,x)
```

```
[Out] ((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/(7*b^4) + (2*a*d^3*f)/(7*b^4))*(a +
b*x)^(7/2) + ((a*((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/b^4 + (2*a*d^3*f)/b
^4))/5 - (6*d*(a*d - b*c)*(b*c*f - 2*a*d*f + b*d*e))/(5*b^4))*(a + b*x)^(5/
2) + (a*(a*(a*((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/b^4 + (2*a*d^3*f)/b^4)
 - (6*d*(a*d - b*c)*(b*c*f - 2*a*d*f + b*d*e))/b^4) + (2*(a*d - b*c)^2*(b*c
*f - 4*a*d*f + 3*b*d*e))/b^4) + (2*(a*d - b*c)^3*(a*f - b*e))/b^4)*(a + b*x
)^(1/2) + ((a*(a*((2*b*d^3*e - 8*a*d^3*f + 6*b*c*d^2*f)/b^4 + (2*a*d^3*f)/b
^4) - (6*d*(a*d - b*c)*(b*c*f - 2*a*d*f + b*d*e))/b^4))/3 + (2*(a*d - b*c)^
2*(b*c*f - 4*a*d*f + 3*b*d*e))/(3*b^4))*(a + b*x)^(3/2) + a^(1/2)*c^3*e*ata
n(((a + b*x)^(1/2)*1i)/a^(1/2))*2i + (2*d^3*f*(a + b*x)^(9/2))/(9*b^4)
```

sympy [A] time = 37.95, size = 274, normalized size = 1.21

$$\frac{2ac^{3}e\,\mathrm{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2c^{3}e\sqrt{a+bx}+\frac{2d^{3}f\,(a+bx)^{\frac{9}{2}}}{9b^{4}}+\frac{2\,(a+bx)^{\frac{7}{2}}\left(-3ad^{3}f+3bcd^{2}f+bd^{3}e\right)}{7b^{4}}+\frac{2\,(a+bx)^{\frac{5}{2}}\left(3a^{2}d^{3}f-6abcd^{2}f-2abd^{3}e+3b^{2}c^{2}df+3b^{2}cd^{2}e\right)}{5b^{4}}+\frac{2\,(a+bx)^{\frac{3}{2}}\left(-a^{3}d^{3}f+3a^{2}bcd^{2}f+a^{2}bd^{3}e-3ab^{2}c^{2}df-3ab^{2}cd^{2}e+b^{3}c^{3}f+3b^{3}c^{2}de\right)}{3b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(f*x+e)*(b*x+a)**(1/2)/x,x)
```

```
[Out] 2*a*c**3*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*c**3*e*sqrt(a + b*x) +
 2*d**3*f*(a + b*x)**(9/2)/(9*b**4) + 2*(a + b*x)**(7/2)*(-3*a*d**3*f + 3*b
*c*d**2*f + b*d**3*e)/(7*b**4) + 2*(a + b*x)**(5/2)*(3*a**2*d**3*f - 6*a*b*
c*d**2*f - 2*a*b*d**3*e + 3*b**2*c**2*d*f + 3*b**2*c*d**2*e)/(5*b**4) + 2*(
a + b*x)**(3/2)*(-a**3*d**3*f + 3*a**2*b*c*d**2*f + a**2*b*d**3*e - 3*a*b**
2*c**2*d*f - 3*a*b**2*c*d**2*e + b**3*c**3*f + 3*b**3*c**2*d*e)/(3*b**4)
```

3.16 $\int \frac{\sqrt{a+bx}\,(c+dx)^2(e+fx)}{x}\,dx$

Optimal. Leaf size=145

$$\frac{2(a+bx)^{3/2}\left(2\left(4a^2d^2f-7abd(2cf+de)+5b^2c(2cf+7de)\right)+3bdx(-4adf+4bcf+7bde)\right)}{105b^3}+2c^2e\sqrt{a+bx}-2\sqrt{a}$$

Rubi [A] time = 0.09, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {153, 147, 50, 63, 208}

$$\frac{2(a+bx)^{3/2}\left(2\left(4a^2d^2f-7abd(2cf+de)+5b^2c(2cf+7de)\right)+3bdx(-4adf+4bcf+7bde)\right)}{105b^3}+2c^2e\sqrt{a+bx}-2\sqrt{a}c^2e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+\frac{2f(a+bx)^{3/2}(c+dx)^2}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(c + d*x)^2*(e + f*x))/x,x]
```

```
[Out] 2*c^2*e*Sqrt[a + b*x] + (2*f*(a + b*x)^(3/2)*(c + d*x)^2)/(7*b) + (2*(a + b
*x)^(3/2)*(2*(4*a^2*d^2*f - 7*a*b*d*(d*e + 2*c*f) + 5*b^2*c*(7*d*e + 2*c*f)
) + 3*b*d*(7*b*d*e + 4*b*c*f - 4*a*d*f)*x))/(105*b^3) - 2*Sqrt[a]*c^2*e*Arc
Tanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
```

```
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\,(c+dx)^2(e+fx)}{x}\,dx = \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2\int \frac{\sqrt{a+bx}\,(c+dx)\left(\frac{7bce}{2}+\frac{1}{2}(7bde+4bcf-4adf)x\right)}{x}\,dx}{7b}$$

$$= \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2\left(4a^2d^2f-7abd(de+2cf)+5b^2c(\right.\right.}{105b^3}$$

$$= 2c^2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2\left(4a^2d^2f-7abd(de\right.\right.}{}$$

$$= 2c^2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2\left(4a^2d^2f-7abd(de\right.\right.}{}$$

$$= 2c^2e\sqrt{a+bx} + \frac{2f(a+bx)^{3/2}(c+dx)^2}{7b} + \frac{2(a+bx)^{3/2}\left(2\left(4a^2d^2f-7abd(de\right.\right.}{}$$

Mathematica [A] time = 0.18, size = 146, normalized size = 1.01

$$\frac{2\left(7be\left(15b^2c^2\sqrt{a+bx}-15\sqrt{a}\,b^2c^2\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)+5d(a+bx)^{3/2}(2bc-ad)+3d^2(a+bx)^{5/2}\right)+f(a+bx)^{3/2}\left(42d(a+bx)(bc-ad)+35(bc-ad)^2+15d^2(a+bx)^2\right)\right)}{105b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(c + d*x)^2*(e + f*x))/x,x]
```

```
[Out] (2*(f*(a + b*x)^(3/2)*(35*(b*c - a*d)^2 + 42*d*(b*c - a*d)*(a + b*x) + 15*d
^2*(a + b*x)^2) + 7*b*e*(15*b^2*c^2*Sqrt[a + b*x] + 5*d*(2*b*c - a*d)*(a +
b*x)^(3/2) + 3*d^2*(a + b*x)^(5/2) - 15*Sqrt[a]*b^2*c^2*ArcTanh[Sqrt[a + b*
x]/Sqrt[a]])))/(105*b^3)
```

IntegrateAlgebraic [A] time = 0.13, size = 202, normalized size = 1.39

$$\frac{2\left(35a^2d^2f(a+bx)^{3/2}+105b^3c^2e\sqrt{a+bx}+35b^2c^2f(a+bx)^{3/2}+70b^2cde(a+bx)^{3/2}+42bcdf(a+bx)^{5/2}-70abcdf(a+bx)^{3/2}+21bd^2e(a+bx)^{5/2}-35abd^2e(a+bx)^{3/2}+15d^2f(a+bx)^{7/2}-42ad^2f(a+bx)^{5/2}\right)}{105b^3}-2\sqrt{a}\,c^2e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x]*(c + d*x)^2*(e + f*x))/x,x]
```

```
[Out] (2*(105*b^3*c^2*e*Sqrt[a + b*x] + 70*b^2*c*d*e*(a + b*x)^(3/2) - 35*a*b*d^2
*e*(a + b*x)^(3/2) + 35*b^2*c^2*f*(a + b*x)^(3/2) - 70*a*b*c*d*f*(a + b*x)^
(3/2) + 35*a^2*d^2*f*(a + b*x)^(3/2) + 21*b*d^2*e*(a + b*x)^(5/2) + 42*b*c*
d*f*(a + b*x)^(5/2) - 42*a*d^2*f*(a + b*x)^(5/2) + 15*d^2*f*(a + b*x)^(7/2)
))/(105*b^3) - 2*Sqrt[a]*c^2*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

fricas [A] time = 1.07, size = 403, normalized size = 2.78

$$\left[\frac{105\sqrt{a}\,b^3c^2e\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(15b^3d^2fx^3+3\left(7b^3d^2e+\left(14b^3cd+ab^2d^2\right)f\right)x^2+7\left(15b^3c^2+10ab^2cd-2a^2bd^2\right)e+\left(35ab^2c^2-28a^2bcd+8a^3d^2\right)f+\left(7\left(10b^3cd+ab^2d^2\right)e+\left(35b^3c^2+14ab^2cd-4a^2bd^2\right)f\right)x\right)\sqrt{bx+a}}{105b^3},\ \frac{2\left(105\sqrt{-a}\,b^3c^2e\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(15b^3d^2fx^3+3\left(7b^3d^2e+\left(14b^3cd+ab^2d^2\right)f\right)x^2+7\left(15b^3c^2+10ab^2cd-2a^2bd^2\right)e+\left(35ab^2c^2-28a^2bcd+8a^3d^2\right)f+\left(7\left(10b^3cd+ab^2d^2\right)e+\left(35b^3c^2+14ab^2cd-4a^2bd^2\right)f\right)x\right)\sqrt{bx+a}\right)}{105b^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/105*(105*sqrt(a)*b^3*c^2*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)
+ 2*(15*b^3*d^2*f*x^3 + 3*(7*b^3*d^2*e + (14*b^3*c*d + a*b^2*d^2)*f)*x^2 +
7*(15*b^3*c^2 + 10*a*b^2*c*d - 2*a^2*b*d^2)*e + (35*a*b^2*c^2 - 28*a^2*b*c*
d + 8*a^3*d^2)*f + (7*(10*b^3*c*d + a*b^2*d^2)*e + (35*b^3*c^2 + 14*a*b^2*c
*d - 4*a^2*b*d^2)*f)*x)*sqrt(b*x + a))/b^3, 2/105*(105*sqrt(-a)*b^3*c^2*e*a
rctan(sqrt(b*x + a)*sqrt(-a)/a) + (15*b^3*d^2*f*x^3 + 3*(7*b^3*d^2*e + (14*
b^3*c*d + a*b^2*d^2)*f)*x^2 + 7*(15*b^3*c^2 + 10*a*b^2*c*d - 2*a^2*b*d^2)*e
 + (35*a*b^2*c^2 - 28*a^2*b*c*d + 8*a^3*d^2)*f + (7*(10*b^3*c*d + a*b^2*d^2
)*e + (35*b^3*c^2 + 14*a*b^2*c*d - 4*a^2*b*d^2)*f)*x)*sqrt(b*x + a))/b^3]
```

giac [A] time = 1.37, size = 201, normalized size = 1.39

$$\frac{2\,ac^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}}+\frac{2\left(35\,(bx+a)^{\frac{3}{2}}b^{20}c^2f+42\,(bx+a)^{\frac{5}{2}}b^{19}cdf-70\,(bx+a)^{\frac{3}{2}}ab^{19}cdf+15\,(bx+a)^{\frac{7}{2}}b^{18}d^2f-42\,(bx+a)^{\frac{5}{2}}ab^{18}d^2f+35\,(bx+a)^{\frac{3}{2}}a^2b^{18}d^2f+105\,\sqrt{bx+a}\,b^{21}c^2e+70\,(bx+a)^{\frac{3}{2}}b^{20}cde+21\,(bx+a)^{\frac{5}{2}}b^{19}d^2e-35\,(bx+a)^{\frac{3}{2}}ab^{19}d^2e\right)}{105\,b^{21}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c^2*arctan(sqrt(b*x + a)/sqrt(-a))*e/sqrt(-a) + 2/105*(35*(b*x + a)^(3/
2)*b^20*c^2*f + 42*(b*x + a)^(5/2)*b^19*c*d*f - 70*(b*x + a)^(3/2)*a*b^19*c
*d*f + 15*(b*x + a)^(7/2)*b^18*d^2*f - 42*(b*x + a)^(5/2)*a*b^18*d^2*f + 35
*(b*x + a)^(3/2)*a^2*b^18*d^2*f + 105*sqrt(b*x + a)*b^21*c^2*e + 70*(b*x +
a)^(3/2)*b^20*c*d*e + 21*(b*x + a)^(5/2)*b^19*d^2*e - 35*(b*x + a)^(3/2)*a*
b^19*d^2*e)/b^21
```

maple [A] time = 0.01, size = 176, normalized size = 1.21

$$\frac{-2\sqrt{a}\;b^3c^2e\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\;b^3c^2e+\frac{2(bx+a)^{\frac{3}{2}}a^2d^2f}{3}-\frac{4(bx+a)^{\frac{3}{2}}abcdf}{3}-\frac{2(bx+a)^{\frac{3}{2}}ab\,d^2e}{3}+\frac{2(bx+a)^{\frac{3}{2}}b^2c^2f}{3}+\frac{4(bx+a)^{\frac{3}{2}}b^2cde}{3}-\frac{4(bx+a)^{\frac{5}{2}}a\,d^2f}{5}+\frac{4(bx+a)^{\frac{5}{2}}bcdf}{5}+\frac{2(bx+a)^{\frac{5}{2}}b\,d^2e}{5}+\frac{2(bx+a)^{\frac{7}{2}}d^2f}{7}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x)
```

```
[Out] 2/b^3*(1/7*d^2*f*(b*x+a)^(7/2)-2/5*(b*x+a)^(5/2)*a*d^2*f+2/5*(b*x+a)^(5/2)*
b*c*d*f+1/5*(b*x+a)^(5/2)*b*d^2*e+1/3*(b*x+a)^(3/2)*a^2*d^2*f-2/3*(b*x+a)^(
3/2)*a*b*c*d*f-1/3*(b*x+a)^(3/2)*a*b*d^2*e+1/3*(b*x+a)^(3/2)*b^2*c^2*f+2/3*
(b*x+a)^(3/2)*b^2*c*d*e+b^3*c^2*e*(b*x+a)^(1/2)-a^(1/2)*b^3*c^2*e*arctanh((
b*x+a)^(1/2)/a^(1/2)))
```

maxima [A] time = 0.98, size = 152, normalized size = 1.05

$$\sqrt{a}\,c^2e\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2\left(105\,\sqrt{bx+a}\,b^3c^2e+15\,(bx+a)^{\frac{7}{2}}d^2f+21\,\left(bd^2e+2\left(bcd-ad^2\right)f\right)(bx+a)^{\frac{5}{2}}+35\left(\left(2\,b^2cd-abd^2\right)e+\left(b^2c^2-2\,abcd+a^2d^2\right)f\right)(bx+a)^{\frac{3}{2}}\right)}{105\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*c^2*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/
105*(105*sqrt(b*x + a)*b^3*c^2*e + 15*(b*x + a)^(7/2)*d^2*f + 21*(b*d^2*e +
 2*(b*c*d - a*d^2)*f)*(b*x + a)^(5/2) + 35*((2*b^2*c*d - a*b*d^2)*e + (b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*f)*(b*x + a)^(3/2))/b^3
```

mupad [B] time = 0.09, size = 263, normalized size = 1.81

$$\left(\frac{2\,b\,d^2\,e-6\,a\,d^2\,f+4\,b\,c\,d\,f}{5\,b^3}+\frac{2\,a\,d^2\,f}{5\,b^3}\right)\,{\left(a+b\,x\right)}^{5/2}+\left(a\,\left(a\,\left(\frac{2\,b\,d^2\,e-6\,a\,d^2\,f+4\,b\,c\,d\,f}{b^3}+\frac{2\,a\,d^2\,f}{b^3}\right)-\frac{2\,\left(a\,d-b\,c\right)\,\left(b\,c\,f-3\,a\,d\,f+2\,b\,d\,e\right)}{b^3}\right)-\frac{2\,{\left(a\,d-b\,c\right)}^2\,\left(a\,f-b\,e\right)}{b^3}\right)\,\sqrt{a+b\,x}+\left(\frac{a\,\left(\frac{2\,b\,d^2\,e-6\,a\,d^2\,f+4\,b\,c\,d\,f}{b^3}+\frac{2\,a\,d^2\,f}{b^3}\right)}{3}-\frac{2\,\left(a\,d-b\,c\right)\,\left(b\,c\,f-3\,a\,d\,f+2\,b\,d\,e\right)}{3\,b^3}\right)\,{\left(a+b\,x\right)}^{3/2}+\frac{2\,d^2\,f\,{\left(a+b\,x\right)}^{7/2}}{7\,b^3}+\sqrt{a}\,c^2\,e\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2)*(c + d*x)^2)/x,x)
```

```
[Out] ((2*b*d^2*e - 6*a*d^2*f + 4*b*c*d*f)/(5*b^3) + (2*a*d^2*f)/(5*b^3))*(a + b*
x)^(5/2) + (a*(a*((2*b*d^2*e - 6*a*d^2*f + 4*b*c*d*f)/b^3 + (2*a*d^2*f)/b^3
) - (2*(a*d - b*c)*(b*c*f - 3*a*d*f + 2*b*d*e))/b^3) - (2*(a*d - b*c)^2*(a*
f - b*e))/b^3)*(a + b*x)^(1/2) + ((a*((2*b*d^2*e - 6*a*d^2*f + 4*b*c*d*f)/b
^3 + (2*a*d^2*f)/b^3))/3 - (2*(a*d - b*c)*(b*c*f - 3*a*d*f + 2*b*d*e))/(3*b
^3))*(a + b*x)^(3/2) + a^(1/2)*c^2*e*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i
+ (2*d^2*f*(a + b*x)^(7/2))/(7*b^3)
```

sympy [A] time = 26.17, size = 167, normalized size = 1.15

$$\frac{2ac^2e\,\mathrm{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2c^2e\sqrt{a+bx}+\frac{2d^2f\,(a+bx)^{\frac{7}{2}}}{7b^3}+\frac{2\,(a+bx)^{\frac{5}{2}}\left(-2ad^2f+2bcdf+bd^2e\right)}{5b^3}+\frac{2\,(a+bx)^{\frac{3}{2}}\left(a^2d^2f-2abcdf-abd^2e+b^2c^2f+2b^2cde\right)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(f*x+e)*(b*x+a)**(1/2)/x,x)
```

```
[Out] 2*a*c**2*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*c**2*e*sqrt(a + b*x) +
 2*d**2*f*(a + b*x)**(7/2)/(7*b**3) + 2*(a + b*x)**(5/2)*(-2*a*d**2*f + 2*b
*c*d*f + b*d**2*e)/(5*b**3) + 2*(a + b*x)**(3/2)*(a**2*d**2*f - 2*a*b*c*d*f
 - a*b*d**2*e + b**2*c**2*f + 2*b**2*c*d*e)/(3*b**3)
```

3.17 $\int \frac{\sqrt{a+bx}\,(c+dx)(e+fx)}{x}\,dx$

Optimal. Leaf size=77

$$-\frac{2(a+bx)^{3/2}(2adf-5b(cf+de)-3bdfx)}{15b^2}+2ce\sqrt{a+bx}-2\sqrt{a}\,ce\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {147, 50, 63, 208}

$$-\frac{2(a+bx)^{3/2}(2adf-5b(cf+de)-3bdfx)}{15b^2}+2ce\sqrt{a+bx}-2\sqrt{a}\,ce\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(c + d*x)*(e + f*x))/x,x]
```

```
[Out] 2*c*e*Sqrt[a + b*x] - (2*(a + b*x)^(3/2)*(2*a*d*f - 5*b*(d*e + c*f) - 3*b*d
*f*x))/(15*b^2) - 2*Sqrt[a]*c*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\,(c+dx)(e+fx)}{x}\,dx = -\frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2} + (ce)\int \frac{\sqrt{a+bx}}{x}\,dx$$

$$= 2ce\sqrt{a+bx} - \frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2} + (ace)\int \frac{1}{x\sqrt{a+b}}$$

$$= 2ce\sqrt{a+bx} - \frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2} + \frac{(2ace)\,\mathrm{Subst}\Big(\int}{}$$

$$= 2ce\sqrt{a+bx} - \frac{2(a+bx)^{3/2}(2adf-5b(de+cf)-3bdfx)}{15b^2} - 2\sqrt{a}\,ce\tanh^{-1}\Big($$

Mathematica [A] time = 0.13, size = 87, normalized size = 1.13

$$\frac{2(a+bx)^{3/2}(-adf+bcf+bde)}{3b^2} + \frac{2df(a+bx)^{5/2}}{5b^2} + 2ce\sqrt{a+bx} - 2\sqrt{a}\,ce\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(c + d*x)*(e + f*x))/x,x]
```

```
[Out] 2*c*e*Sqrt[a + b*x] + (2*(b*d*e + b*c*f - a*d*f)*(a + b*x)^(3/2))/(3*b^2) +
 (2*d*f*(a + b*x)^(5/2))/(5*b^2) - 2*Sqrt[a]*c*e*ArcTanh[Sqrt[a + b*x]/Sqrt
[a]]
```

IntegrateAlgebraic [A] time = 0.08, size = 91, normalized size = 1.18

$$\frac{2\sqrt{a+bx}\,\left(5bcf(a+bx)+5bde(a+bx)+3df(a+bx)^2-5adf(a+bx)+15b^2ce\right)}{15b^2} - 2\sqrt{a}\,ce\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x]*(c + d*x)*(e + f*x))/x,x]
```

```
[Out] (2*Sqrt[a + b*x]*(15*b^2*c*e + 5*b*d*e*(a + b*x) + 5*b*c*f*(a + b*x) - 5*a*
d*f*(a + b*x) + 3*d*f*(a + b*x)^2))/(15*b^2) - 2*Sqrt[a]*c*e*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]]
```

fricas [A] time = 0.93, size = 217, normalized size = 2.82

$$\left[\frac{15\sqrt{a}\,b^2ce\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(3b^2dfx^2+5\left(3b^2c+abd\right)e+\left(5abc-2a^2d\right)f+\left(5b^2de+\left(5b^2c+abd\right)f\right)x\right)\sqrt{bx+a}}{15b^2},\ \frac{2\left(15\sqrt{-a}\,b^2ce\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(3b^2dfx^2+5\left(3b^2c+abd\right)e+\left(5abc-2a^2d\right)f+\left(5b^2de+\left(5b^2c+abd\right)f\right)x\right)\sqrt{bx+a}\right)}{15b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/15*(15*sqrt(a)*b^2*c*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*
(3*b^2*d*f*x^2 + 5*(3*b^2*c + a*b*d)*e + (5*a*b*c - 2*a^2*d)*f + (5*b^2*d*e
 + (5*b^2*c + a*b*d)*f)*x)*sqrt(b*x + a))/b^2, 2/15*(15*sqrt(-a)*b^2*c*e*ar
ctan(sqrt(b*x + a)*sqrt(-a)/a) + (3*b^2*d*f*x^2 + 5*(3*b^2*c + a*b*d)*e + (
5*a*b*c - 2*a^2*d)*f + (5*b^2*d*e + (5*b^2*c + a*b*d)*f)*x)*sqrt(b*x + a))/
b^2]
```

giac [A] time = 1.26, size = 105, normalized size = 1.36

$$\frac{2\,ac\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}} + \frac{2\left(5\,(bx+a)^{\frac{3}{2}}b^9cf + 3\,(bx+a)^{\frac{5}{2}}b^8df - 5\,(bx+a)^{\frac{3}{2}}ab^8df + 15\,\sqrt{bx+a}\,b^{10}ce + 5\,(bx+a)^{\frac{3}{2}}b^9de\right)}{15\,b^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*c*arctan(sqrt(b*x + a)/sqrt(-a))*e/sqrt(-a) + 2/15*(5*(b*x + a)^(3/2)*b
^9*c*f + 3*(b*x + a)^(5/2)*b^8*d*f - 5*(b*x + a)^(3/2)*a*b^8*d*f + 15*sqrt(
b*x + a)*b^10*c*e + 5*(b*x + a)^(3/2)*b^9*d*e)/b^10
```

maple [A] time = 0.01, size = 89, normalized size = 1.16

$$\frac{-2\sqrt{a}\, b^2 ce \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\; b^2 ce-\frac{2(bx+a)^{\frac{3}{2}} adf}{3}+\frac{2(bx+a)^{\frac{3}{2}} bcf}{3}+\frac{2(bx+a)^{\frac{3}{2}} bde}{3}+\frac{2(bx+a)^{\frac{5}{2}} df}{5}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x)
```

```
[Out] 2/b^2*(1/5*d*f*(b*x+a)^(5/2)-1/3*(b*x+a)^(3/2)*a*d*f+1/3*(b*x+a)^(3/2)*b*c*
f+1/3*(b*x+a)^(3/2)*b*d*e+b^2*c*e*(b*x+a)^(1/2)-a^(1/2)*b^2*c*e*arctanh((b*
x+a)^(1/2)/a^(1/2)))
```

maxima [A] time = 0.98, size = 90, normalized size = 1.17

$$\sqrt{a}\, ce \log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2\left(15\sqrt{bx+a}\, b^2 ce+3\,(bx+a)^{\frac{5}{2}} df+5\left(bde+(bc-ad)f\right)(bx+a)^{\frac{3}{2}}\right)}{15\, b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*c*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/15
*(15*sqrt(b*x + a)*b^2*c*e + 3*(b*x + a)^(5/2)*d*f + 5*(b*d*e + (b*c - a*d)
*f)*(b*x + a)^(3/2))/b^2
```

mupad [B] time = 2.49, size = 136, normalized size = 1.77

$$\left(a\left(\frac{2bcf-4adf+2bde}{b^2}+\frac{2adf}{b^2}\right)+\frac{2\,(ad-bc)\left(af-be\right)}{b^2}\right)\sqrt{a+bx}+\left(\frac{2bcf-4adf+2bde}{3\,b^2}+\frac{2adf}{3\,b^2}\right)(a+bx)^{3/2}+\frac{2df\,(a+bx)^{5/2}}{5\,b^2}+\sqrt{a}\, ce \operatorname{atan}\left(\frac{\sqrt{a+bx}\,1\mathrm{i}}{\sqrt{a}}\right)2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2)*(c + d*x))/x,x)
```

```
[Out] (a*((2*b*c*f - 4*a*d*f + 2*b*d*e)/b^2 + (2*a*d*f)/b^2) + (2*(a*d - b*c)*(a*
f - b*e))/b^2)*(a + b*x)^(1/2) + ((2*b*c*f - 4*a*d*f + 2*b*d*e)/(3*b^2) + (
2*a*d*f)/(3*b^2))*(a + b*x)^(3/2) + (2*d*f*(a + b*x)^(5/2))/(5*b^2) + a^(1/
2)*c*e*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i
```

sympy [A] time = 25.97, size = 92, normalized size = 1.19

$$\frac{2ace \operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2ce\sqrt{a+bx}+\frac{2df\,(a+bx)^{\frac{5}{2}}}{5b^2}+\frac{2\,(a+bx)^{\frac{3}{2}}\left(-adf+bcf+bde\right)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(f*x+e)*(b*x+a)**(1/2)/x,x)
```

```
[Out] 2*a*c*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*c*e*sqrt(a + b*x) + 2*d*f
*(a + b*x)**(5/2)/(5*b**2) + 2*(a + b*x)**(3/2)*(-a*d*f + b*c*f + b*d*e)/(3
*b**2)
```

3.18 $\int \frac{\sqrt{a+bx}(e+fx)}{x} dx$

Optimal. Leaf size=54

$$2e\sqrt{a+bx} - 2\sqrt{a}\, e \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) + \frac{2f(a+bx)^{3/2}}{3b}$$

Rubi [A] time = 0.02, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {80, 50, 63, 208}

$$2e\sqrt{a+bx} - 2\sqrt{a}\, e \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) + \frac{2f(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/x,x]
```

```
[Out] 2*e*Sqrt[a + b*x] + (2*f*(a + b*x)^(3/2))/(3*b) - 2*Sqrt[a]*e*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+bx}(e+fx)}{x}\,dx &= \frac{2f(a+bx)^{3/2}}{3b}+e\int \frac{\sqrt{a+bx}}{x}\,dx\\ &= 2e\sqrt{a+bx}+\frac{2f(a+bx)^{3/2}}{3b}+(ae)\int \frac{1}{x\sqrt{a+bx}}\,dx\\ &= 2e\sqrt{a+bx}+\frac{2f(a+bx)^{3/2}}{3b}+\frac{(2ae)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+bx}\right)}{b}\\ &= 2e\sqrt{a+bx}+\frac{2f(a+bx)^{3/2}}{3b}-2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\end{aligned}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 1.02

$$e\left(2\sqrt{a+bx}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)+\frac{2f(a+bx)^{3/2}}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/x,x]
```

```
[Out] (2*f*(a + b*x)^(3/2))/(3*b) + e*(2*Sqrt[a + b*x] - 2*Sqrt[a]*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]])
```

IntegrateAlgebraic [A] time = 0.04, size = 57, normalized size = 1.06

$$\frac{2\left(3be\sqrt{a+bx}+f(a+bx)^{3/2}\right)}{3b}-2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x]*(e + f*x))/x,x]
```

```
[Out] (2*(3*b*e*Sqrt[a + b*x] + f*(a + b*x)^(3/2)))/(3*b) - 2*Sqrt[a]*e*ArcTanh[S
qrt[a + b*x]/Sqrt[a]]
```

fricas [A] time = 0.78, size = 111, normalized size = 2.06

$$\left[\frac{3\sqrt{a}\,be\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a}+2a}{x}\right)+2\left(bfx+3be+af\right)\sqrt{bx+a}}{3b},\ \frac{2\left(3\sqrt{-a}\,be\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)+\left(bfx+3be+af\right)\sqrt{bx+a}\right)}{3b}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/3*(3*sqrt(a)*b*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*(b*f*x
 + 3*b*e + a*f)*sqrt(b*x + a))/b, 2/3*(3*sqrt(-a)*b*e*arctan(sqrt(b*x + a)*
sqrt(-a)/a) + (b*f*x + 3*b*e + a*f)*sqrt(b*x + a))/b]
```

giac [A] time = 1.20, size = 57, normalized size = 1.06

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}}+\frac{2\left((bx+a)^{\frac{3}{2}}b^2f+3\sqrt{bx+a}\,b^3e\right)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*e/sqrt(-a) + 2/3*((b*x + a)^(3/2)*b^2*f
+ 3*sqrt(b*x + a)*b^3*e)/b^3
```

maple [A] time = 0.01, size = 46, normalized size = 0.85

$$\frac{-2\sqrt{a}\,be\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2\sqrt{bx+a}\,be+\frac{2(bx+a)^{\frac{3}{2}}f}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x,x)
```

```
[Out] 2/b*(1/3*f*(b*x+a)^(3/2)+(b*x+a)^(1/2)*b*e-a^(1/2)*b*e*arctanh((b*x+a)^(1/2
)/a^(1/2)))
```

maxima [A] time = 0.97, size = 60, normalized size = 1.11

$$\sqrt{a}\,e\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)+\frac{2\left(3\sqrt{bx+a}\,be+(bx+a)^{\frac{3}{2}}f\right)}{3\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] sqrt(a)*e*log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a))) + 2/3*(3
*sqrt(b*x + a)*b*e + (b*x + a)^(3/2)*f)/b
```

mupad [B] time = 0.07, size = 45, normalized size = 0.83

$$2\,e\,\sqrt{a+b\,x}+\frac{2\,f\,{\left(a+b\,x\right)}^{3/2}}{3\,b}+\sqrt{a}\,e\,\mathrm{atan}\left(\frac{\sqrt{a+b\,x}\,1{}\mathrm{i}}{\sqrt{a}}\right)\,2{}\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2))/x,x)
```

```
[Out] 2*e*(a + b*x)^(1/2) + a^(1/2)*e*atan(((a + b*x)^(1/2)*1i)/a^(1/2))*2i + (2*
f*(a + b*x)^(3/2))/(3*b)
```

sympy [A] time = 6.08, size = 54, normalized size = 1.00

$$\frac{2ae\,\mathrm{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{\sqrt{-a}}+2e\sqrt{a+bx}+\frac{2f\,(a+bx)^{\frac{3}{2}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x,x)
```

```
[Out] 2*a*e*atan(sqrt(a + b*x)/sqrt(-a))/sqrt(-a) + 2*e*sqrt(a + b*x) + 2*f*(a +
b*x)**(3/2)/(3*b)
```

3.19 $\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2\sqrt{bc-ad}\,(de-cf)\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}}-\frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}+\frac{2f\sqrt{a+bx}}{d}$$

Rubi [A] time = 0.12, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {154, 156, 63, 208, 205}

$$\frac{2\sqrt{bc-ad}\,(de-cf)\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}}-\frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}+\frac{2f\sqrt{a+bx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)),x]
```

```
[Out] (2*f*Sqrt[a + b*x])/d + (2*Sqrt[b*c - a*d]*(d*e - c*f)*ArcTan[(Sqrt[d]*Sqrt
[a + b*x])/Sqrt[b*c - a*d]])/(c*d^(3/2)) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*
x]/Sqrt[a]])/c
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 154

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegersQ[2*m, 2*n, 2*p]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{a+bx}\,(e+fx)}{x(c+dx)}\,dx = \frac{2f\sqrt{a+bx}}{d} + \frac{2\int \frac{\frac{ade}{2}+\frac{1}{2}(bde-bcf+adf)x}{x\sqrt{a+bx}\,(c+dx)}\,dx}{d}$$

$$= \frac{2f\sqrt{a+bx}}{d} + \frac{(ae)\int \frac{1}{x\sqrt{a+bx}}\,dx}{c} + \frac{((bc-ad)(de-cf))\int \frac{1}{\sqrt{a+bx}\,(c+dx)}\,dx}{cd}$$

$$= \frac{2f\sqrt{a+bx}}{d} + \frac{(2ae)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{bc} + \frac{(2(bc-ad)(de-cf))\,\text{Subst}\left(\right.}{bc}$$

$$= \frac{2f\sqrt{a+bx}}{d} + \frac{2\sqrt{bc-ad}\,(de-cf)\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}} - \frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}$$

Mathematica [A] time = 0.23, size = 100, normalized size = 0.99

$$\frac{-\frac{2\sqrt{bc-ad}\,(cf-de)\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{3/2}} + \frac{2cf\sqrt{a+bx}}{d} - 2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)),x]
```

```
[Out] ((2*c*f*Sqrt[a + b*x])/d - (2*Sqrt[b*c - a*d]*(-(d*e) + c*f)*ArcTan[(Sqrt[d
]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/d^(3/2) - 2*Sqrt[a]*e*ArcTanh[Sqrt[a + b
*x]/Sqrt[a]])/c
```

IntegrateAlgebraic [A] time = 0.17, size = 101, normalized size = 1.00

$$-\frac{2\sqrt{bc-ad}\,(cf-de)\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{cd^{3/2}} - \frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c} + \frac{2f\sqrt{a+bx}}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)),x]
```

```
[Out] (2*f*Sqrt[a + b*x])/d - (2*Sqrt[b*c - a*d]*(-(d*e) + c*f)*ArcTan[(Sqrt[d]*S
qrt[a + b*x])/Sqrt[b*c - a*d]])/(c*d^(3/2)) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a +
 b*x]/Sqrt[a]])/c
```

fricas [A] time = 1.47, size = 450, normalized size = 4.46

$$\left[\frac{\sqrt{a}\,de\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}\,cf-(de-cf)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{bdx-bc+2ad-2\sqrt{bx+a}\,d\sqrt{-\frac{bc-ad}{d}}}{dx+c}\right)}{cd}, \frac{2\sqrt{-a}\,de\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+2\sqrt{bx+a}\,cf-(de-cf)\sqrt{-\frac{bc-ad}{d}}\log\left(\frac{bdx-bc+2ad-2\sqrt{bx+a}\,d\sqrt{-\frac{bc-ad}{d}}}{dx+c}\right)}{cd}, \frac{\sqrt{a}\,de\log\left(\frac{bx-2\sqrt{bx+a}\,\sqrt{a}+2a}{x}\right)+2\sqrt{bx+a}\,cf-2(de-cf)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{\sqrt{bx+a}\,d\sqrt{\frac{bc-ad}{d}}}{bc-ad}\right)}{cd}, \frac{2\left(\sqrt{-a}\,de\arctan\left(\frac{\sqrt{bx+a}\,\sqrt{-a}}{a}\right)+\sqrt{bx+a}\,cf-(de-cf)\sqrt{\frac{bc-ad}{d}}\arctan\left(-\frac{\sqrt{bx+a}\,d\sqrt{\frac{bc-ad}{d}}}{bc-ad}\right)\right)}{cd}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*d*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a
)*c*f - (d*e - c*f)*sqrt(-(b*c - a*d)/d)*log((b*d*x - b*c + 2*a*d - 2*sqrt(
b*x + a)*d*sqrt(-(b*c - a*d)/d))/(d*x + c)))/(c*d), (2*sqrt(-a)*d*e*arctan(
sqrt(b*x + a)*sqrt(-a)/a) + 2*sqrt(b*x + a)*c*f - (d*e - c*f)*sqrt(-(b*c -
a*d)/d)*log((b*d*x - b*c + 2*a*d - 2*sqrt(b*x + a)*d*sqrt(-(b*c - a*d)/d))/
(d*x + c)))/(c*d), (sqrt(a)*d*e*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x
) + 2*sqrt(b*x + a)*c*f - 2*(d*e - c*f)*sqrt((b*c - a*d)/d)*arctan(-sqrt(b*
```

```
x + a)*d*sqrt((b*c - a*d)/d)/(b*c - a*d)))/(c*d), 2*(sqrt(-a)*d*e*arctan(sq
rt(b*x + a)*sqrt(-a)/a) + sqrt(b*x + a)*c*f - (d*e - c*f)*sqrt((b*c - a*d)/
d)*arctan(-sqrt(b*x + a)*d*sqrt((b*c - a*d)/d)/(b*c - a*d)))/(c*d)]
```

giac [A] time = 1.22, size = 112, normalized size = 1.11

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}\,c}+\frac{2\,\sqrt{bx+a}\,f}{d}-\frac{2\left(bc^2f-acdf-bcde+ad^2e\right)\arctan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*e/(sqrt(-a)*c) + 2*sqrt(b*x + a)*f/d - 2
*(b*c^2*f - a*c*d*f - b*c*d*e + a*d^2*e)*arctan(sqrt(b*x + a)*d/sqrt(b*c*d
- a*d^2))/(sqrt(b*c*d - a*d^2)*c*d)
```

maple [A] time = 0.02, size = 103, normalized size = 1.02

$$-\frac{2\sqrt{a}\,e\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{c}+\frac{2\sqrt{bx+a}\,f}{d}-\frac{2\left(acdf-a\,d^2e-b\,c^2f+bcde\right)\mathrm{arctanh}\left(\frac{\sqrt{bx+a}\,d}{\sqrt{(ad-bc)d}}\right)}{\sqrt{(ad-bc)\,d}\,cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x)
```

```
[Out] 2*f*(b*x+a)^(1/2)/d-2*e*arctanh((b*x+a)^(1/2)/a^(1/2))*a^(1/2)/c-2/d*(a*c*d
*f-a*d^2*e-b*c^2*f+b*c*d*e)/c/((a*d-b*c)*d)^(1/2)*arctanh((b*x+a)^(1/2)*d/(
(a*d-b*c)*d)^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 2.82, size = 2355, normalized size = 23.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2))/(x*(c + d*x)),x)
```

```
[Out] (2*f*(a + b*x)^(1/2))/d - (a^(1/2)*e*atan(((a^(1/2)*e*((8*(a + b*x)^(1/2)*(
b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2*b
^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2*e*
f - 2*a^2*b^2*c*d^3*e*f))/d + (a^(1/2)*e*((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2
*d^3*f))/d + (8*a^(1/2)*e*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(a + b*x)^(1/2))/
(c*d)))/c)*1i)/c + (a^(1/2)*e*((8*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 2*a^2*b^2*
d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3
*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2*e*f - 2*a^2*b^2*c*d^3*e*f)
)/d - (a^(1/2)*e*((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f))/d - (8*a^(1/2)*
e*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(a + b*x)^(1/2))/(c*d)))/c)*1i)/c)/((16*(
a^2*b^3*d^3*e^3 - a*b^4*c*d^2*e^3 - a*b^4*c^3*e*f^2 + a^3*b^2*d^3*e^2*f - 3
```

```
*a^2*b^3*c*d^2*e^2*f + 2*a^2*b^3*c^2*d*e*f^2 - a^3*b^2*c*d^2*e*f^2 + 2*a*b^
4*c^2*d*e^2*f))/d - (a^(1/2)*e*((8*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 2*a^2*b^2
*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^
3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2*e*f - 2*a^2*b^2*c*d^3*e*f
))/d + (a^(1/2)*e*((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f))/d + (8*a^(1/2)
*e*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(a + b*x)^(1/2))/(c*d)))/c))/c + (a^(1/2
)*e*((8*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2
- 2*b^4*c^3*d*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d
*f^2 + 4*a*b^3*c^2*d^2*e*f - 2*a^2*b^2*c*d^3*e*f))/d - (a^(1/2)*e*((8*(a*b^
3*c^3*d^2*f - a^2*b^2*c^2*d^3*f))/d - (8*a^(1/2)*e*(b^3*c^3*d^3 - 2*a*b^2*c
^2*d^4)*(a + b*x)^(1/2))/(c*d)))/c))/c))*2i)/c - (atan(((((8*(a + b*x)^(1/2
)*(b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^
2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2
*e*f - 2*a^2*b^2*c*d^3*e*f))/d + (((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f)
)/d + (8*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2
)*(a + b*x)^(1/2))/(c*d^4))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3))*(
c*f - d*e)*(d^3*(a*d - b*c))^(1/2)*1i)/(c*d^3) + (((8*(a + b*x)^(1/2)*(b^4*
c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a^2*b^2*c
^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^2*e*f -
2*a^2*b^2*c*d^3*e*f))/d - (((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f))/d - (
8*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2)*(a +
b*x)^(1/2))/(c*d^4))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3))*(c*f - d
*e)*(d^3*(a*d - b*c))^(1/2)*1i)/(c*d^3))/((16*(a^2*b^3*d^3*e^3 - a*b^4*c*d^
2*e^3 - a*b^4*c^3*e*f^2 + a^3*b^2*d^3*e^2*f - 3*a^2*b^3*c*d^2*e^2*f + 2*a^2
*b^3*c^2*d*e*f^2 - a^3*b^2*c*d^2*e*f^2 + 2*a*b^4*c^2*d*e^2*f))/d - (((8*(a
+ b*x)^(1/2)*(b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3
*d*e*f + a^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*
b^3*c^2*d^2*e*f - 2*a^2*b^2*c*d^3*e*f))/d + (((8*(a*b^3*c^3*d^2*f - a^2*b^2
*c^2*d^3*f))/d + (8*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(c*f - d*e)*(d^3*(a*d -
 b*c))^(1/2)*(a + b*x)^(1/2))/(c*d^4))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))
/(c*d^3))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3) + (((8*(a + b*x)^(1/
2)*(b^4*c^4*f^2 + 2*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 - 2*b^4*c^3*d*e*f + a
^2*b^2*c^2*d^2*f^2 - 2*a*b^3*c*d^3*e^2 - 2*a*b^3*c^3*d*f^2 + 4*a*b^3*c^2*d^
2*e*f - 2*a^2*b^2*c*d^3*e*f))/d - (((8*(a*b^3*c^3*d^2*f - a^2*b^2*c^2*d^3*f
))/d - (8*(b^3*c^3*d^3 - 2*a*b^2*c^2*d^4)*(c*f - d*e)*(d^3*(a*d - b*c))^(1/
2)*(a + b*x)^(1/2))/(c*d^4))*(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3))*
(c*f - d*e)*(d^3*(a*d - b*c))^(1/2))/(c*d^3)))*(c*f - d*e)*(d^3*(a*d - b*c)
)^(1/2)*2i)/(c*d^3)
```

sympy [A] time = 24.25, size = 100, normalized size = 0.99

$$\frac{2ae\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-a}}\right)}{c\sqrt{-a}}+\frac{2f\sqrt{a+bx}}{d}+\frac{2\left(ad-bc\right)\left(cf-de\right)\operatorname{atan}\left(\frac{\sqrt{a+bx}}{\sqrt{-\frac{ad-bc}{d}}}\right)}{cd^2\sqrt{-\frac{ad-bc}{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x/(d*x+c),x)
```

```
[Out] 2*a*e*atan(sqrt(a + b*x)/sqrt(-a))/(c*sqrt(-a)) + 2*f*sqrt(a + b*x)/d + 2*(
a*d - b*c)*(c*f - d*e)*atan(sqrt(a + b*x)/sqrt(-(a*d - b*c)/d))/(c*d**2*sqr
t(-(a*d - b*c)/d))
```

3.20 $\int \frac{\sqrt{a+bx}(e+fx)}{x(c+dx)^2} dx$

Optimal. Leaf size=128

$$-\frac{\left(2ad^2e-bc(cf+de)\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2}+\frac{\sqrt{a+bx}\,(de-cf)}{cd(c+dx)}$$

Rubi [A] time = 0.12, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {149, 156, 63, 208, 205}

$$-\frac{\left(2ad^2e-bc(cf+de)\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}\sqrt{bc-ad}}-\frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2}+\frac{\sqrt{a+bx}\,(de-cf)}{cd(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^2),x]
```

```
[Out] ((d*e - c*f)*Sqrt[a + b*x])/(c*d*(c + d*x)) - ((2*a*d^2*e - b*c*(d*e + c*f)
)*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(c^2*d^(3/2)*Sqrt[b*c -
a*d]) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/c^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx}\,(e+fx)}{x(c+dx)^2}\,dx &= \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} - \frac{\int \frac{-ade-\frac{1}{2}b(de+cf)x}{x\sqrt{a+bx}\,(c+dx)}\,dx}{cd}\\
&= \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} + \frac{(ae)\int \frac{1}{x\sqrt{a+bx}}\,dx}{c^2} - \frac{\left(2ad^2e-bc(de+cf)\right)\int \frac{1}{\sqrt{a+bx}\,(c+dx)}\,dx}{2c^2d}\\
&= \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} + \frac{(2ae)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+bx}\right)}{bc^2} - \frac{\left(2ad^2e-bc(de+cf)\right)\,\text{S}}{}\\
&= \frac{(de-cf)\sqrt{a+bx}}{cd(c+dx)} - \frac{\left(2ad^2e-bc(de+cf)\right)\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}\sqrt{bc-ad}} - \frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right.}{c^2}
\end{aligned}
$$

Mathematica [A] time = 0.20, size = 122, normalized size = 0.95

$$
\frac{\frac{\left(bc(cf+de)-2ad^2e\right)\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{d^{3/2}\sqrt{bc-ad}} + \frac{c\sqrt{a+bx}\,(de-cf)}{d(c+dx)} - 2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^2),x]
```

```
[Out] ((c*(d*e - c*f)*Sqrt[a + b*x])/(d*(c + d*x)) + ((-2*a*d^2*e + b*c*(d*e + c*
f))*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(d^(3/2)*Sqrt[b*c - a*
d]) - 2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/c^2
```

IntegrateAlgebraic [A] time = 0.48, size = 139, normalized size = 1.09

$$
\frac{\left(-2ad^2e+bc^2f+bcde\right)\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}\sqrt{bc-ad}} - \frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^2} - \frac{b\sqrt{a+bx}\,(cf-de)}{cd(d(a+bx)-ad+bc)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^2),x]
```

```
[Out] -((b*(-(d*e) + c*f)*Sqrt[a + b*x])/(c*d*(b*c - a*d + d*(a + b*x)))) + ((b*c
*d*e - 2*a*d^2*e + b*c^2*f)*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]
)/(c^2*d^(3/2)*Sqrt[b*c - a*d]) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a + b*x]/Sqrt[a
]])/c^2
```

fricas [B] time = 1.36, size = 1008, normalized size = 7.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*((b*c^3*f + (b*c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^3
)*e)*x)*sqrt(-b*c*d + a*d^2)*log((b*d*x - b*c + 2*a*d - 2*sqrt(-b*c*d + a*d
^2)*sqrt(b*x + a))/(d*x + c)) - 2*((b*c*d^3 - a*d^4)*e*x + (b*c^2*d^2 - a*c
*d^3)*e)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*((b*c^2*d
^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)*f)*sqrt(b*x + a))/(b*c^4*d^2 - a*c^
3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x), 1/2*(4*((b*c*d^3 - a*d^4)*e*x + (b*c^2*
```

```
d^2 - a*c*d^3)*e)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - (b*c^3*f + (b
*c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^3)*e)*x)*sqrt(-b*c*d
+ a*d^2)*log((b*d*x - b*c + 2*a*d - 2*sqrt(-b*c*d + a*d^2)*sqrt(b*x + a))/(
d*x + c)) + 2*((b*c^2*d^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)*f)*sqrt(b*x
+ a))/(b*c^4*d^2 - a*c^3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x), -((b*c^3*f + (b*
c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^3)*e)*x)*sqrt(b*c*d -
a*d^2)*arctan(sqrt(b*c*d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) - ((b*c*d^3
- a*d^4)*e*x + (b*c^2*d^2 - a*c*d^3)*e)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*
sqrt(a) + 2*a)/x) - ((b*c^2*d^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)*f)*sqr
t(b*x + a))/(b*c^4*d^2 - a*c^3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x), -((b*c^3*f
 + (b*c^2*d - 2*a*c*d^2)*e + (b*c^2*d*f + (b*c*d^2 - 2*a*d^3)*e)*x)*sqrt(b*
c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) - 2*((
b*c*d^3 - a*d^4)*e*x + (b*c^2*d^2 - a*c*d^3)*e)*sqrt(-a)*arctan(sqrt(b*x +
a)*sqrt(-a)/a) - ((b*c^2*d^2 - a*c*d^3)*e - (b*c^3*d - a*c^2*d^2)*f)*sqrt(b
*x + a))/(b*c^4*d^2 - a*c^3*d^3 + (b*c^3*d^3 - a*c^2*d^4)*x)]
```

giac [**A**] time = 1.32, size = 142, normalized size = 1.11

$$\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}\,c^2}+\frac{\left(bc^2f+bcde-2\,ad^2e\right)\arctan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2}\,c^2d}-\frac{\sqrt{bx+a}\,bcf-\sqrt{bx+a}\,bde}{(bc+(bx+a)d-ad)cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*e/(sqrt(-a)*c^2) + (b*c^2*f + b*c*d*e -
2*a*d^2*e)*arctan(sqrt(b*x + a)*d/sqrt(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)
*c^2*d) - (sqrt(b*x + a)*b*c*f - sqrt(b*x + a)*b*d*e)/((b*c + (b*x + a)*d -
 a*d)*c*d)
```

maple [**A**] time = 0.02, size = 137, normalized size = 1.07

$$2\left(-\frac{\sqrt{a}\;e\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b\,c^2}-\frac{\frac{\left(cf-de\right)\sqrt{bx+a}\;bc}{2(-ad+bc+(bx+a)d)d}-\frac{\left(2a\,d^2e-b\,c^2f-bcde\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}\;d}{\sqrt{(ad-bc)d}}\right)}{2\sqrt{(ad-bc)d}\;d}}{b\,c^2}\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x)
```

```
[Out] 2*b*(-a^(1/2)/b*e/c^2*arctanh((b*x+a)^(1/2)/a^(1/2))-1/c^2/b*(1/2*b*c*(c*f-
d*e)/d*(b*x+a)^(1/2)/((b*x+a)*d-a*d+b*c)-1/2*(2*a*d^2*e-b*c^2*f-b*c*d*e)/d/
((a*d-b*c)*d)^(1/2)*arctanh((b*x+a)^(1/2)/((a*d-b*c)*d)^(1/2)*d)))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [**B**] time = 2.95, size = 1814, normalized size = 14.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2))/(x*(c + d*x)^2),x)
```

```
[Out] (atan((((((2*(2*a*b^3*c^4*d^3*e - 2*a*b^3*c^5*d^2*f))/(c^3*d) + ((4*b^3*c^
5*d^3 - 8*a*b^2*c^4*d^4)*(d^3*(a*d - b*c))^(1/2)*(a + b*x)^(1/2)*(b*c^2*f -
 2*a*d^2*e + b*c*d*e))/(c^2*d*(a*c^2*d^4 - b*c^3*d^3)))*(d^3*(a*d - b*c))^(
1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^4 - b*c^3*d^3)) + (2*(a +
 b*x)^(1/2)*(b^4*c^4*f^2 + 8*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 + 2*b^4*c^3*
d*e*f - 4*a*b^3*c*d^3*e^2 - 4*a*b^3*c^2*d^2*e*f))/(c^2*d))*(d^3*(a*d - b*c)
)^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e)*1i)/(2*(a*c^2*d^4 - b*c^3*d^3)) - (
((((2*(2*a*b^3*c^4*d^3*e - 2*a*b^3*c^5*d^2*f))/(c^3*d) - ((4*b^3*c^5*d^3 -
8*a*b^2*c^4*d^4)*(d^3*(a*d - b*c))^(1/2)*(a + b*x)^(1/2)*(b*c^2*f - 2*a*d^2
*e + b*c*d*e))/(c^2*d*(a*c^2*d^4 - b*c^3*d^3)))*(d^3*(a*d - b*c))^(1/2)*(b*
c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^4 - b*c^3*d^3)) - (2*(a + b*x)^(1
/2)*(b^4*c^4*f^2 + 8*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^2 + 2*b^4*c^3*d*e*f -
4*a*b^3*c*d^3*e^2 - 4*a*b^3*c^2*d^2*e*f))/(c^2*d))*(d^3*(a*d - b*c))^(1/2)*
(b*c^2*f - 2*a*d^2*e + b*c*d*e)*1i)/(2*(a*c^2*d^4 - b*c^3*d^3)))/((4*(a*b^4
*c*d^2*e^3 - 2*a^2*b^3*d^3*e^3 + a*b^4*c^3*e*f^2 - 2*a^2*b^3*c*d^2*e^2*f +
2*a*b^4*c^2*d*e^2*f))/(c^3*d) + (((((2*(2*a*b^3*c^4*d^3*e - 2*a*b^3*c^5*d^2
*f))/(c^3*d) + ((4*b^3*c^5*d^3 - 8*a*b^2*c^4*d^4)*(d^3*(a*d - b*c))^(1/2)*(
a + b*x)^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(c^2*d*(a*c^2*d^4 - b*c^3*d
^3)))*(d^3*(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^
4 - b*c^3*d^3)) + (2*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 8*a^2*b^2*d^4*e^2 + b^4
*c^2*d^2*e^2 + 2*b^4*c^3*d*e*f - 4*a*b^3*c*d^3*e^2 - 4*a*b^3*c^2*d^2*e*f))/
(c^2*d))*(d^3*(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2
*d^4 - b*c^3*d^3)) + (((((2*(2*a*b^3*c^4*d^3*e - 2*a*b^3*c^5*d^2*f))/(c^3*d
) - ((4*b^3*c^5*d^3 - 8*a*b^2*c^4*d^4)*(d^3*(a*d - b*c))^(1/2)*(a + b*x)^(1
/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(c^2*d*(a*c^2*d^4 - b*c^3*d^3)))*(d^3*
(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^4 - b*c^3*d
^3)) - (2*(a + b*x)^(1/2)*(b^4*c^4*f^2 + 8*a^2*b^2*d^4*e^2 + b^4*c^2*d^2*e^
2 + 2*b^4*c^3*d*e*f - 4*a*b^3*c*d^3*e^2 - 4*a*b^3*c^2*d^2*e*f))/(c^2*d))*(d
^3*(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e))/(2*(a*c^2*d^4 - b*c^
3*d^3))))*(d^3*(a*d - b*c))^(1/2)*(b*c^2*f - 2*a*d^2*e + b*c*d*e)*1i)/(a*c^
2*d^4 - b*c^3*d^3) - (2*a^(1/2)*e*atanh((4*a^(1/2)*b^4*e*f^2*(a + b*x)^(1/2
))/(4*a*b^4*e*f^2 + (4*a*b^4*d^2*e^3)/c^2 - (16*a^2*b^3*d^2*e^2*f)/c^2 + (8
*a*b^4*d*e^2*f)/c) + (8*a^(1/2)*b^4*e^2*f*(a + b*x)^(1/2))/(8*a*b^4*e^2*f +
 (4*a*b^4*d*e^3)/c - (16*a^2*b^3*d*e^2*f)/c + (4*a*b^4*c*e*f^2)/d) + (4*a^(
1/2)*b^4*d*e^3*(a + b*x)^(1/2))/(4*a*b^4*d*e^3 + 8*a*b^4*c*e^2*f - 16*a^2*b
^3*d*e^2*f + (4*a*b^4*c^2*e*f^2)/d) - (16*a^(3/2)*b^3*d*e^2*f*(a + b*x)^(1/
2))/(4*a*b^4*d*e^3 + 8*a*b^4*c*e^2*f - 16*a^2*b^3*d*e^2*f + (4*a*b^4*c^2*e*
f^2)/d)))/c^2 - ((b*c*f - b*d*e)*(a + b*x)^(1/2))/(c*d*(b*c - a*d + d*(a +
b*x)))
```

sympy [B] time = 142.24, size = 1149, normalized size = 8.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x/(d*x+c)**2,x)
```

```
[Out] 2*a*b*d*e*sqrt(a + b*x)/(2*a*b*c**2*d + 2*a*b*c*d**2*x - 2*b**2*c**3 - 2*b*
*2*c**2*d*x) - a*b*f*sqrt(1/(d*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(1/(d*(a
*d - b*c)**3)) + 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)) - b**2*c**2*sqrt(1/(d
*(a*d - b*c)**3)) + sqrt(a + b*x))/2 + a*b*f*sqrt(1/(d*(a*d - b*c)**3))*log
(a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)
) + b**2*c**2*sqrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/2 - 2*a*b*f*sqrt(
a + b*x)/(2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c**2 - 2*b**2*c*d*x) + a*b*d*e*
sqrt(1/(d*(a*d - b*c)**3))*log(-a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) + 2*a*
b*c*d*sqrt(1/(d*(a*d - b*c)**3)) - b**2*c**2*sqrt(1/(d*(a*d - b*c)**3)) + s
qrt(a + b*x))/(2*c) - a*b*d*e*sqrt(1/(d*(a*d - b*c)**3))*log(a**2*d**2*sqrt
```

```
(1/(d*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)) + b**2*c**2*s
qrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/(2*c) - 2*a*e*atan(sqrt(a + b*x)
/sqrt(-a + b*c/d))/(c**2*sqrt(-a + b*c/d)) + 2*a*e*atan(sqrt(a + b*x)/sqrt(
-a))/(c**2*sqrt(-a)) + 2*b**2*c*f*sqrt(a + b*x)/(2*a*b*c*d**2 + 2*a*b*d**3*
x - 2*b**2*c**2*d - 2*b**2*c*d**2*x) + b**2*c*f*sqrt(1/(d*(a*d - b*c)**3))*
log(-a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)
**3)) - b**2*c**2*sqrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/(2*d) - b**2*
c*f*sqrt(1/(d*(a*d - b*c)**3))*log(a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) - 2
*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)) + b**2*c**2*sqrt(1/(d*(a*d - b*c)**3))
+ sqrt(a + b*x))/(2*d) - b**2*e*sqrt(1/(d*(a*d - b*c)**3))*log(-a**2*d**2*s
qrt(1/(d*(a*d - b*c)**3)) + 2*a*b*c*d*sqrt(1/(d*(a*d - b*c)**3)) - b**2*c**
2*sqrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/2 + b**2*e*sqrt(1/(d*(a*d - b
*c)**3))*log(a**2*d**2*sqrt(1/(d*(a*d - b*c)**3)) - 2*a*b*c*d*sqrt(1/(d*(a*
d - b*c)**3)) + b**2*c**2*sqrt(1/(d*(a*d - b*c)**3)) + sqrt(a + b*x))/2 - 2
*b**2*e*sqrt(a + b*x)/(2*a*b*c*d + 2*a*b*d**2*x - 2*b**2*c**2 - 2*b**2*c*d*
x) + 2*b*f*atan(sqrt(a + b*x)/sqrt(-a + b*c/d))/(d**2*sqrt(-a + b*c/d))
```

3.21 $\int \frac{\sqrt{a+bx}\,(e+fx)}{x(c+dx)^3}\,dx$

Optimal. Leaf size=205

$$-\frac{\left(-8a^2d^3e+12abcd^2e-b^2c^2(cf+3de)\right)\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{4c^3d^{3/2}(bc-ad)^{3/2}}-\frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^3}-\frac{\sqrt{a+bx}\,\left(4ad^2e-bc(cf+\right.}{4c^2d(c+dx)(bc-ad}$$

Rubi [A] time = 0.28, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {149, 151, 156, 63, 208, 205}

$$-\frac{\left(-8a^2d^3e+12abcd^2e-b^2c^2(cf+3de)\right)\tan^{-1}\left(\frac{\sqrt{d}\,\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{4c^3d^{3/2}(bc-ad)^{3/2}}-\frac{\sqrt{a+bx}\,\left(4ad^2e-bc(cf+3de)\right)}{4c^2d(c+dx)(bc-ad)}-\frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^3}+\frac{\sqrt{a+bx}\,(de-cf)}{2cd(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^3),x]
```

```
[Out] ((d*e - c*f)*Sqrt[a + b*x])/(2*c*d*(c + d*x)^2) - ((4*a*d^2*e - b*c*(3*d*e
+ c*f))*Sqrt[a + b*x])/(4*c^2*d*(b*c - a*d)*(c + d*x)) - ((12*a*b*c*d^2*e -
 8*a^2*d^3*e - b^2*c^2*(3*d*e + c*f))*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b
*c - a*d]])/(4*c^3*d^(3/2)*(b*c - a*d)^(3/2)) - (2*Sqrt[a]*e*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]])/c^3
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
```

```
 + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{a+bx}\,(e+fx)}{x(c+dx)^3}\,dx &= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{\int \frac{-2ade-\frac{1}{2}b(3de+cf)x}{x\sqrt{a+bx}\,(c+dx)^2}\,dx}{2cd} \\
&= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{\left(4ad^2e-bc(3de+cf)\right)\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} + \frac{\int \frac{2ad(bc-ad)e-\frac{1}{4}b\left(4ad^2e-bc(3de+cf)\right)x}{x\sqrt{a+bx}\,(c+dx)}}{2c^2d(bc-ad)} \\
&= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{\left(4ad^2e-bc(3de+cf)\right)\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} + \frac{(ae)\int \frac{1}{x\sqrt{a+bx}}\,dx}{c^3} - \frac{\left(12abcd^2e\right.}{} \\
&= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{\left(4ad^2e-bc(3de+cf)\right)\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} + \frac{(2ae)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a}\right.}{bc^3} \\
&= \frac{(de-cf)\sqrt{a+bx}}{2cd(c+dx)^2} - \frac{\left(4ad^2e-bc(3de+cf)\right)\sqrt{a+bx}}{4c^2d(bc-ad)(c+dx)} - \frac{\left(12abcd^2e-8a^2d^3e-b^2c^2(3de\right.}{4c^3d^{3/2}(bc-}
\end{aligned}
$$

Mathematica [A] time = 0.53, size = 259, normalized size = 1.26

$$
\frac{\frac{2\left(\frac{\left(8a^2d^3e-12abcd^2e+b^2c^2(cf+3de)\right)\left(\sqrt{d}\sqrt{a+bx}-\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)\right)}{4d^{3/2}}+2e(bc-ad)^2\left(\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)-\sqrt{a+bx}\right)\right)}{c^2(bc-ad)}-\frac{(a+bx)^{3/2}\left(4ad^2e+bc(cf-5de)\right)}{2c(c+dx)(bc-ad)}+\frac{(a+bx)^{3/2}(de-cf)}{(c+dx)^2}}{2c(ad-bc)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^3),x]
```

```
[Out] (((d*e - c*f)*(a + b*x)^(3/2))/(c + d*x)^2 - ((4*a*d^2*e + b*c*(-5*d*e + c*
f))*(a + b*x)^(3/2))/(2*c*(b*c - a*d)*(c + d*x)) + (2*(((-12*a*b*c*d^2*e +
8*a^2*d^3*e + b^2*c^2*(3*d*e + c*f))*(Sqrt[d]*Sqrt[a + b*x] - Sqrt[b*c - a*
d]*ArcTan[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]]))/(4*d^(3/2)) + 2*(b*c -
 a*d)^2*e*(-Sqrt[a + b*x] + Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])))/(c^2*
(b*c - a*d)))/(2*c*(-(b*c) + a*d))
```

IntegrateAlgebraic [A] time = 1.26, size = 248, normalized size = 1.21

$$
-\frac{b\sqrt{a+bx}\left(-4a^2d^3e-bc^2df(a+bx)-abc^2df-3bcd^2e(a+bx)+9abcd^2e+4ad^3e(a+bx)+b^2c^3f-5b^2c^2de\right)}{4c^2d(bc-ad)(d(a+bx)-ad+bc)^2}+\frac{\left(8a^2d^3e-12abcd^2e+b^2c^3f+3b^2c^2de\right)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{4c^3d^{3/2}(bc-ad)^{3/2}}-\frac{2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{c^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x]*(e + f*x))/(x*(c + d*x)^3),x]
```

```
[Out] -1/4*(b*Sqrt[a + b*x]*(-5*b^2*c^2*d*e + 9*a*b*c*d^2*e - 4*a^2*d^3*e + b^2*c
^3*f - a*b*c^2*d*f - 3*b*c*d^2*e*(a + b*x) + 4*a*d^3*e*(a + b*x) - b*c^2*d*
f*(a + b*x)))/(c^2*d*(b*c - a*d)*(b*c - a*d + d*(a + b*x))^2) + ((3*b^2*c^2
*d*e - 12*a*b*c*d^2*e + 8*a^2*d^3*e + b^2*c^3*f)*ArcTan[(Sqrt[d]*Sqrt[a + b
*x])/Sqrt[b*c - a*d]])/(4*c^3*d^(3/2)*(b*c - a*d)^(3/2)) - (2*Sqrt[a]*e*Arc
Tanh[Sqrt[a + b*x]/Sqrt[a]])/c^3
```

fricas [B] time = 3.64, size = 2211, normalized size = 10.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] [1/8*((b^2*c^5*f + (b^2*c^3*d^2*f + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d
^5)*e)*x^2 + (3*b^2*c^4*d - 12*a*b*c^3*d^2 + 8*a^2*c^2*d^3)*e + 2*(b^2*c^4*
d*f + (3*b^2*c^3*d^2 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*e)*x)*sqrt(-b*c*d + a*
d^2)*log((b*d*x - b*c + 2*a*d + 2*sqrt(-b*c*d + a*d^2)*sqrt(b*x + a))/(d*x
+ c)) + 8*((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*d^6)*e*x^2 + 2*(b^2*c^3*d^3 - 2
*a*b*c^2*d^4 + a^2*c*d^5)*e*x + (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)
*e)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*((5*b^2*c^4*d^
2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4)*e - (b^2*c^5*d - 3*a*b*c^4*d^2 + 2*a^2*
c^3*d^3)*f + ((3*b^2*c^3*d^3 - 7*a*b*c^2*d^4 + 4*a^2*c*d^5)*e + (b^2*c^4*d^
2 - a*b*c^3*d^3)*f)*x)*sqrt(b*x + a))/(b^2*c^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^
5*d^4 + (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*c^3*d^6)*x^2 + 2*(b^2*c^6*d^3 -
2*a*b*c^5*d^4 + a^2*c^4*d^5)*x), 1/8*(16*((b^2*c^2*d^4 - 2*a*b*c*d^5 + a^2*
d^6)*e*x^2 + 2*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*e*x + (b^2*c^4*d^2
 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*e)*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a
) + (b^2*c^5*f + (b^2*c^3*d^2*f + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d^5
)*e)*x^2 + (3*b^2*c^4*d - 12*a*b*c^3*d^2 + 8*a^2*c^2*d^3)*e + 2*(b^2*c^4*d*
f + (3*b^2*c^3*d^2 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*e)*x)*sqrt(-b*c*d + a*d^
2)*log((b*d*x - b*c + 2*a*d + 2*sqrt(-b*c*d + a*d^2)*sqrt(b*x + a))/(d*x +
c)) + 2*((5*b^2*c^4*d^2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4)*e - (b^2*c^5*d -
3*a*b*c^4*d^2 + 2*a^2*c^3*d^3)*f + ((3*b^2*c^3*d^3 - 7*a*b*c^2*d^4 + 4*a^2*
c*d^5)*e + (b^2*c^4*d^2 - a*b*c^3*d^3)*f)*x)*sqrt(b*x + a))/(b^2*c^7*d^2 -
2*a*b*c^6*d^3 + a^2*c^5*d^4 + (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*c^3*d^6)*x
^2 + 2*(b^2*c^6*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x), -1/4*((b^2*c^5*f + (
b^2*c^3*d^2*f + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d^5)*e)*x^2 + (3*b^2*
c^4*d - 12*a*b*c^3*d^2 + 8*a^2*c^2*d^3)*e + 2*(b^2*c^4*d*f + (3*b^2*c^3*d^2
 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*e)*x)*sqrt(b*c*d - a*d^2)*arctan(sqrt(b*c*
d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) - 4*((b^2*c^2*d^4 - 2*a*b*c*d^5 + a
^2*d^6)*e*x^2 + 2*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*e*x + (b^2*c^4*
d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*e)*sqrt(a)*log((b*x - 2*sqrt(b*x + a)*sq
rt(a) + 2*a)/x) - ((5*b^2*c^4*d^2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4)*e - (b^
2*c^5*d - 3*a*b*c^4*d^2 + 2*a^2*c^3*d^3)*f + ((3*b^2*c^3*d^3 - 7*a*b*c^2*d^
4 + 4*a^2*c*d^5)*e + (b^2*c^4*d^2 - a*b*c^3*d^3)*f)*x)*sqrt(b*x + a))/(b^2*
c^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4 + (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*
c^3*d^6)*x^2 + 2*(b^2*c^6*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x), -1/4*((b^2
*c^5*f + (b^2*c^3*d^2*f + (3*b^2*c^2*d^3 - 12*a*b*c*d^4 + 8*a^2*d^5)*e)*x^2
 + (3*b^2*c^4*d - 12*a*b*c^3*d^2 + 8*a^2*c^2*d^3)*e + 2*(b^2*c^4*d*f + (3*b
^2*c^3*d^2 - 12*a*b*c^2*d^3 + 8*a^2*c*d^4)*e)*x)*sqrt(b*c*d - a*d^2)*arctan
(sqrt(b*c*d - a*d^2)*sqrt(b*x + a)/(b*d*x + a*d)) - 8*((b^2*c^2*d^4 - 2*a*b
*c*d^5 + a^2*d^6)*e*x^2 + 2*(b^2*c^3*d^3 - 2*a*b*c^2*d^4 + a^2*c*d^5)*e*x +
 (b^2*c^4*d^2 - 2*a*b*c^3*d^3 + a^2*c^2*d^4)*e)*sqrt(-a)*arctan(sqrt(b*x +
a)*sqrt(-a)/a) - ((5*b^2*c^4*d^2 - 11*a*b*c^3*d^3 + 6*a^2*c^2*d^4)*e - (b^2
*c^5*d - 3*a*b*c^4*d^2 + 2*a^2*c^3*d^3)*f + ((3*b^2*c^3*d^3 - 7*a*b*c^2*d^4
 + 4*a^2*c*d^5)*e + (b^2*c^4*d^2 - a*b*c^3*d^3)*f)*x)*sqrt(b*x + a))/(b^2*c
^7*d^2 - 2*a*b*c^6*d^3 + a^2*c^5*d^4 + (b^2*c^5*d^4 - 2*a*b*c^4*d^5 + a^2*c
^3*d^6)*x^2 + 2*(b^2*c^6*d^3 - 2*a*b*c^5*d^4 + a^2*c^4*d^5)*x)]
```

giac [A] time = 1.40, size = 301, normalized size = 1.47

$$\frac{\left(b^2c^3f+3\,b^2c^2de-12\,abcd^2e+8\,a^2d^3e\right)\arctan\left(\frac{\sqrt{bx+a}\,d}{\sqrt{bcd-ad^2}}\right)}{4\left(bc^4d-ac^3d^2\right)\sqrt{bcd-ad^2}}+\frac{2\,a\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)e}{\sqrt{-a}\,c^3}-\frac{\sqrt{bx+a}\,b^3c^3f-(bx+a)^{\frac{3}{2}}b^2c^2df-\sqrt{bx+a}\,ab^2c^2df-5\sqrt{bx+a}\,b^3c^2de-3\,(bx+a)^{\frac{3}{2}}b^2cd^2e+9\sqrt{bx+a}\,ab^2cd^2e+4\,(bx+a)^{\frac{3}{2}}abd^3e-4\sqrt{bx+a}\,a^2bd^3e}{4\left(bc^3d-ac^2d^2\right)(bc+(bx+a)d-ad)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e + 8*a^2*d^3*e)*arctan(sqrt(
b*x + a)*d/sqrt(b*c*d - a*d^2))/((b*c^4*d - a*c^3*d^2)*sqrt(b*c*d - a*d^2))
 + 2*a*arctan(sqrt(b*x + a)/sqrt(-a))*e/(sqrt(-a)*c^3) - 1/4*(sqrt(b*x + a)
*b^3*c^3*f - (b*x + a)^(3/2)*b^2*c^2*d*f - sqrt(b*x + a)*a*b^2*c^2*d*f - 5*
sqrt(b*x + a)*b^3*c^2*d*e - 3*(b*x + a)^(3/2)*b^2*c*d^2*e + 9*sqrt(b*x + a)
*a*b^2*c*d^2*e + 4*(b*x + a)^(3/2)*a*b*d^3*e - 4*sqrt(b*x + a)*a^2*b*d^3*e)
/((b*c^3*d - a*c^2*d^2)*(b*c + (b*x + a)*d - a*d)^2)
```

maple [A] time = 0.02, size = 221, normalized size = 1.08

$$2\left(-\frac{\sqrt{a}\,e\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b^2c^3}-\frac{-\frac{\left(8a^2d^3e-12abc\,d^2e+c^3b^2f+3b^2c^2de\right)\mathrm{arctanh}\left(\frac{\sqrt{bx+a}\,d}{\sqrt{(ad-bc)d}}\right)}{8(ad-bc)\sqrt{(ad-bc)d}\,d}+\frac{-\frac{\left(4a\,d^2e-b\,c^2f-3bcde\right)(bx+a)^{\frac{3}{2}}bc}{8(ad-bc)}+\frac{\left(4a\,d^2e+b\,c^2f-5bcde\right)\sqrt{bx+a}\,bc}{8d}}{(-ad+bc+(bx+a)d)^2}}{b^2c^3}\right)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x)
```

```
[Out] 2*b^2*(-a^(1/2)/b^2*e/c^3*arctanh((b*x+a)^(1/2)/a^(1/2))-1/c^3/b^2*((-1/8*b
*c*(4*a*d^2*e-b*c^2*f-3*b*c*d*e)/(a*d-b*c)*(b*x+a)^(3/2)+1/8*(4*a*d^2*e+b*c
^2*f-5*b*c*d*e)*b*c/d*(b*x+a)^(1/2))/(-a*d+b*c+(b*x+a)*d)^2-1/8*(8*a^2*d^3*
e-12*a*b*c*d^2*e+b^2*c^3*f+3*b^2*c^2*d*e)/(a*d-b*c)/d/((a*d-b*c)*d)^(1/2)*a
rctanh((b*x+a)^(1/2)/((a*d-b*c)*d)^(1/2)*d)))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)^(1/2)/x/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more
 details)Is a*d-b*c positive or negative?
```

mupad [B] time = 4.63, size = 4839, normalized size = 23.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(a + b*x)^(1/2))/(x*(c + d*x)^3),x)
```

```
[Out] (atan((((d^3*(a*d - b*c)^3)^(1/2)*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*
b^2*d^6*e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2
 - 72*a*b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f -
24*a*b^5*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) - ((d^
3*(a*d - b*c)^3)^(1/2)*((5*a*b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^
7*d^4*e + 4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3
 - 2*a*b*c^7*d^2) - ((d^3*(a*d - b*c)^3)^(1/2)*(a + b*x)^(1/2)*(8*a^2*d^3*e
 + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e)*(64*b^5*c^9*d^3 - 256*a*b^4*
c^8*d^4 + 320*a^2*b^3*c^7*d^5 - 128*a^3*b^2*c^6*d^6))/(64*(b^2*c^6*d + a^2*
```

```
c^4*d^3 - 2*a*b*c^5*d^2)*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a
^2*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e))
/(8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^
2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e)*1i)/(8*(a^3*c^3*d^6 -
 b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)) + ((d^3*(a*d - b*c)^3)^(
1/2)*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^2*d^6*e^2 + 9*b^6*c^4*d^2*e
^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 72*a*b^5*c^3*d^3*e^2 - 320
*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f - 24*a*b^5*c^4*d^2*e*f))/(8*(b^
2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) + ((d^3*(a*d - b*c)^3)^(1/2)*((5*a*
b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*d^4*e + 4*a^3*b^3*c^6*d^5*e
 + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) + ((d^3*(a*
d - b*c)^3)^(1/2)*(a + b*x)^(1/2)*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e
- 12*a*b*c*d^2*e)*(64*b^5*c^9*d^3 - 256*a*b^4*c^8*d^4 + 320*a^2*b^3*c^7*d^5
 - 128*a^3*b^2*c^6*d^6))/(64*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)*(a^3
*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*d^3*e
+ b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e))/(8*(a^3*c^3*d^6 - b^3*c^6*d^
3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c
^2*d*e - 12*a*b*c*d^2*e)*1i)/(8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^
4 - 3*a^2*b*c^4*d^5)))/(((a*b^6*c^5*e*f^2)/4 - 12*a^2*b^5*c^2*d^3*e^3 - 8*a
^4*b^3*d^5*e^3 + (9*a*b^6*c^3*d^2*e^3)/4 + 18*a^3*b^4*c*d^4*e^3 - 4*a^2*b^5
*c^3*d^2*e^2*f + 2*a^3*b^4*c^2*d^3*e^2*f + (3*a*b^6*c^4*d*e^2*f)/2)/(b^2*c^
8*d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) - ((d^3*(a*d - b*c)^3)^(1/2)*(((a + b*x)
^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^2*d^6*e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d
*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 72*a*b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e
^2 + 16*a^2*b^4*c^3*d^3*e*f - 24*a*b^5*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^
4*d^3 - 2*a*b*c^5*d^2)) - ((d^3*(a*d - b*c)^3)^(1/2)*((5*a*b^5*c^8*d^3*e -
a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*d^4*e + 4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d
^3*f)/(b^2*c^8*d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) - ((d^3*(a*d - b*c)^3)^(1/2
)*(a + b*x)^(1/2)*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e
)*(64*b^5*c^9*d^3 - 256*a*b^4*c^8*d^4 + 320*a^2*b^3*c^7*d^5 - 128*a^3*b^2*c
^6*d^6))/(64*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)*(a^3*c^3*d^6 - b^3*c
^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*
b^2*c^2*d*e - 12*a*b*c*d^2*e))/(8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*
d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*
c*d^2*e))/(8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5
)) + ((d^3*(a*d - b*c)^3)^(1/2)*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^
2*d^6*e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2 -
 72*a*b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f - 24
*a*b^5*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) + ((d^3*
(a*d - b*c)^3)^(1/2)*((5*a*b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*
d^4*e + 4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3 -
 2*a*b*c^7*d^2) + ((d^3*(a*d - b*c)^3)^(1/2)*(a + b*x)^(1/2)*(8*a^2*d^3*e +
 b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e)*(64*b^5*c^9*d^3 - 256*a*b^4*c^
8*d^4 + 320*a^2*b^3*c^7*d^5 - 128*a^3*b^2*c^6*d^6))/(64*(b^2*c^6*d + a^2*c^
4*d^3 - 2*a*b*c^5*d^2)*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2
*b*c^4*d^5)))*(8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e))/(
8*(a^3*c^3*d^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)))*(8*a^2*
d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e))/(8*(a^3*c^3*d^6 - b^3*
c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5))))*(d^3*(a*d - b*c)^3)^(1/2)*(
8*a^2*d^3*e + b^2*c^3*f + 3*b^2*c^2*d*e - 12*a*b*c*d^2*e)*1i)/(4*(a^3*c^3*d
^6 - b^3*c^6*d^3 + 3*a*b^2*c^5*d^4 - 3*a^2*b*c^4*d^5)) - (((a + b*x)^(1/2)*
(b^2*c^2*f + 4*a*b*d^2*e - 5*b^2*c*d*e))/(4*c^2*d) + ((a + b*x)^(3/2)*(b^2*
c^2*f - 4*a*b*d^2*e + 3*b^2*c*d*e))/(4*c^2*(a*d - b*c)))/(d^2*(a + b*x)^2 -
 (2*a*d^2 - 2*b*c*d)*(a + b*x) + a^2*d^2 + b^2*c^2 - 2*a*b*c*d) + (a^(1/2)*
e*atan(((a^(1/2)*e*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^2*d^6*e^2 + 9
*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 72*a*b^5*c^3
*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f - 24*a*b^5*c^4*d^
2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) + (a^(1/2)*e*((5*a*b^
5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*d^4*e + 4*a^3*b^3*c^6*d^5*e +
```

```
a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) + (a^(1/2)*e*
(a + b*x)^(1/2)*(64*b^5*c^9*d^3 - 256*a*b^4*c^8*d^4 + 320*a^2*b^3*c^7*d^5 -
 128*a^3*b^2*c^6*d^6))/(8*c^3*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2))))/
c^3)*1i)/c^3 + (a^(1/2)*e*(((a + b*x)^(1/2)*(b^6*c^6*f^2 + 128*a^4*b^2*d^6*
e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4*c^2*d^4*e^2 - 72*a*
b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^3*d^3*e*f - 24*a*b^5
*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d^2)) - (a^(1/2)*e*(
(5*a*b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*d^4*e + 4*a^3*b^3*c^6*
d^5*e + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) - (a^(
1/2)*e*(a + b*x)^(1/2)*(64*b^5*c^9*d^3 - 256*a*b^4*c^8*d^4 + 320*a^2*b^3*c^
7*d^5 - 128*a^3*b^2*c^6*d^6))/(8*c^3*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*d
^2))))/c^3)*1i)/c^3)/(((a*b^6*c^5*e*f^2)/4 - 12*a^2*b^5*c^2*d^3*e^3 - 8*a^4
*b^3*d^5*e^3 + (9*a*b^6*c^3*d^2*e^3)/4 + 18*a^3*b^4*c*d^4*e^3 - 4*a^2*b^5*c
^3*d^2*e^2*f + 2*a^3*b^4*c^2*d^3*e^2*f + (3*a*b^6*c^4*d*e^2*f)/2)/(b^2*c^8*
d + a^2*c^6*d^3 - 2*a*b*c^7*d^2) + (a^(1/2)*e*(((a + b*x)^(1/2)*(b^6*c^6*f^
2 + 128*a^4*b^2*d^6*e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2*b^4
*c^2*d^4*e^2 - 72*a*b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^4*c^
3*d^3*e*f - 24*a*b^5*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*c^5*
d^2)) + (a^(1/2)*e*((5*a*b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^7*d^
4*e + 4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3 - 2
*a*b*c^7*d^2) + (a^(1/2)*e*(a + b*x)^(1/2)*(64*b^5*c^9*d^3 - 256*a*b^4*c^8*
d^4 + 320*a^2*b^3*c^7*d^5 - 128*a^3*b^2*c^6*d^6))/(8*c^3*(b^2*c^6*d + a^2*c
^4*d^3 - 2*a*b*c^5*d^2))))/c^3))/c^3 - (a^(1/2)*e*(((a + b*x)^(1/2)*(b^6*c^
6*f^2 + 128*a^4*b^2*d^6*e^2 + 9*b^6*c^4*d^2*e^2 + 6*b^6*c^5*d*e*f + 256*a^2
*b^4*c^2*d^4*e^2 - 72*a*b^5*c^3*d^3*e^2 - 320*a^3*b^3*c*d^5*e^2 + 16*a^2*b^
4*c^3*d^3*e*f - 24*a*b^5*c^4*d^2*e*f))/(8*(b^2*c^6*d + a^2*c^4*d^3 - 2*a*b*
c^5*d^2)) - (a^(1/2)*e*((5*a*b^5*c^8*d^3*e - a*b^5*c^9*d^2*f - 9*a^2*b^4*c^
7*d^4*e + 4*a^3*b^3*c^6*d^5*e + a^2*b^4*c^8*d^3*f)/(b^2*c^8*d + a^2*c^6*d^3
 - 2*a*b*c^7*d^2) - (a^(1/2)*e*(a + b*x)^(1/2)*(64*b^5*c^9*d^3 - 256*a*b^4*
c^8*d^4 + 320*a^2*b^3*c^7*d^5 - 128*a^3*b^2*c^6*d^6))/(8*c^3*(b^2*c^6*d + a
^2*c^4*d^3 - 2*a*b*c^5*d^2))))/c^3))/c^3))*2i)/c^3
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x+e)*(b*x+a)**(1/2)/x/(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.22 $\int \frac{x^3(1+ax)}{\sqrt{ax}\,\sqrt{1-ax}}\,dx$

Optimal. Leaf size=111

$$-\frac{\sqrt{1-ax}\,(ax)^{7/2}}{4a^4}-\frac{5\sqrt{1-ax}\,(ax)^{5/2}}{8a^4}-\frac{25\sqrt{1-ax}\,(ax)^{3/2}}{32a^4}-\frac{75\sqrt{1-ax}\,\sqrt{ax}}{64a^4}-\frac{75\sin^{-1}(1-2ax)}{128a^4}$$

Rubi [A] time = 0.04, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {16, 80, 50, 53, 619, 216}

$$-\frac{\sqrt{1-ax}\,(ax)^{7/2}}{4a^4}-\frac{5\sqrt{1-ax}\,(ax)^{5/2}}{8a^4}-\frac{25\sqrt{1-ax}\,(ax)^{3/2}}{32a^4}-\frac{75\sqrt{1-ax}\,\sqrt{ax}}{64a^4}-\frac{75\sin^{-1}(1-2ax)}{128a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-75*Sqrt[a*x]*Sqrt[1 - a*x])/(64*a^4) - (25*(a*x)^(3/2)*Sqrt[1 - a*x])/(32
*a^4) - (5*(a*x)^(5/2)*Sqrt[1 - a*x])/(8*a^4) - ((a*x)^(7/2)*Sqrt[1 - a*x])
/(4*a^4) - (75*ArcSin[1 - 2*a*x])/(128*a^4)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
+ 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx &= \frac{\int \frac{(ax)^{5/2}(1+ax)}{\sqrt{1-ax}}\,dx}{a^3}\\
&= -\frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{15\int \frac{(ax)^{5/2}}{\sqrt{1-ax}}\,dx}{8a^3}\\
&= -\frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{25\int \frac{(ax)^{3/2}}{\sqrt{1-ax}}\,dx}{16a^3}\\
&= -\frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{75\int \frac{\sqrt{ax}}{\sqrt{1-ax}}\,dx}{64a^3}\\
&= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{75\int \frac{1}{\sqrt{ax}\sqrt{1}}}{128a^3}\\
&= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} + \frac{75\int \frac{1}{\sqrt{ax-a^2}}}{128a^3}\\
&= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} - \frac{75\,\mathrm{Subst}\Big(\int}{}\\
&= -\frac{75\sqrt{ax}\sqrt{1-ax}}{64a^4} - \frac{25(ax)^{3/2}\sqrt{1-ax}}{32a^4} - \frac{5(ax)^{5/2}\sqrt{1-ax}}{8a^4} - \frac{(ax)^{7/2}\sqrt{1-ax}}{4a^4} - \frac{75\sin^{-1}(1-}{128a^4}
\end{aligned}
$$

Mathematica [A] time = 0.09, size = 89, normalized size = 0.80

$$\frac{\sqrt{a}\,x\left(16a^4x^4+24a^3x^3+10a^2x^2+25ax-75\right)+75\sqrt{x}\sqrt{1-ax}\,\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{64a^{7/2}\sqrt{-ax(ax-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (Sqrt[a]*x*(-75 + 25*a*x + 10*a^2*x^2 + 24*a^3*x^3 + 16*a^4*x^4) + 75*Sqrt[
x]*Sqrt[1 - a*x]*ArcSin[Sqrt[a]*Sqrt[x]])/(64*a^(7/2)*Sqrt[-(a*x*(-1 + a*x)
)])
```

IntegrateAlgebraic [A] time = 0.10, size = 116, normalized size = 1.05

$$-\frac{75\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax}}\right)}{64a^4}-\frac{\sqrt{1-ax}\left(\frac{75(1-ax)^3}{a^3x^3}+\frac{275(1-ax)^2}{a^2x^2}+\frac{365(1-ax)}{ax}+181\right)}{64a^4\sqrt{ax}\left(\frac{1-ax}{ax}+1\right)^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^3*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] -1/64*(Sqrt[1 - a*x]*(181 + (365*(1 - a*x))/(a*x) + (275*(1 - a*x)^2)/(a^2*
x^2) + (75*(1 - a*x)^3)/(a^3*x^3)))/(a^4*Sqrt[a*x]*(1 + (1 - a*x)/(a*x))^4)
 - (75*ArcTan[Sqrt[1 - a*x]/Sqrt[a*x]])/(64*a^4)
```

fricas [A] time = 1.32, size = 65, normalized size = 0.59

$$-\frac{\left(16\,a^{3}x^{3}+40\,a^{2}x^{2}+50\,ax+75\right)\sqrt{ax}\,\sqrt{-ax+1}\,+75\,\arctan\left(\frac{\sqrt{ax}\,\sqrt{-ax+1}}{ax}\right)}{64\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/64*((16*a^3*x^3 + 40*a^2*x^2 + 50*a*x + 75)*sqrt(a*x)*sqrt(-a*x + 1) + 7
5*arctan(sqrt(a*x)*sqrt(-a*x + 1)/(a*x)))/a^4
```

giac [A] time = 1.25, size = 63, normalized size = 0.57

$$-\frac{\left(2\left(4\,ax\left(\frac{2\,x}{a^{2}}+\frac{5}{a^{3}}\right)+\frac{25}{a^{3}}\right)ax+\frac{75}{a^{3}}\right)\sqrt{ax}\,\sqrt{-ax+1}\,-\frac{75\,\arcsin\left(\sqrt{ax}\right)}{a^{3}}}{64\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/64*((2*(4*a*x*(2*x/a^2 + 5/a^3) + 25/a^3)*a*x + 75/a^3)*sqrt(a*x)*sqrt(-
a*x + 1) - 75*arcsin(sqrt(a*x))/a^3)/a
```

maple [C] time = 0.04, size = 132, normalized size = 1.19

$$-\frac{\sqrt{-ax+1}\,\left(32\sqrt{-\left(ax-1\right)ax}\,a^{3}x^{3}\mathrm{csgn}(a)+80\sqrt{-\left(ax-1\right)ax}\,a^{2}x^{2}\mathrm{csgn}(a)+100\sqrt{-\left(ax-1\right)ax}\,ax\,\mathrm{csgn}(a)-75\arctan\left(\frac{(2ax-1)\mathrm{csgn}(a)}{2\sqrt{-(ax-1)ax}}\right)+150\sqrt{-\left(ax-1\right)ax}\,\mathrm{csgn}(a)\right)x\,\mathrm{csgn}(a)}{128\sqrt{ax}\,\sqrt{-\left(ax-1\right)ax}\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -1/128*(-a*x+1)^(1/2)*x*(32*csgn(a)*x^3*a^3*(-x*(a*x-1)*a)^(1/2)+80*csgn(a)
*x^2*a^2*(-x*(a*x-1)*a)^(1/2)+100*csgn(a)*(-x*(a*x-1)*a)^(1/2)*x*a+150*csgn
(a)*(-x*(a*x-1)*a)^(1/2)-75*arctan(1/2*csgn(a)*(2*a*x-1)/(-x*(a*x-1)*a)^(1/
2)))*csgn(a)/a^3/(a*x)^(1/2)/(-x*(a*x-1)*a)^(1/2)
```

maxima [A] time = 0.97, size = 105, normalized size = 0.95

$$-\frac{\sqrt{-a^{2}x^{2}+ax}\,x^{3}}{4\,a}-\frac{5\,\sqrt{-a^{2}x^{2}+ax}\,x^{2}}{8\,a^{2}}-\frac{25\,\sqrt{-a^{2}x^{2}+ax}\,x}{32\,a^{3}}-\frac{75\,\arcsin\left(-\frac{2\,a^{2}x-a}{a}\right)}{128\,a^{4}}-\frac{75\,\sqrt{-a^{2}x^{2}+ax}}{64\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*sqrt(-a^2*x^2 + a*x)*x^3/a - 5/8*sqrt(-a^2*x^2 + a*x)*x^2/a^2 - 25/32*
sqrt(-a^2*x^2 + a*x)*x/a^3 - 75/128*arcsin(-(2*a^2*x - a)/a)/a^4 - 75/64*sq
rt(-a^2*x^2 + a*x)/a^4
```

mupad [B] time = 7.78, size = 345, normalized size = 3.11

$$\frac{75\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{32\,a^{4}}-\frac{\frac{5\,\sqrt{a\,x}}{4\,\left(\sqrt{1-a\,x}-1\right)}+\frac{85\,{\left(a\,x\right)}^{3/2}}{12\,{\left(\sqrt{1-a\,x}-1\right)}^{3}}+\frac{33\,{\left(a\,x\right)}^{5/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^{5}}-\frac{33\,{\left(a\,x\right)}^{7/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^{7}}-\frac{85\,{\left(a\,x\right)}^{9/2}}{12\,{\left(\sqrt{1-a\,x}-1\right)}^{9}}-\frac{5\,{\left(a\,x\right)}^{11/2}}{4\,{\left(\sqrt{1-a\,x}-1\right)}^{11}}}{a^{4}\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^{2}}+1\right)}^{6}}-\frac{\frac{35\,\sqrt{a\,x}}{32\,\left(\sqrt{1-a\,x}-1\right)}+\frac{805\,{\left(a\,x\right)}^{3/2}}{96\,{\left(\sqrt{1-a\,x}-1\right)}^{3}}+\frac{2681\,{\left(a\,x\right)}^{5/2}}{96\,{\left(\sqrt{1-a\,x}-1\right)}^{5}}+\frac{5053\,{\left(a\,x\right)}^{7/2}}{96\,{\left(\sqrt{1-a\,x}-1\right)}^{7}}-\frac{5053\,{\left(a\,x\right)}^{9/2}}{96\,{\left(\sqrt{1-a\,x}-1\right)}^{9}}-\frac{2681\,{\left(a\,x\right)}^{11/2}}{96\,{\left(\sqrt{1-a\,x}-1\right)}^{11}}-\frac{805\,{\left(a\,x\right)}^{13/2}}{96\,{\left(\sqrt{1-a\,x}-1\right)}^{13}}-\frac{35\,{\left(a\,x\right)}^{15/2}}{32\,{\left(\sqrt{1-a\,x}-1\right)}^{15}}}{a^{4}\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^{2}}+1\right)}^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a*x + 1))/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] (75*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/(32*a^4) - ((5*(a*x)^(1/2))/(4
*((1 - a*x)^(1/2) - 1)) + (85*(a*x)^(3/2))/(12*((1 - a*x)^(1/2) - 1)^3) + (
```

```
33*(a*x)^(5/2))/(2*((1 - a*x)^(1/2) - 1)^5) - (33*(a*x)^(7/2))/(2*((1 - a*x
)^(1/2) - 1)^7) - (85*(a*x)^(9/2))/(12*((1 - a*x)^(1/2) - 1)^9) - (5*(a*x)^
(11/2))/(4*((1 - a*x)^(1/2) - 1)^11))/(a^4*((a*x)/((1 - a*x)^(1/2) - 1)^2 +
 1)^6) - ((35*(a*x)^(1/2))/(32*((1 - a*x)^(1/2) - 1)) + (805*(a*x)^(3/2))/(
96*((1 - a*x)^(1/2) - 1)^3) + (2681*(a*x)^(5/2))/(96*((1 - a*x)^(1/2) - 1)^
5) + (5053*(a*x)^(7/2))/(96*((1 - a*x)^(1/2) - 1)^7) - (5053*(a*x)^(9/2))/(
96*((1 - a*x)^(1/2) - 1)^9) - (2681*(a*x)^(11/2))/(96*((1 - a*x)^(1/2) - 1)
^11) - (805*(a*x)^(13/2))/(96*((1 - a*x)^(1/2) - 1)^13) - (35*(a*x)^(15/2))
/(32*((1 - a*x)^(1/2) - 1)^15))/(a^4*((a*x)/((1 - a*x)^(1/2) - 1)^2 + 1)^8)
```

sympy [C] time = 35.80, size = 484, normalized size = 4.36

$$a\left(\begin{cases} -\frac{35i\,\mathrm{acosh}\left(\sqrt{a}\sqrt{x}\right)}{64a^{5}} - \frac{ix^{\frac{9}{2}}}{4\sqrt{a}\sqrt{ax-1}} - \frac{ix^{\frac{7}{2}}}{24a^{\frac{3}{2}}\sqrt{ax-1}} - \frac{7ix^{\frac{5}{2}}}{96a^{\frac{5}{2}}\sqrt{ax-1}} - \frac{35ix^{\frac{3}{2}}}{192a^{\frac{7}{2}}\sqrt{ax-1}} + \frac{35i\sqrt{x}}{64a^{\frac{9}{2}}\sqrt{ax-1}} & \text{for } |ax|>1 \\ \frac{35\,\mathrm{asin}\left(\sqrt{a}\sqrt{x}\right)}{64a^{5}} + \frac{x^{\frac{9}{2}}}{4\sqrt{a}\sqrt{-ax+1}} + \frac{x^{\frac{7}{2}}}{24a^{\frac{3}{2}}\sqrt{-ax+1}} + \frac{7x^{\frac{5}{2}}}{96a^{\frac{5}{2}}\sqrt{-ax+1}} + \frac{35x^{\frac{3}{2}}}{192a^{\frac{7}{2}}\sqrt{-ax+1}} - \frac{35\sqrt{x}}{64a^{\frac{9}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{5i\,\mathrm{acosh}\left(\sqrt{a}\sqrt{x}\right)}{8a^{4}} - \frac{ix^{\frac{7}{2}}}{3\sqrt{a}\sqrt{ax-1}} - \frac{ix^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{ax-1}} - \frac{5ix^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{ax-1}} + \frac{5i\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{ax-1}} & \text{for } |ax|>1 \\ \frac{5\,\mathrm{asin}\left(\sqrt{a}\sqrt{x}\right)}{8a^{4}} + \frac{x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-ax+1}} + \frac{x^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{-ax+1}} + \frac{5x^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{-ax+1}} - \frac{5\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-35*I*acosh(sqrt(a)*sqrt(x))/(64*a**5) - I*x**(9/2)/(4*sqrt(a)
*sqrt(a*x - 1)) - I*x**(7/2)/(24*a**(3/2)*sqrt(a*x - 1)) - 7*I*x**(5/2)/(96
*a**(5/2)*sqrt(a*x - 1)) - 35*I*x**(3/2)/(192*a**(7/2)*sqrt(a*x - 1)) + 35*
I*sqrt(x)/(64*a**(9/2)*sqrt(a*x - 1)), Abs(a*x) > 1), (35*asin(sqrt(a)*sqrt
(x))/(64*a**5) + x**(9/2)/(4*sqrt(a)*sqrt(-a*x + 1)) + x**(7/2)/(24*a**(3/2
)*sqrt(-a*x + 1)) + 7*x**(5/2)/(96*a**(5/2)*sqrt(-a*x + 1)) + 35*x**(3/2)/(
192*a**(7/2)*sqrt(-a*x + 1)) - 35*sqrt(x)/(64*a**(9/2)*sqrt(-a*x + 1)), Tru
e)) + Piecewise((-5*I*acosh(sqrt(a)*sqrt(x))/(8*a**4) - I*x**(7/2)/(3*sqrt(
a)*sqrt(a*x - 1)) - I*x**(5/2)/(12*a**(3/2)*sqrt(a*x - 1)) - 5*I*x**(3/2)/(
24*a**(5/2)*sqrt(a*x - 1)) + 5*I*sqrt(x)/(8*a**(7/2)*sqrt(a*x - 1)), Abs(a*
x) > 1), (5*asin(sqrt(a)*sqrt(x))/(8*a**4) + x**(7/2)/(3*sqrt(a)*sqrt(-a*x
+ 1)) + x**(5/2)/(12*a**(3/2)*sqrt(-a*x + 1)) + 5*x**(3/2)/(24*a**(5/2)*sqr
t(-a*x + 1)) - 5*sqrt(x)/(8*a**(7/2)*sqrt(-a*x + 1)), True))
```

3.23 $\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}} dx$

Optimal. Leaf size=87

$$-\frac{\sqrt{1-ax}\,(ax)^{5/2}}{3a^3}-\frac{11\sqrt{1-ax}\,(ax)^{3/2}}{12a^3}-\frac{11\sqrt{1-ax}\,\sqrt{ax}}{8a^3}-\frac{11\sin^{-1}(1-2ax)}{16a^3}$$

Rubi [A] time = 0.03, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = $\{16, 80, 50, 53, 619, 216\}$

$$-\frac{\sqrt{1-ax}\,(ax)^{5/2}}{3a^3}-\frac{11\sqrt{1-ax}\,(ax)^{3/2}}{12a^3}-\frac{11\sqrt{1-ax}\,\sqrt{ax}}{8a^3}-\frac{11\sin^{-1}(1-2ax)}{16a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-11*Sqrt[a*x]*Sqrt[1 - a*x])/(8*a^3) - (11*(a*x)^(3/2)*Sqrt[1 - a*x])/(12*
a^3) - ((a*x)^(5/2)*Sqrt[1 - a*x])/(3*a^3) - (11*ArcSin[1 - 2*a*x])/(16*a^3
)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
```

```
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx &= \frac{\int \frac{(ax)^{3/2}(1+ax)}{\sqrt{1-ax}}\,dx}{a^2}\\
&= -\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}+\frac{11\int \frac{(ax)^{3/2}}{\sqrt{1-ax}}\,dx}{6a^2}\\
&= -\frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3}-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}+\frac{11\int \frac{\sqrt{ax}}{\sqrt{1-ax}}\,dx}{8a^2}\\
&= -\frac{11\sqrt{ax}\sqrt{1-ax}}{8a^3}-\frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3}-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}+\frac{11\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx}{16a^2}\\
&= -\frac{11\sqrt{ax}\sqrt{1-ax}}{8a^3}-\frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3}-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}+\frac{11\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx}{16a^2}\\
&= -\frac{11\sqrt{ax}\sqrt{1-ax}}{8a^3}-\frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3}-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}-\frac{11\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,a-2a\right.}{16a^4}\\
&= -\frac{11\sqrt{ax}\sqrt{1-ax}}{8a^3}-\frac{11(ax)^{3/2}\sqrt{1-ax}}{12a^3}-\frac{(ax)^{5/2}\sqrt{1-ax}}{3a^3}-\frac{11\sin^{-1}(1-2ax)}{16a^3}\end{aligned}$$

Mathematica [A] time = 0.03, size = 81, normalized size = 0.93

$$\frac{\sqrt{a}\,x\left(8a^3x^3+14a^2x^2+11ax-33\right)+33\sqrt{x}\sqrt{1-ax}\,\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{24a^{5/2}\sqrt{-ax(ax-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (Sqrt[a]*x*(-33 + 11*a*x + 14*a^2*x^2 + 8*a^3*x^3) + 33*Sqrt[x]*Sqrt[1 - a*
x]*ArcSin[Sqrt[a]*Sqrt[x]])/(24*a^(5/2)*Sqrt[-(a*x*(-1 + a*x))])
```

IntegrateAlgebraic [A] time = 0.09, size = 100, normalized size = 1.15

$$-\frac{11\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax}}\right)}{8a^3}-\frac{\sqrt{1-ax}\left(\frac{33(1-ax)^2}{a^2x^2}+\frac{88(1-ax)}{ax}+63\right)}{24a^3\sqrt{ax}\left(\frac{1-ax}{ax}+1\right)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x^2*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] -1/24*(Sqrt[1 - a*x]*(63 + (88*(1 - a*x))/(a*x) + (33*(1 - a*x)^2)/(a^2*x^2
)))/(a^3*Sqrt[a*x]*(1 + (1 - a*x)/(a*x))^3) - (11*ArcTan[Sqrt[1 - a*x]/Sqrt
[a*x]])/(8*a^3)
```

fricas [A] time = 1.29, size = 57, normalized size = 0.66

$$-\frac{\left(8\,a^2x^2+22\,ax+33\right)\sqrt{ax}\,\sqrt{-ax+1}\,+33\,\arctan\left(\frac{\sqrt{ax}\,\sqrt{-ax+1}}{ax}\right)}{24\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/24*((8*a^2*x^2 + 22*a*x + 33)*sqrt(a*x)*sqrt(-a*x + 1) + 33*arctan(sqrt(
a*x)*sqrt(-a*x + 1)/(a*x)))/a^3
```

giac [A] time = 1.34, size = 53, normalized size = 0.61

$$-\frac{\left(2\,ax\left(\frac{4\,x}{a}+\frac{11}{a^2}\right)+\frac{33}{a^2}\right)\sqrt{ax}\,\sqrt{-ax+1}-\frac{33\,\arcsin\left(\sqrt{ax}\right)}{a^2}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/24*((2*a*x*(4*x/a + 11/a^2) + 33/a^2)*sqrt(a*x)*sqrt(-a*x + 1) - 33*arcs
in(sqrt(a*x))/a^2)/a
```

maple [C] time = 0.02, size = 111, normalized size = 1.28

$$-\frac{\sqrt{-ax+1}\,\left(16\sqrt{-\left(ax-1\right)ax}\,a^2x^2\mathrm{csgn}(a)+44\sqrt{-\left(ax-1\right)ax}\,ax\,\mathrm{csgn}(a)-33\arctan\left(\frac{(2ax-1)\mathrm{csgn}(a)}{2\sqrt{-(ax-1)ax}}\right)+66\sqrt{-\left(ax-1\right)ax}\,\mathrm{csgn}(a)\right)x\,\mathrm{csgn}(a)}{48\sqrt{ax}\,\sqrt{-\left(ax-1\right)ax}\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -1/48*(-a*x+1)^(1/2)*x*(16*(-(a*x-1)*a*x)^(1/2)*a^2*x^2*csgn(a)+44*(-(a*x-1
)*a*x)^(1/2)*a*x*csgn(a)+66*(-(a*x-1)*a*x)^(1/2)*csgn(a)-33*arctan(1/2*(2*a
*x-1)/(-(a*x-1)*a*x)^(1/2)*csgn(a)))*csgn(a)/a^2/(a*x)^(1/2)/(-(a*x-1)*a*x)
^(1/2)
```

maxima [A] time = 0.99, size = 83, normalized size = 0.95

$$-\frac{\sqrt{-a^2x^2+ax}\,x^2}{3\,a}-\frac{11\,\sqrt{-a^2x^2+ax}\,x}{12\,a^2}-\frac{11\,\arcsin\left(-\frac{2\,a^2x-a}{a}\right)}{16\,a^3}-\frac{11\,\sqrt{-a^2x^2+ax}}{8\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(-a^2*x^2 + a*x)*x^2/a - 11/12*sqrt(-a^2*x^2 + a*x)*x/a^2 - 11/16*
arcsin(-(2*a^2*x - a)/a)/a^3 - 11/8*sqrt(-a^2*x^2 + a*x)/a^3
```

mupad [B] time = 5.92, size = 269, normalized size = 3.09

$$\frac{11\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{4\,a^3}-\frac{\frac{5\,\sqrt{a\,x}}{4\,\left(\sqrt{1-a\,x}-1\right)}+\frac{85\,{\left(a\,x\right)}^{3/2}}{12\,{\left(\sqrt{1-a\,x}-1\right)}^3}+\frac{33\,{\left(a\,x\right)}^{5/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^5}-\frac{33\,{\left(a\,x\right)}^{7/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^7}-\frac{85\,{\left(a\,x\right)}^{9/2}}{12\,{\left(\sqrt{1-a\,x}-1\right)}^9}-\frac{5\,{\left(a\,x\right)}^{11/2}}{4\,{\left(\sqrt{1-a\,x}-1\right)}^{11}}}{a^3\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^6}-\frac{\frac{3\,\sqrt{a\,x}}{2\,\left(\sqrt{1-a\,x}-1\right)}+\frac{11\,{\left(a\,x\right)}^{3/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^3}-\frac{11\,{\left(a\,x\right)}^{5/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^5}-\frac{3\,{\left(a\,x\right)}^{7/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^7}}{a^3\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a*x + 1))/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] (11*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/(4*a^3) - ((5*(a*x)^(1/2))/(4*
((1 - a*x)^(1/2) - 1)) + (85*(a*x)^(3/2))/(12*((1 - a*x)^(1/2) - 1)^3) + (3
3*(a*x)^(5/2))/(2*((1 - a*x)^(1/2) - 1)^5) - (33*(a*x)^(7/2))/(2*((1 - a*x)
^(1/2) - 1)^7) - (85*(a*x)^(9/2))/(12*((1 - a*x)^(1/2) - 1)^9) - (5*(a*x)^(
11/2))/(4*((1 - a*x)^(1/2) - 1)^11))/(a^3*((a*x)/((1 - a*x)^(1/2) - 1)^2 +
1)^6) - ((3*(a*x)^(1/2))/(2*((1 - a*x)^(1/2) - 1)) + (11*(a*x)^(3/2))/(2*((
1 - a*x)^(1/2) - 1)^3) - (11*(a*x)^(5/2))/(2*((1 - a*x)^(1/2) - 1)^5) - (3*
```

```
(a*x)^(7/2))/(2*((1 - a*x)^(1/2) - 1)^7))/(a^3*((a*x)/((1 - a*x)^(1/2) - 1)
^2 + 1)^4)
```

sympy [C] time = 25.60, size = 393, normalized size = 4.52

$$a\left(\begin{cases} -\frac{5i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{8a^4} - \frac{ix^{\frac{7}{2}}}{3\sqrt{a}\sqrt{ax-1}} - \frac{ix^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{ax-1}} - \frac{5ix^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{ax-1}} + \frac{5i\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{ax-1}} & \text{for } |ax| > 1 \\ \frac{5\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{8a^4} + \frac{x^{\frac{7}{2}}}{3\sqrt{a}\sqrt{-ax+1}} + \frac{x^{\frac{5}{2}}}{12a^{\frac{3}{2}}\sqrt{-ax+1}} + \frac{5x^{\frac{3}{2}}}{24a^{\frac{5}{2}}\sqrt{-ax+1}} - \frac{5\sqrt{x}}{8a^{\frac{7}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}\right) + \begin{cases} -\frac{3i\operatorname{acosh}\left(\sqrt{a}\sqrt{x}\right)}{4a^3} - \frac{ix^{\frac{5}{2}}}{2\sqrt{a}\sqrt{ax-1}} - \frac{ix^{\frac{3}{2}}}{4a^{\frac{3}{2}}\sqrt{ax-1}} + \frac{3i\sqrt{x}}{4a^{\frac{5}{2}}\sqrt{ax-1}} & \text{for } |ax| > 1 \\ \frac{3\operatorname{asin}\left(\sqrt{a}\sqrt{x}\right)}{4a^3} + \frac{x^{\frac{5}{2}}}{2\sqrt{a}\sqrt{-ax+1}} + \frac{x^{\frac{3}{2}}}{4a^{\frac{3}{2}}\sqrt{-ax+1}} - \frac{3\sqrt{x}}{4a^{\frac{5}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-5*I*acosh(sqrt(a)*sqrt(x))/(8*a**4) - I*x**(7/2)/(3*sqrt(a)*s
qrt(a*x - 1)) - I*x**(5/2)/(12*a**(3/2)*sqrt(a*x - 1)) - 5*I*x**(3/2)/(24*a
**(5/2)*sqrt(a*x - 1)) + 5*I*sqrt(x)/(8*a**(7/2)*sqrt(a*x - 1)), Abs(a*x) >
 1), (5*asin(sqrt(a)*sqrt(x))/(8*a**4) + x**(7/2)/(3*sqrt(a)*sqrt(-a*x + 1)
) + x**(5/2)/(12*a**(3/2)*sqrt(-a*x + 1)) + 5*x**(3/2)/(24*a**(5/2)*sqrt(-a
*x + 1)) - 5*sqrt(x)/(8*a**(7/2)*sqrt(-a*x + 1)), True)) + Piecewise((-3*I*
acosh(sqrt(a)*sqrt(x))/(4*a**3) - I*x**(5/2)/(2*sqrt(a)*sqrt(a*x - 1)) - I*
x**(3/2)/(4*a**(3/2)*sqrt(a*x - 1)) + 3*I*sqrt(x)/(4*a**(5/2)*sqrt(a*x - 1)
), Abs(a*x) > 1), (3*asin(sqrt(a)*sqrt(x))/(4*a**3) + x**(5/2)/(2*sqrt(a)*s
qrt(-a*x + 1)) + x**(3/2)/(4*a**(3/2)*sqrt(-a*x + 1)) - 3*sqrt(x)/(4*a**(5/
2)*sqrt(-a*x + 1)), True))
```

3.24 $\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=63

$$-\frac{\sqrt{1-ax}\,(ax)^{3/2}}{2a^2}-\frac{7\sqrt{1-ax}\,\sqrt{ax}}{4a^2}-\frac{7\sin^{-1}(1-2ax)}{8a^2}$$

Rubi [A] time = 0.02, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {16, 80, 50, 53, 619, 216}

$$-\frac{\sqrt{1-ax}\,(ax)^{3/2}}{2a^2}-\frac{7\sqrt{1-ax}\,\sqrt{ax}}{4a^2}-\frac{7\sin^{-1}(1-2ax)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-7*Sqrt[a*x]*Sqrt[1 - a*x])/(4*a^2) - ((a*x)^(3/2)*Sqrt[1 - a*x])/(2*a^2)
- (7*ArcSin[1 - 2*a*x])/(8*a^2)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(1+ax)}{\sqrt{ax}\sqrt{1-ax}}\,dx &= \frac{\int \frac{\sqrt{ax}(1+ax)}{\sqrt{1-ax}}\,dx}{a}\\
&= -\frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2}+\frac{7\int \frac{\sqrt{ax}}{\sqrt{1-ax}}\,dx}{4a}\\
&= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2}-\frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2}+\frac{7\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx}{8a}\\
&= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2}-\frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2}+\frac{7\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx}{8a}\\
&= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2}-\frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2}-\frac{7\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,a-2a^2x\right)}{8a^3}\\
&= -\frac{7\sqrt{ax}\sqrt{1-ax}}{4a^2}-\frac{(ax)^{3/2}\sqrt{1-ax}}{2a^2}-\frac{7\sin^{-1}(1-2ax)}{8a^2}
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 73, normalized size = 1.16

$$
\frac{\sqrt{a}\,x\left(2a^2x^2+5ax-7\right)+7\sqrt{x}\sqrt{1-ax}\,\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)}{4a^{3/2}\sqrt{-ax(ax-1)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (Sqrt[a]*x*(-7 + 5*a*x + 2*a^2*x^2) + 7*Sqrt[x]*Sqrt[1 - a*x]*ArcSin[Sqrt[a
]*Sqrt[x]])/(4*a^(3/2)*Sqrt[-(a*x*(-1 + a*x))])
```

IntegrateAlgebraic [A] time = 0.08, size = 84, normalized size = 1.33

$$
-\frac{\sqrt{1-ax}\left(\frac{7(1-ax)}{ax}+9\right)}{4a^2\sqrt{ax}\left(\frac{1-ax}{ax}+1\right)^2}-\frac{7\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax}}\right)}{4a^2}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(1 + a*x))/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] -1/4*(Sqrt[1 - a*x]*(9 + (7*(1 - a*x))/(a*x)))/(a^2*Sqrt[a*x]*(1 + (1 - a*x
)/(a*x))^2) - (7*ArcTan[Sqrt[1 - a*x]/Sqrt[a*x]])/(4*a^2)
```

fricas [A] time = 1.31, size = 49, normalized size = 0.78

$$
-\frac{(2\,ax+7)\sqrt{ax}\sqrt{-ax+1}+7\,\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax}\right)}{4\,a^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*((2*a*x + 7)*sqrt(a*x)*sqrt(-a*x + 1) + 7*arctan(sqrt(a*x)*sqrt(-a*x +
 1)/(a*x)))/a^2
```

giac [A] time = 1.28, size = 40, normalized size = 0.63

$$-\frac{\sqrt{ax}\,\sqrt{-ax+1}\left(2\,x+\frac{7}{a}\right)-\frac{7\,\arcsin\left(\sqrt{ax}\right)}{a}}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(a*x)*sqrt(-a*x + 1)*(2*x + 7/a) - 7*arcsin(sqrt(a*x))/a)/a
```

maple [C] time = 0.02, size = 90, normalized size = 1.43

$$-\frac{\sqrt{-ax+1}\,\left(4\sqrt{-\left(ax-1\right)ax}\,ax\,\mathrm{csgn}\left(a\right)-7\arctan\left(\frac{\left(2ax-1\right)\mathrm{csgn}\left(a\right)}{2\sqrt{-\left(ax-1\right)ax}}\right)+14\sqrt{-\left(ax-1\right)ax}\,\,\mathrm{csgn}\left(a\right)\right)x\,\mathrm{csgn}\left(a\right)}{8\sqrt{ax}\,\sqrt{-\left(ax-1\right)ax}\,\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -1/8*(-a*x+1)^(1/2)*x/a*(4*(-(a*x-1)*a*x)^(1/2)*a*x*csgn(a)+14*(-(a*x-1)*a*
x)^(1/2)*csgn(a)-7*arctan(1/2*(2*a*x-1)/(-(a*x-1)*a*x)^(1/2)*csgn(a)))*csgn
(a)/(a*x)^(1/2)/(-(a*x-1)*a*x)^(1/2)
```

maxima [A] time = 0.96, size = 61, normalized size = 0.97

$$-\frac{\sqrt{-a^{2}x^{2}+ax}\,x}{2\,a}-\frac{7\,\arcsin\left(-\frac{2\,a^{2}x-a}{a}\right)}{8\,a^{2}}-\frac{7\,\sqrt{-a^{2}x^{2}+ax}}{4\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + a*x)*x/a - 7/8*arcsin(-(2*a^2*x - a)/a)/a^2 - 7/4*sqrt
(-a^2*x^2 + a*x)/a^2
```

mupad [B] time = 4.53, size = 191, normalized size = 3.03

$$\frac{7\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{2\,a^2}-\frac{\frac{2\,\sqrt{a\,x}}{\sqrt{1-a\,x}-1}-\frac{2\,{\left(a\,x\right)}^{3/2}}{{\left(\sqrt{1-a\,x}-1\right)}^3}}{a^2\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^2}-\frac{\frac{3\,\sqrt{a\,x}}{2\,\left(\sqrt{1-a\,x}-1\right)}+\frac{11\,{\left(a\,x\right)}^{3/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^3}-\frac{11\,{\left(a\,x\right)}^{5/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^5}-\frac{3\,{\left(a\,x\right)}^{7/2}}{2\,{\left(\sqrt{1-a\,x}-1\right)}^7}}{a^2\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + 1))/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] (7*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/(2*a^2) - ((2*(a*x)^(1/2))/((1
- a*x)^(1/2) - 1) - (2*(a*x)^(3/2))/((1 - a*x)^(1/2) - 1)^3)/(a^2*((a*x)/((
1 - a*x)^(1/2) - 1)^2 + 1)^2) - ((3*(a*x)^(1/2))/(2*((1 - a*x)^(1/2) - 1))
+ (11*(a*x)^(3/2))/(2*((1 - a*x)^(1/2) - 1)^3) - (11*(a*x)^(5/2))/(2*((1 -
a*x)^(1/2) - 1)^5) - (3*(a*x)^(7/2))/(2*((1 - a*x)^(1/2) - 1)^7))/(a^2*((a*
x)/((1 - a*x)^(1/2) - 1)^2 + 1)^4)
```

sympy [C] time = 20.77, size = 269, normalized size = 4.27

$$a\left(\begin{cases}-\frac{3i\,\mathrm{acosh}\left(\sqrt{a}\,\sqrt{x}\right)}{4a^{3}}-\frac{ix^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{ax-1}}-\frac{ix^{\frac{3}{2}}}{4a^{\frac{3}{2}}\,\sqrt{ax-1}}+\frac{3i\sqrt{x}}{4a^{\frac{5}{2}}\,\sqrt{ax-1}} & \text{for}\ |ax|>1\\ \frac{3\,\mathrm{asin}\left(\sqrt{a}\,\sqrt{x}\right)}{4a^{3}}+\frac{x^{\frac{5}{2}}}{2\sqrt{a}\,\sqrt{-ax+1}}+\frac{x^{\frac{3}{2}}}{4a^{\frac{3}{2}}\,\sqrt{-ax+1}}-\frac{3\sqrt{x}}{4a^{\frac{5}{2}}\,\sqrt{-ax+1}} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{i\,\mathrm{acosh}\left(\sqrt{a}\,\sqrt{x}\right)}{a^{2}}-\frac{i\sqrt{x}\,\sqrt{ax-1}}{a^{\frac{3}{2}}} & \text{for}\ |ax|>1\\ \frac{\mathrm{asin}\left(\sqrt{a}\,\sqrt{x}\right)}{a^{2}}+\frac{x^{\frac{3}{2}}}{\sqrt{a}\,\sqrt{-ax+1}}-\frac{\sqrt{x}}{a^{\frac{3}{2}}\,\sqrt{-ax+1}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-3*I*acosh(sqrt(a)*sqrt(x))/(4*a**3) - I*x**(5/2)/(2*sqrt(a)*s
qrt(a*x - 1)) - I*x**(3/2)/(4*a**(3/2)*sqrt(a*x - 1)) + 3*I*sqrt(x)/(4*a**(
5/2)*sqrt(a*x - 1)), Abs(a*x) > 1), (3*asin(sqrt(a)*sqrt(x))/(4*a**3) + x**
(5/2)/(2*sqrt(a)*sqrt(-a*x + 1)) + x**(3/2)/(4*a**(3/2)*sqrt(-a*x + 1)) - 3
*sqrt(x)/(4*a**(5/2)*sqrt(-a*x + 1)), True)) + Piecewise((-I*acosh(sqrt(a)*
sqrt(x))/a**2 - I*sqrt(x)*sqrt(a*x - 1)/a**(3/2), Abs(a*x) > 1), (asin(sqrt
(a)*sqrt(x))/a**2 + x**(3/2)/(sqrt(a)*sqrt(-a*x + 1)) - sqrt(x)/(a**(3/2)*s
qrt(-a*x + 1)), True))
```

3.25 $\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=37

$$-\frac{\sqrt{ax}\sqrt{1-ax}}{a}-\frac{3\sin^{-1}(1-2ax)}{2a}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = $\{80, 53, 619, 216\}$

$$-\frac{\sqrt{ax}\sqrt{1-ax}}{a}-\frac{3\sin^{-1}(1-2ax)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] -((Sqrt[a*x]*Sqrt[1 - a*x])/a) - (3*ArcSin[1 - 2*a*x])/(2*a)
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1+ax}{\sqrt{ax}\sqrt{1-ax}}\,dx &= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}+\frac{3}{2}\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx\\
&= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}+\frac{3}{2}\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx\\
&= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}-\frac{3\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx,x,a-2a^2x\right)}{2a^2}\\
&= -\frac{\sqrt{ax}\sqrt{1-ax}}{a}-\frac{3\sin^{-1}(1-2ax)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 61, normalized size = 1.65

$$\frac{\sqrt{a}\,x(ax-1)+3\sqrt{x}\,\sqrt{1-ax}\,\sin^{-1}\left(\sqrt{a}\,\sqrt{x}\right)}{\sqrt{a}\,\sqrt{-ax(ax-1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (Sqrt[a]*x*(-1 + a*x) + 3*Sqrt[x]*Sqrt[1 - a*x]*ArcSin[Sqrt[a]*Sqrt[x]])/(S
qrt[a]*Sqrt[-(a*x*(-1 + a*x))])
```

IntegrateAlgebraic [A] time = 0.11, size = 51, normalized size = 1.38

$$-\frac{\sqrt{ax}\,\sqrt{1-ax}}{a}-\frac{6\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax}+1}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + a*x)/(Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] -((Sqrt[a*x]*Sqrt[1 - a*x])/a) - (6*ArcTan[Sqrt[1 - a*x]/(1 + Sqrt[a*x])])/
a
```

fricas [A] time = 1.31, size = 43, normalized size = 1.16

$$-\frac{\sqrt{ax}\,\sqrt{-ax+1}+3\arctan\left(\frac{\sqrt{ax}\,\sqrt{-ax+1}}{ax}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(a*x)*sqrt(-a*x + 1) + 3*arctan(sqrt(a*x)*sqrt(-a*x + 1)/(a*x)))/a
```

giac [A] time = 1.24, size = 28, normalized size = 0.76

$$-\frac{\sqrt{ax}\,\sqrt{-ax+1}-3\arcsin\left(\sqrt{ax}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(sqrt(a*x)*sqrt(-a*x + 1) - 3*arcsin(sqrt(a*x)))/a
```

maple [C] time = 0.02, size = 70, normalized size = 1.89

$$-\frac{\sqrt{-ax+1}\,\left(-3\arctan\left(\frac{(2ax-1)\mathrm{csgn}(a)}{2\sqrt{-(ax-1)ax}}\right)+2\sqrt{-(ax-1)\,ax}\,\mathrm{csgn}(a)\right)x\,\mathrm{csgn}(a)}{2\sqrt{ax}\,\sqrt{-(ax-1)\,ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -1/2*(-a*x+1)^(1/2)*x*(2*(-(a*x-1)*a*x)^(1/2)*csgn(a)-3*arctan(1/2*(2*a*x-1
)/(-(a*x-1)*a*x)^(1/2)*csgn(a)))*csgn(a)/(a*x)^(1/2)/(-(a*x-1)*a*x)^(1/2)
```

maxima [A] time = 0.95, size = 41, normalized size = 1.11

$$-\frac{3\arcsin\left(-\frac{2a^2x-a}{a}\right)}{2a}-\frac{\sqrt{-a^2x^2+ax}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*arcsin(-(2*a^2*x - a)/a)/a - sqrt(-a^2*x^2 + a*x)/a
```

mupad [B] time = 3.45, size = 118, normalized size = 3.19

$$\frac{2\,\mathrm{atan}\left(\frac{\sqrt{a\,x}}{\sqrt{1-a\,x}-1}\right)}{a}-\frac{4\,\mathrm{atan}\left(\frac{a\,\left(\sqrt{1-a\,x}-1\right)}{\sqrt{a\,x}\,\sqrt{a^2}}\right)}{\sqrt{a^2}}-\frac{\frac{2\,\sqrt{a\,x}}{\sqrt{1-a\,x}-1}-\frac{2\,{\left(a\,x\right)}^{3/2}}{{\left(\sqrt{1-a\,x}-1\right)}^3}}{a\,{\left(\frac{a\,x}{{\left(\sqrt{1-a\,x}-1\right)}^2}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/((a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] (2*atan((a*x)^(1/2)/((1 - a*x)^(1/2) - 1)))/a - (4*atan((a*((1 - a*x)^(1/2)
 - 1))/((a*x)^(1/2)*(a^2)^(1/2))))/(a^2)^(1/2) - ((2*(a*x)^(1/2))/((1 - a*x
)^(1/2) - 1) - (2*(a*x)^(3/2))/((1 - a*x)^(1/2) - 1)^3)/(a*((a*x)/((1 - a*x
)^(1/2) - 1)^2 + 1)^2)
```

sympy [C] time = 11.71, size = 133, normalized size = 3.59

$$a\left(\begin{cases} -\frac{i\,\mathrm{acosh}\left(\sqrt{a}\,\sqrt{x}\right)}{a^2}-\frac{i\sqrt{x}\,\sqrt{ax-1}}{a^{\frac{3}{2}}} & \text{for } |ax|>1 \\ \frac{\mathrm{asin}\left(\sqrt{a}\,\sqrt{x}\right)}{a^2}+\frac{x^{\frac{3}{2}}}{\sqrt{a}\,\sqrt{-ax+1}}-\frac{\sqrt{x}}{a^{\frac{3}{2}}\sqrt{-ax+1}} & \text{otherwise} \end{cases}\right)+\begin{cases} -\frac{2i\,\mathrm{acosh}\left(\sqrt{a}\,\sqrt{x}\right)}{a} & \text{for } |ax|>1 \\ \frac{2\,\mathrm{asin}\left(\sqrt{a}\,\sqrt{x}\right)}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-I*acosh(sqrt(a)*sqrt(x))/a**2 - I*sqrt(x)*sqrt(a*x - 1)/a**(3
/2), Abs(a*x) > 1), (asin(sqrt(a)*sqrt(x))/a**2 + x**(3/2)/(sqrt(a)*sqrt(-a
*x + 1)) - sqrt(x)/(a**(3/2)*sqrt(-a*x + 1)), True)) + Piecewise((-2*I*acos
h(sqrt(a)*sqrt(x))/a, Abs(a*x) > 1), (2*asin(sqrt(a)*sqrt(x))/a, True))
```

3.26 $\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\sqrt{1-ax}}{\sqrt{ax}}-\sin^{-1}(1-2ax)$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {16, 78, 53, 619, 216}

$$-\frac{2\sqrt{1-ax}}{\sqrt{ax}}-\sin^{-1}(1-2ax)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*Sqrt[1 - a*x])/Sqrt[a*x] - ArcSin[1 - 2*a*x]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 53

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 619

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4
*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b
 + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+ax}{x\sqrt{ax}\sqrt{1-ax}}\,dx &= a\int \frac{1+ax}{(ax)^{3/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}} + a\int \frac{1}{\sqrt{ax}\sqrt{1-ax}}\,dx \\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}} + a\int \frac{1}{\sqrt{ax-a^2x^2}}\,dx \\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{a^2}}}\,dx, x, a-2a^2x\right)}{a} \\
&= -\frac{2\sqrt{1-ax}}{\sqrt{ax}} - \sin^{-1}(1-2ax)
\end{aligned}
$$

Mathematica [A] time = 0.03, size = 53, normalized size = 1.83

$$
\frac{2\left(ax+\sqrt{a}\sqrt{x}\sqrt{1-ax}\,\sin^{-1}\left(\sqrt{a}\sqrt{x}\right)-1\right)}{\sqrt{-ax(ax-1)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (2*(-1 + a*x + Sqrt[a]*Sqrt[x]*Sqrt[1 - a*x]*ArcSin[Sqrt[a]*Sqrt[x]]))/Sqrt
[-(a*x*(-1 + a*x))]
```

IntegrateAlgebraic [A] time = 0.10, size = 45, normalized size = 1.55

$$
-\frac{2\sqrt{1-ax}}{\sqrt{ax}} - 4\tan^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{ax}+1}\right)
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + a*x)/(x*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*Sqrt[1 - a*x])/Sqrt[a*x] - 4*ArcTan[Sqrt[1 - a*x]/(1 + Sqrt[a*x])]
```

fricas [B] time = 0.81, size = 47, normalized size = 1.62

$$
-\frac{2\left(ax\arctan\left(\frac{\sqrt{ax}\sqrt{-ax+1}}{ax}\right)+\sqrt{ax}\sqrt{-ax+1}\right)}{ax}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*(a*x*arctan(sqrt(a*x)*sqrt(-a*x + 1)/(a*x)) + sqrt(a*x)*sqrt(-a*x + 1))/
(a*x)
```

giac [A] time = 1.23, size = 44, normalized size = 1.52

$$
-\frac{\sqrt{-ax+1}-1}{\sqrt{ax}} + \frac{\sqrt{ax}}{\sqrt{-ax+1}-1} + 2\arcsin\left(\sqrt{ax}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -(sqrt(-a*x + 1) - 1)/sqrt(a*x) + sqrt(a*x)/(sqrt(-a*x + 1) - 1) + 2*arcsin
(sqrt(a*x))
```

maple [C] time = 0.02, size = 69, normalized size = 2.38

$$\frac{\left(ax\arctan\left(\frac{(2ax-1)\mathrm{csgn}(a)}{2\sqrt{-(ax-1)ax}}\right)-2\sqrt{-(ax-1)\,ax}\;\mathrm{csgn}(a)\right)\sqrt{-ax+1}\;\mathrm{csgn}(a)}{\sqrt{ax}\;\sqrt{-(ax-1)\,ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] (arctan(1/2*(2*a*x-1)/(-(a*x-1)*a*x)^(1/2)*csgn(a))*x*a-2*(-(a*x-1)*a*x)^(1
/2)*csgn(a))*(-a*x+1)^(1/2)*csgn(a)/(a*x)^(1/2)/(-(a*x-1)*a*x)^(1/2)
```

maxima [A] time = 0.95, size = 41, normalized size = 1.41

$$-\frac{2\sqrt{-a^2x^2+ax}}{ax}-\arcsin\left(-\frac{2\,a^2x-a}{a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*sqrt(-a^2*x^2 + a*x)/(a*x) - arcsin(-(2*a^2*x - a)/a)
```

mupad [B] time = 2.98, size = 47, normalized size = 1.62

$$-\frac{2\sqrt{1-a\,x}}{\sqrt{a\,x}}-\frac{4\,a\,\mathrm{atan}\left(\frac{a\left(\sqrt{1-a\,x}-1\right)}{\sqrt{a\,x}\;\sqrt{a^2}}\right)}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] - (2*(1 - a*x)^(1/2))/(a*x)^(1/2) - (4*a*atan((a*((1 - a*x)^(1/2) - 1))/((a
*x)^(1/2)*(a^2)^(1/2))))/(a^2)^(1/2)
```

sympy [C] time = 25.62, size = 71, normalized size = 2.45

$$a\left(\begin{cases}-\frac{2i\,\mathrm{acosh}\left(\sqrt{a}\,\sqrt{x}\right)}{a} & \text{for } |ax|>1\\ \frac{2\,\mathrm{asin}\left(\sqrt{a}\,\sqrt{x}\right)}{a} & \text{otherwise}\end{cases}\right)+\begin{cases}-2\sqrt{-1+\frac{1}{ax}} & \text{for } \frac{1}{|ax|}>1\\ -2i\sqrt{1-\frac{1}{ax}} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-2*I*acosh(sqrt(a)*sqrt(x))/a, Abs(a*x) > 1), (2*asin(sqrt(a)*
sqrt(x))/a, True)) + Piecewise((-2*sqrt(-1 + 1/(a*x)), 1/Abs(a*x) > 1), (-2
*I*sqrt(1 - 1/(a*x)), True))
```

3.27 $\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=45

$$-\frac{10a\sqrt{1-ax}}{3\sqrt{ax}}-\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {16, 78, 37}

$$-\frac{10a\sqrt{1-ax}}{3\sqrt{ax}}-\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x^2*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a*Sqrt[1 - a*x])/(3*(a*x)^(3/2)) - (10*a*Sqrt[1 - a*x])/(3*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{1+ax}{x^2\sqrt{ax}\sqrt{1-ax}}\,dx &= a^2\int \frac{1+ax}{(ax)^{5/2}\sqrt{1-ax}}\,dx\\
&= -\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}}+\frac{1}{3}\left(5a^2\right)\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}\,dx\\
&= -\frac{2a\sqrt{1-ax}}{3(ax)^{3/2}}-\frac{10a\sqrt{1-ax}}{3\sqrt{ax}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.64

$$-\frac{2\sqrt{-ax(ax-1)}\,(5ax+1)}{3ax^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x^2*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(1 + 5*a*x))/(3*a*x^2)
```

IntegrateAlgebraic [A] time = 0.04, size = 37, normalized size = 0.82

$$-\frac{2a\sqrt{1-ax}\left(\frac{1-ax}{ax}+6\right)}{3\sqrt{ax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + a*x)/(x^2*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a*Sqrt[1 - a*x]*(6 + (1 - a*x)/(a*x)))/(3*Sqrt[a*x])
```

fricas [A] time = 0.80, size = 27, normalized size = 0.60

$$-\frac{2\,(5\,ax+1)\sqrt{ax}\,\sqrt{-ax+1}}{3\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(5*a*x + 1)*sqrt(a*x)*sqrt(-a*x + 1)/(a*x^2)
```

giac [B] time = 1.27, size = 88, normalized size = 1.96

$$-\frac{\frac{a^2\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}}+\frac{21\,a^2\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}}-\frac{\left(a^2+\frac{21\,a\left(\sqrt{-ax+1}-1\right)^2}{x}\right)(ax)^{\frac{3}{2}}}{\left(\sqrt{-ax+1}-1\right)^3}}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/12*(a^2*(sqrt(-a*x + 1) - 1)^3/(a*x)^(3/2) + 21*a^2*(sqrt(-a*x + 1) - 1)
/sqrt(a*x) - (a^2 + 21*a*(sqrt(-a*x + 1) - 1)^2/x)*(a*x)^(3/2)/(sqrt(-a*x +
 1) - 1)^3)/a
```

maple [A] time = 0.00, size = 25, normalized size = 0.56

$$-\frac{2\,(5ax+1)\,\sqrt{-ax+1}}{3\sqrt{ax}\;x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -2/3*(5*a*x+1)/x/(a*x)^(1/2)*(-a*x+1)^(1/2)
```

maxima [A] time = 0.96, size = 42, normalized size = 0.93

$$-\frac{10\,\sqrt{-a^2x^2+ax}}{3\,x}-\frac{2\,\sqrt{-a^2x^2+ax}}{3\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^2/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -10/3*sqrt(-a^2*x^2 + a*x)/x - 2/3*sqrt(-a^2*x^2 + a*x)/(a*x^2)
```

mupad [B] time = 2.75, size = 24, normalized size = 0.53

$$-\frac{\sqrt{1-a\,x}\,\left(\frac{10\,a\,x}{3}+\frac{2}{3}\right)}{x\,\sqrt{a\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^2*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] -((1 - a*x)^(1/2)*((10*a*x)/3 + 2/3))/(x*(a*x)^(1/2))
```

sympy [C] time = 15.24, size = 107, normalized size = 2.38

$$a\left(\begin{cases} -2\sqrt{-1+\frac{1}{ax}} & \text{for}\ \frac{1}{|ax|}>1 \\ -2i\sqrt{1-\frac{1}{ax}} & \text{otherwise} \end{cases}\right)+\begin{cases} -\frac{4a\sqrt{-1+\frac{1}{ax}}}{3}-\frac{2\sqrt{-1+\frac{1}{ax}}}{3x} & \text{for}\ \frac{1}{|ax|}>1 \\ -\frac{4ia\sqrt{1-\frac{1}{ax}}}{3}-\frac{2i\sqrt{1-\frac{1}{ax}}}{3x} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x**2/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-2*sqrt(-1 + 1/(a*x)), 1/Abs(a*x) > 1), (-2*I*sqrt(1 - 1/(a*x)
), True)) + Piecewise((-4*a*sqrt(-1 + 1/(a*x))/3 - 2*sqrt(-1 + 1/(a*x))/(3*
x), 1/Abs(a*x) > 1), (-4*I*a*sqrt(1 - 1/(a*x))/3 - 2*I*sqrt(1 - 1/(a*x))/(3
*x), True))
```

3.28 $\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=73

$$-\frac{12a^2\sqrt{1-ax}}{5\sqrt{ax}}-\frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}}-\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}$$

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {16, 78, 45, 37}

$$-\frac{12a^2\sqrt{1-ax}}{5\sqrt{ax}}-\frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}}-\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x^3*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a^2*Sqrt[1 - a*x])/(5*(a*x)^(5/2)) - (6*a^2*Sqrt[1 - a*x])/(5*(a*x)^(3/
2)) - (12*a^2*Sqrt[1 - a*x])/(5*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rubi steps

$$\begin{aligned}\int \frac{1+ax}{x^3\sqrt{ax}\sqrt{1-ax}}\,dx &= a^3\int \frac{1+ax}{(ax)^{7/2}\sqrt{1-ax}}\,dx\\ &= -\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}+\frac{1}{5}\left(9a^3\right)\int \frac{1}{(ax)^{5/2}\sqrt{1-ax}}\,dx\\ &= -\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}-\frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}}+\frac{1}{5}\left(6a^3\right)\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}\,dx\\ &= -\frac{2a^2\sqrt{1-ax}}{5(ax)^{5/2}}-\frac{6a^2\sqrt{1-ax}}{5(ax)^{3/2}}-\frac{12a^2\sqrt{1-ax}}{5\sqrt{ax}}\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.51

$$-\frac{2\sqrt{-ax(ax-1)}\left(6a^2x^2+3ax+1\right)}{5ax^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x^3*Sqrt[a*x]*Sqrt[1 - a*x]),x]
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(1 + 3*a*x + 6*a^2*x^2))/(5*a*x^3)
```

IntegrateAlgebraic [A] time = 0.04, size = 55, normalized size = 0.75

$$-\frac{2a^2\sqrt{1-ax}\left(\frac{(1-ax)^2}{a^2x^2}+\frac{5(1-ax)}{ax}+10\right)}{5\sqrt{ax}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + a*x)/(x^3*Sqrt[a*x]*Sqrt[1 - a*x]),x]
[Out] (-2*a^2*Sqrt[1 - a*x]*(10 + (5*(1 - a*x))/(a*x) + (1 - a*x)^2/(a^2*x^2)))/(
5*Sqrt[a*x])
```

fricas [A] time = 1.33, size = 35, normalized size = 0.48

$$-\frac{2\left(6\,a^2x^2+3\,ax+1\right)\sqrt{ax}\,\sqrt{-ax+1}}{5\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
[Out] -2/5*(6*a^2*x^2 + 3*a*x + 1)*sqrt(a*x)*sqrt(-a*x + 1)/(a*x^3)
```

giac [B] time = 1.24, size = 130, normalized size = 1.78

$$-\frac{\frac{a^3\left(\sqrt{-ax+1}-1\right)^5}{(ax)^{\frac{5}{2}}}+\frac{15\,a^3\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}}+\frac{110\,a^3\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}}-\frac{\left(a^3+\frac{15\,a^2\left(\sqrt{-ax+1}-1\right)^2}{x}+\frac{110\,a\left(\sqrt{-ax+1}-1\right)^4}{x^2}\right)(ax)^{\frac{5}{2}}}{\left(\sqrt{-ax+1}-1\right)^5}}{80\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
[Out] -1/80*(a^3*(sqrt(-a*x + 1) - 1)^5/(a*x)^(5/2) + 15*a^3*(sqrt(-a*x + 1) - 1)
^3/(a*x)^(3/2) + 110*a^3*(sqrt(-a*x + 1) - 1)/sqrt(a*x) - (a^3 + 15*a^2*(sq
```

```
rt(-a*x + 1) - 1)^2/x + 110*a*(sqrt(-a*x + 1) - 1)^4/x^2)*(a*x)^(5/2)/(sqrt
(-a*x + 1) - 1)^5)/a
```

maple [A] time = 0.00, size = 33, normalized size = 0.45

$$-\frac{2\left(6a^2x^2+3ax+1\right)\sqrt{-ax+1}}{5\sqrt{ax}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -2/5*(6*a^2*x^2+3*a*x+1)/x^2/(a*x)^(1/2)*(-a*x+1)^(1/2)
```

maxima [A] time = 0.96, size = 62, normalized size = 0.85

$$-\frac{12\sqrt{-a^2x^2+ax}\,a}{5\,x}-\frac{6\sqrt{-a^2x^2+ax}}{5\,x^2}-\frac{2\sqrt{-a^2x^2+ax}}{5\,ax^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^3/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -12/5*sqrt(-a^2*x^2 + a*x)*a/x - 6/5*sqrt(-a^2*x^2 + a*x)/x^2 - 2/5*sqrt(-a
^2*x^2 + a*x)/(a*x^3)
```

mupad [B] time = 2.73, size = 32, normalized size = 0.44

$$-\frac{\sqrt{1-a\,x}\,\left(\frac{12\,a^2\,x^2}{5}+\frac{6\,a\,x}{5}+\frac{2}{5}\right)}{x^2\,\sqrt{a\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^3*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] -((1 - a*x)^(1/2)*((6*a*x)/5 + (12*a^2*x^2)/5 + 2/5))/(x^2*(a*x)^(1/2))
```

sympy [C] time = 17.84, size = 189, normalized size = 2.59

$$a\left(\begin{cases}-\frac{4a\sqrt{-1+\frac{1}{ax}}}{3}-\frac{2\sqrt{-1+\frac{1}{ax}}}{3x} & \text{for}\ \frac{1}{|ax|}>1\\ -\frac{4ia\sqrt{1-\frac{1}{ax}}}{3}-\frac{2i\sqrt{1-\frac{1}{ax}}}{3x} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{15}-\frac{8a\sqrt{-1+\frac{1}{ax}}}{15x}-\frac{2\sqrt{-1+\frac{1}{ax}}}{5x^2} & \text{for}\ \frac{1}{|ax|}>1\\ -\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{15}-\frac{8ia\sqrt{1-\frac{1}{ax}}}{15x}-\frac{2i\sqrt{1-\frac{1}{ax}}}{5x^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x**3/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-4*a*sqrt(-1 + 1/(a*x))/3 - 2*sqrt(-1 + 1/(a*x))/(3*x), 1/Abs(
a*x) > 1), (-4*I*a*sqrt(1 - 1/(a*x))/3 - 2*I*sqrt(1 - 1/(a*x))/(3*x), True)
) + Piecewise((-16*a**2*sqrt(-1 + 1/(a*x))/15 - 8*a*sqrt(-1 + 1/(a*x))/(15*
x) - 2*sqrt(-1 + 1/(a*x))/(5*x**2), 1/Abs(a*x) > 1), (-16*I*a**2*sqrt(1 - 1
/(a*x))/15 - 8*I*a*sqrt(1 - 1/(a*x))/(15*x) - 2*I*sqrt(1 - 1/(a*x))/(5*x**2
), True))
```

3.29 $\int \frac{1+ax}{x^4\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=97

$$-\frac{208a^3\sqrt{1-ax}}{105\sqrt{ax}}-\frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}}-\frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}}-\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}}$$

Rubi [A] time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {16, 78, 45, 37}

$$-\frac{208a^3\sqrt{1-ax}}{105\sqrt{ax}}-\frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}}-\frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}}-\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x^4*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a^3*Sqrt[1 - a*x])/(7*(a*x)^(7/2)) - (26*a^3*Sqrt[1 - a*x])/(35*(a*x)^(
5/2)) - (104*a^3*Sqrt[1 - a*x])/(105*(a*x)^(3/2)) - (208*a^3*Sqrt[1 - a*x])
/(105*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+ax}{x^4\sqrt{ax}\,\sqrt{1-ax}}\,dx &= a^4\int \frac{1+ax}{(ax)^{9/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}} + \frac{1}{7}\left(13a^4\right)\int \frac{1}{(ax)^{7/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}} - \frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}} + \frac{1}{35}\left(52a^4\right)\int \frac{1}{(ax)^{5/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}} - \frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}} - \frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}} + \frac{1}{105}\left(104a^4\right)\int \frac{1}{(ax)^{3/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^3\sqrt{1-ax}}{7(ax)^{7/2}} - \frac{26a^3\sqrt{1-ax}}{35(ax)^{5/2}} - \frac{104a^3\sqrt{1-ax}}{105(ax)^{3/2}} - \frac{208a^3\sqrt{1-ax}}{105\sqrt{ax}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 45, normalized size = 0.46

$$
-\frac{2\sqrt{-ax(ax-1)}\left(104a^3x^3+52a^2x^2+39ax+15\right)}{105ax^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x^4*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(15 + 39*a*x + 52*a^2*x^2 + 104*a^3*x^3))/(105*
a*x^4)
```

IntegrateAlgebraic [A] time = 0.05, size = 72, normalized size = 0.74

$$
-\frac{2a^3\sqrt{1-ax}\left(\frac{15(1-ax)^3}{a^3x^3}+\frac{84(1-ax)^2}{a^2x^2}+\frac{175(1-ax)}{ax}+210\right)}{105\sqrt{ax}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + a*x)/(x^4*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a^3*Sqrt[1 - a*x]*(210 + (175*(1 - a*x))/(a*x) + (84*(1 - a*x)^2)/(a^2*
x^2) + (15*(1 - a*x)^3)/(a^3*x^3)))/(105*Sqrt[a*x])
```

fricas [A] time = 0.83, size = 43, normalized size = 0.44

$$
-\frac{2\left(104\,a^3x^3+52\,a^2x^2+39\,ax+15\right)\sqrt{ax}\,\sqrt{-ax+1}}{105\,ax^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/105*(104*a^3*x^3 + 52*a^2*x^2 + 39*a*x + 15)*sqrt(a*x)*sqrt(-a*x + 1)/(a
*x^4)
```

giac [B] time = 1.41, size = 175, normalized size = 1.80

$$
-\frac{\frac{15\,a^4\left(\sqrt{-ax+1}-1\right)^7}{(ax)^{\frac{7}{2}}}+\frac{231\,a^4\left(\sqrt{-ax+1}-1\right)^5}{(ax)^{\frac{5}{2}}}+\frac{1435\,a^4\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}}+\frac{7875\,a^4\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}}-\frac{\left(15\,a^4+\frac{231\,a^3\left(\sqrt{-ax+1}-1\right)^2}{x}+\frac{1435\,a^2\left(\sqrt{-ax+1}-1\right)^4}{x^2}+\frac{7875\,a\left(\sqrt{-ax+1}-1\right)^6}{x^3}\right)(ax)^{\frac{7}{2}}}{\left(\sqrt{-ax+1}-1\right)^7}}{6720\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6720*(15*a^4*(sqrt(-a*x + 1) - 1)^7/(a*x)^(7/2) + 231*a^4*(sqrt(-a*x + 1
) - 1)^5/(a*x)^(5/2) + 1435*a^4*(sqrt(-a*x + 1) - 1)^3/(a*x)^(3/2) + 7875*a
^4*(sqrt(-a*x + 1) - 1)/sqrt(a*x) - (15*a^4 + 231*a^3*(sqrt(-a*x + 1) - 1)^
2/x + 1435*a^2*(sqrt(-a*x + 1) - 1)^4/x^2 + 7875*a*(sqrt(-a*x + 1) - 1)^6/x
^3)*(a*x)^(7/2)/(sqrt(-a*x + 1) - 1)^7)/a
```

maple [A] time = 0.01, size = 41, normalized size = 0.42

$$-\frac{2\left(104a^3x^3+52a^2x^2+39ax+15\right)\sqrt{-ax+1}}{105\sqrt{ax}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -2/105*(104*a^3*x^3+52*a^2*x^2+39*a*x+15)/x^3/(a*x)^(1/2)*(-a*x+1)^(1/2)
```

maxima [A] time = 0.97, size = 84, normalized size = 0.87

$$-\frac{208\sqrt{-a^2x^2+ax}\,a^2}{105\,x}-\frac{104\sqrt{-a^2x^2+ax}\,a}{105\,x^2}-\frac{26\sqrt{-a^2x^2+ax}}{35\,x^3}-\frac{2\sqrt{-a^2x^2+ax}}{7\,ax^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^4/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -208/105*sqrt(-a^2*x^2 + a*x)*a^2/x - 104/105*sqrt(-a^2*x^2 + a*x)*a/x^2 -
26/35*sqrt(-a^2*x^2 + a*x)/x^3 - 2/7*sqrt(-a^2*x^2 + a*x)/(a*x^4)
```

mupad [B] time = 2.77, size = 40, normalized size = 0.41

$$-\frac{\sqrt{1-a\,x}\,\left(\frac{208\,a^3\,x^3}{105}+\frac{104\,a^2\,x^2}{105}+\frac{26\,a\,x}{35}+\frac{2}{7}\right)}{x^3\,\sqrt{a\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^4*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] -((1 - a*x)^(1/2)*((26*a*x)/35 + (104*a^2*x^2)/105 + (208*a^3*x^3)/105 + 2/
7))/(x^3*(a*x)^(1/2))
```

sympy [C] time = 23.15, size = 274, normalized size = 2.82

$$a\left(\begin{cases}-\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{15}-\frac{8a\sqrt{-1+\frac{1}{ax}}}{15x}-\frac{2\sqrt{-1+\frac{1}{ax}}}{5x^2} & \text{for}\ \frac{1}{|ax|}>1\\ -\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{15}-\frac{8ia\sqrt{1-\frac{1}{ax}}}{15x}-\frac{2i\sqrt{1-\frac{1}{ax}}}{5x^2} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{32a^3\sqrt{-1+\frac{1}{ax}}}{35}-\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{35x}-\frac{12a\sqrt{-1+\frac{1}{ax}}}{35x^2}-\frac{2\sqrt{-1+\frac{1}{ax}}}{7x^3} & \text{for}\ \frac{1}{|ax|}>1\\ -\frac{32ia^3\sqrt{1-\frac{1}{ax}}}{35}-\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{35x}-\frac{12ia\sqrt{1-\frac{1}{ax}}}{35x^2}-\frac{2i\sqrt{1-\frac{1}{ax}}}{7x^3} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x**4/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-16*a**2*sqrt(-1 + 1/(a*x))/15 - 8*a*sqrt(-1 + 1/(a*x))/(15*x)
 - 2*sqrt(-1 + 1/(a*x))/(5*x**2), 1/Abs(a*x) > 1), (-16*I*a**2*sqrt(1 - 1/(
a*x))/15 - 8*I*a*sqrt(1 - 1/(a*x))/(15*x) - 2*I*sqrt(1 - 1/(a*x))/(5*x**2),
 True)) + Piecewise((-32*a**3*sqrt(-1 + 1/(a*x))/35 - 16*a**2*sqrt(-1 + 1/(
a*x))/(35*x) - 12*a*sqrt(-1 + 1/(a*x))/(35*x**2) - 2*sqrt(-1 + 1/(a*x))/(7*
x**3), 1/Abs(a*x) > 1), (-32*I*a**3*sqrt(1 - 1/(a*x))/35 - 16*I*a**2*sqrt(1
 - 1/(a*x))/(35*x) - 12*I*a*sqrt(1 - 1/(a*x))/(35*x**2) - 2*I*sqrt(1 - 1/(a
*x))/(7*x**3), True))
```

3.30 $\int \frac{1+ax}{x^5\sqrt{ax}\sqrt{1-ax}}\,dx$

Optimal. Leaf size=121

$$-\frac{544a^4\sqrt{1-ax}}{315\sqrt{ax}}-\frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}}-\frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}}-\frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}}-\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}$$

Rubi [A] time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {16, 78, 45, 37}

$$-\frac{544a^4\sqrt{1-ax}}{315\sqrt{ax}}-\frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}}-\frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}}-\frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}}-\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)/(x^5*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a^4*Sqrt[1 - a*x])/(9*(a*x)^(9/2)) - (34*a^4*Sqrt[1 - a*x])/(63*(a*x)^(
7/2)) - (68*a^4*Sqrt[1 - a*x])/(105*(a*x)^(5/2)) - (272*a^4*Sqrt[1 - a*x])/
(315*(a*x)^(3/2)) - (544*a^4*Sqrt[1 - a*x])/(315*Sqrt[a*x])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rubi steps

$$
\begin{aligned}
\int \frac{1+ax}{x^5\sqrt{ax}\,\sqrt{1-ax}}\,dx &= a^5\int \frac{1+ax}{(ax)^{11/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}} + \frac{1}{9}\left(17a^5\right)\int \frac{1}{(ax)^{9/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}} - \frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}} + \frac{1}{21}\left(34a^5\right)\int \frac{1}{(ax)^{7/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}} - \frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}} - \frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}} + \frac{1}{105}\left(136a^5\right)\int \frac{1}{(ax)^{5/2}\sqrt{1-ax}}\,dx \\
&= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}} - \frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}} - \frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}} - \frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}} + \frac{1}{315}\left(272a^5\right)\int \frac{}{(a\cdots} \\
&= -\frac{2a^4\sqrt{1-ax}}{9(ax)^{9/2}} - \frac{34a^4\sqrt{1-ax}}{63(ax)^{7/2}} - \frac{68a^4\sqrt{1-ax}}{105(ax)^{5/2}} - \frac{272a^4\sqrt{1-ax}}{315(ax)^{3/2}} - \frac{544a^4\sqrt{1-ax}}{315\sqrt{ax}}
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 53, normalized size = 0.44

$$
-\frac{2\sqrt{-ax(ax-1)}\left(272a^4x^4+136a^3x^3+102a^2x^2+85ax+35\right)}{315ax^5}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + a*x)/(x^5*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*Sqrt[-(a*x*(-1 + a*x))]*(35 + 85*a*x + 102*a^2*x^2 + 136*a^3*x^3 + 272*
a^4*x^4))/(315*a*x^5)
```

IntegrateAlgebraic [A] time = 0.05, size = 88, normalized size = 0.73

$$
-\frac{2a^4\sqrt{1-ax}\left(\frac{35(1-ax)^4}{a^4x^4}+\frac{225(1-ax)^3}{a^3x^3}+\frac{567(1-ax)^2}{a^2x^2}+\frac{735(1-ax)}{ax}+630\right)}{315\sqrt{ax}}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + a*x)/(x^5*Sqrt[a*x]*Sqrt[1 - a*x]),x]
```

```
[Out] (-2*a^4*Sqrt[1 - a*x]*(630 + (735*(1 - a*x))/(a*x) + (567*(1 - a*x)^2)/(a^2
*x^2) + (225*(1 - a*x)^3)/(a^3*x^3) + (35*(1 - a*x)^4)/(a^4*x^4)))/(315*Sqr
t[a*x])
```

fricas [A] time = 0.91, size = 51, normalized size = 0.42

$$
-\frac{2\left(272\,a^4x^4+136\,a^3x^3+102\,a^2x^2+85\,ax+35\right)\sqrt{ax}\,\sqrt{-ax+1}}{315\,ax^5}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/315*(272*a^4*x^4 + 136*a^3*x^3 + 102*a^2*x^2 + 85*a*x + 35)*sqrt(a*x)*sq
rt(-a*x + 1)/(a*x^5)
```

giac [B] time = 1.32, size = 217, normalized size = 1.79

$$
-\frac{\frac{35\,a^5\left(\sqrt{-ax+1}-1\right)^9}{(ax)^{\frac{9}{2}}}+\frac{585\,a^5\left(\sqrt{-ax+1}-1\right)^7}{(ax)^{\frac{7}{2}}}+\frac{4032\,a^5\left(\sqrt{-ax+1}-1\right)^5}{(ax)^{\frac{5}{2}}}+\frac{17640\,a^5\left(\sqrt{-ax+1}-1\right)^3}{(ax)^{\frac{3}{2}}}+\frac{83790\,a^5\left(\sqrt{-ax+1}-1\right)}{\sqrt{ax}}-\frac{\left(35\,a^5+\frac{585\,a^4\left(\sqrt{-ax+1}-1\right)^2}{x}+\frac{4032\,a^3\left(\sqrt{-ax+1}-1\right)^4}{x^2}+\frac{17640\,a^2\left(\sqrt{-ax+1}-1\right)^6}{x^3}+\frac{83790\,a\left(\sqrt{-ax+1}-1\right)^8}{x^4}\right)(ax)^{\frac{9}{2}}}{\left(\sqrt{-ax+1}-1\right)^9}}{80640\,a}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/80640*(35*a^5*(sqrt(-a*x + 1) - 1)^9/(a*x)^(9/2) + 585*a^5*(sqrt(-a*x +
1) - 1)^7/(a*x)^(7/2) + 4032*a^5*(sqrt(-a*x + 1) - 1)^5/(a*x)^(5/2) + 17640
*a^5*(sqrt(-a*x + 1) - 1)^3/(a*x)^(3/2) + 83790*a^5*(sqrt(-a*x + 1) - 1)/sq
rt(a*x) - (35*a^5 + 585*a^4*(sqrt(-a*x + 1) - 1)^2/x + 4032*a^3*(sqrt(-a*x
+ 1) - 1)^4/x^2 + 17640*a^2*(sqrt(-a*x + 1) - 1)^6/x^3 + 83790*a*(sqrt(-a*x
 + 1) - 1)^8/x^4)*(a*x)^(9/2)/(sqrt(-a*x + 1) - 1)^9)/a
```

maple [A] time = 0.01, size = 49, normalized size = 0.40

$$-\frac{2\left(272a^4x^4+136a^3x^3+102a^2x^2+85ax+35\right)\sqrt{-ax+1}}{315\sqrt{ax}\;x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x)
```

```
[Out] -2/315*(272*a^4*x^4+136*a^3*x^3+102*a^2*x^2+85*a*x+35)/x^4/(a*x)^(1/2)*(-a*
x+1)^(1/2)
```

maxima [A] time = 0.97, size = 106, normalized size = 0.88

$$-\frac{544\sqrt{-a^2x^2+ax}\,a^3}{315\,x}-\frac{272\sqrt{-a^2x^2+ax}\,a^2}{315\,x^2}-\frac{68\sqrt{-a^2x^2+ax}\,a}{105\,x^3}-\frac{34\sqrt{-a^2x^2+ax}}{63\,x^4}-\frac{2\sqrt{-a^2x^2+ax}}{9\,ax^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x^5/(a*x)^(1/2)/(-a*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -544/315*sqrt(-a^2*x^2 + a*x)*a^3/x - 272/315*sqrt(-a^2*x^2 + a*x)*a^2/x^2
- 68/105*sqrt(-a^2*x^2 + a*x)*a/x^3 - 34/63*sqrt(-a^2*x^2 + a*x)/x^4 - 2/9*
sqrt(-a^2*x^2 + a*x)/(a*x^5)
```

mupad [B] time = 2.83, size = 48, normalized size = 0.40

$$-\frac{\sqrt{1-a\,x}\,\left(\frac{544\,a^4\,x^4}{315}+\frac{272\,a^3\,x^3}{315}+\frac{68\,a^2\,x^2}{105}+\frac{34\,a\,x}{63}+\frac{2}{9}\right)}{x^4\,\sqrt{a\,x}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x + 1)/(x^5*(a*x)^(1/2)*(1 - a*x)^(1/2)),x)
```

```
[Out] -((1 - a*x)^(1/2)*((34*a*x)/63 + (68*a^2*x^2)/105 + (272*a^3*x^3)/315 + (54
4*a^4*x^4)/315 + 2/9))/(x^4*(a*x)^(1/2))
```

sympy [C] time = 33.86, size = 359, normalized size = 2.97

$$a\left(\begin{cases}-\frac{32a^3\sqrt{-1+\frac{1}{ax}}}{35}-\frac{16a^2\sqrt{-1+\frac{1}{ax}}}{35x}-\frac{12a\sqrt{-1+\frac{1}{ax}}}{35x^2}-\frac{2\sqrt{-1+\frac{1}{ax}}}{7x^3} & \text{for}\ \frac{1}{|ax|}>1\\ -\frac{32ia^3\sqrt{1-\frac{1}{ax}}}{35}-\frac{16ia^2\sqrt{1-\frac{1}{ax}}}{35x}-\frac{12ia\sqrt{1-\frac{1}{ax}}}{35x^2}-\frac{2i\sqrt{1-\frac{1}{ax}}}{7x^3} & \text{otherwise}\end{cases}\right)+\begin{cases}-\frac{256a^4\sqrt{-1+\frac{1}{ax}}}{315}-\frac{128a^3\sqrt{-1+\frac{1}{ax}}}{315x}-\frac{32a^2\sqrt{-1+\frac{1}{ax}}}{105x^2}-\frac{16a\sqrt{-1+\frac{1}{ax}}}{63x^3}-\frac{2\sqrt{-1+\frac{1}{ax}}}{9x^4} & \text{for}\ \frac{1}{|ax|}>1\\ -\frac{256ia^4\sqrt{1-\frac{1}{ax}}}{315}-\frac{128ia^3\sqrt{1-\frac{1}{ax}}}{315x}-\frac{32ia^2\sqrt{1-\frac{1}{ax}}}{105x^2}-\frac{16ia\sqrt{1-\frac{1}{ax}}}{63x^3}-\frac{2i\sqrt{1-\frac{1}{ax}}}{9x^4} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)/x**5/(a*x)**(1/2)/(-a*x+1)**(1/2),x)
```

```
[Out] a*Piecewise((-32*a**3*sqrt(-1 + 1/(a*x))/35 - 16*a**2*sqrt(-1 + 1/(a*x))/(3
5*x) - 12*a*sqrt(-1 + 1/(a*x))/(35*x**2) - 2*sqrt(-1 + 1/(a*x))/(7*x**3), 1
/Abs(a*x) > 1), (-32*I*a**3*sqrt(1 - 1/(a*x))/35 - 16*I*a**2*sqrt(1 - 1/(a*
x))/(35*x) - 12*I*a*sqrt(1 - 1/(a*x))/(35*x**2) - 2*I*sqrt(1 - 1/(a*x))/(7*
```

```
x**3), True)) + Piecewise((-256*a**4*sqrt(-1 + 1/(a*x))/315 - 128*a**3*sqrt
(-1 + 1/(a*x))/(315*x) - 32*a**2*sqrt(-1 + 1/(a*x))/(105*x**2) - 16*a*sqrt(
-1 + 1/(a*x))/(63*x**3) - 2*sqrt(-1 + 1/(a*x))/(9*x**4), 1/Abs(a*x) > 1), (
-256*I*a**4*sqrt(1 - 1/(a*x))/315 - 128*I*a**3*sqrt(1 - 1/(a*x))/(315*x) -
32*I*a**2*sqrt(1 - 1/(a*x))/(105*x**2) - 16*I*a*sqrt(1 - 1/(a*x))/(63*x**3)
 - 2*I*sqrt(1 - 1/(a*x))/(9*x**4), True))
```

3.31 $\int \frac{-1+2ax}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx$

Optimal. Leaf size=39

$$2a\tan^{-1}\left(\sqrt{x-1}\,\sqrt{x+1}\right)-\frac{\sqrt{x-1}\,\sqrt{x+1}}{x}$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {151, 12, 92, 203}

$$2a\tan^{-1}\left(\sqrt{x-1}\,\sqrt{x+1}\right)-\frac{\sqrt{x-1}\,\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + 2*a*x)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 2*a*ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{-1+2ax}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x} + \int \frac{2a}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx$$
$$= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x} + (2a)\int \frac{1}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx$$
$$= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x} + (2a)\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x}\,\sqrt{1+x}\right)$$
$$= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x} + 2a\tan^{-1}\left(\sqrt{-1+x}\,\sqrt{1+x}\right)$$

Mathematica [A] time = 0.02, size = 48, normalized size = 1.23

$$\frac{2a\sqrt{x^2-1}\,x\tan^{-1}\left(\sqrt{x^2-1}\right)-x^2+1}{\sqrt{x-1}\,x\sqrt{x+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + 2*a*x)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
[Out] (1 - x^2 + 2*a*x*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]])/(Sqrt[-1 + x]*x*Sqr
t[1 + x])
```

IntegrateAlgebraic [A] time = 0.06, size = 49, normalized size = 1.26

$$4a\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{x+1}}\right)-\frac{2\sqrt{x-1}}{\sqrt{x+1}\left(\frac{x-1}{x+1}+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + 2*a*x)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
[Out] (-2*Sqrt[-1 + x])/(Sqrt[1 + x]*(1 + (-1 + x)/(1 + x))) + 4*a*ArcTan[Sqrt[-1
 + x]/Sqrt[1 + x]]
```

fricas [A] time = 0.81, size = 40, normalized size = 1.03

$$\frac{4\,ax\arctan\left(\sqrt{x+1}\,\sqrt{x-1}\,-x\right)-\sqrt{x+1}\,\sqrt{x-1}\,-x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-1)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
[Out] (4*a*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

giac [A] time = 1.27, size = 43, normalized size = 1.10

$$-4\,a\arctan\left(\frac{1}{2}\left(\sqrt{x+1}\,-\sqrt{x-1}\right)^2\right)-\frac{8}{\left(\sqrt{x+1}\,-\sqrt{x-1}\right)^4+4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-1)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
[Out] -4*a*arctan(1/2*(sqrt(x + 1) - sqrt(x - 1))^2) - 8/((sqrt(x + 1) - sqrt(x -
 1))^4 + 4)
```

maple [A] time = 0.02, size = 44, normalized size = 1.13

$$\frac{\left(-2ax\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)-\sqrt{x^2-1}\right)\sqrt{x-1}\,\sqrt{x+1}}{\sqrt{x^2-1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x-1)/x^2/(x-1)^(1/2)/(x+1)^(1/2),x)
```

```
[Out] (-2*x*a*arctan(1/(x^2-1)^(1/2))-(x^2-1)^(1/2))*(x-1)^(1/2)*(x+1)^(1/2)/x/(x
^2-1)^(1/2)
```

maxima [A] time = 0.96, size = 21, normalized size = 0.54

$$-2\,a\arcsin\left(\frac{1}{|x|}\right)-\frac{\sqrt{x^2-1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-1)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*a*arcsin(1/abs(x)) - sqrt(x^2 - 1)/x
```

mupad [B] time = 4.08, size = 65, normalized size = 1.67

$$-\frac{\sqrt{x-1}\,\sqrt{x+1}}{x}-a\left(\ln\left(\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)\right)2\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*a*x - 1)/(x^2*(x - 1)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] - a*(log(((x - 1)^(1/2) - 1i)^2/((x + 1)^(1/2) - 1)^2 + 1) - log(((x - 1)^(
1/2) - 1i)/((x + 1)^(1/2) - 1)))*2i - ((x - 1)^(1/2)*(x + 1)^(1/2))/x
```

sympy [C] time = 35.80, size = 117, normalized size = 3.00

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{x^2}\right)}{2\pi^{\frac{3}{2}}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{x^2}\right)}{2\pi^{\frac{3}{2}}}+\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{2i\pi}}{x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*a*x-1)/x**2/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), x
**(-2))/(2*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4,
3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/x**2)/(2*pi**(3/2)) + meijerg(((
5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), x**(-2))/(4*pi
**(3/2)) + I*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1
, 1, 0)), exp_polar(2*I*pi)/x**2)/(4*pi**(3/2))
```

3.32 $\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx$

Optimal. Leaf size=39

$$2a\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

Rubi [A] time = 0.01, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.139, Rules used = {188, 151, 12, 92, 203}

$$2a\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a^2*x^2 - (1 - a*x)^2)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
```

```
[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + 2*a*ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]
```

Rule 188

```
Int[(u_)^(m_.)*(v_)^(n_.)*(w_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[ExpandToS
um[u, x]^m*ExpandToSum[v, x]^n*ExpandToSum[w, x]^p*ExpandToSum[z, x]^q, x]
/; FreeQ[{m, n, p, q}, x] && LinearQ[{u, v, w, z}, x] &&  !LinearMatchQ[{u,
 v, w, z}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{a^2x^2-(1-ax)^2}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx &= \int \frac{-1+2ax}{\sqrt{-1+x}\,x^2\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+\int \frac{2a}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+(2a)\int \frac{1}{\sqrt{-1+x}\,x\sqrt{1+x}}\,dx\\
&= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+(2a)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sqrt{-1+x}\,\sqrt{1+x}\right)\\
&= -\frac{\sqrt{-1+x}\,\sqrt{1+x}}{x}+2a\tan^{-1}\left(\sqrt{-1+x}\,\sqrt{1+x}\right)
\end{aligned}
$$

Mathematica [A] time = 0.01, size = 48, normalized size = 1.23

$$
\frac{2a\sqrt{x^2-1}\,x\tan^{-1}\left(\sqrt{x^2-1}\right)-x^2+1}{\sqrt{x-1}\,x\sqrt{x+1}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a^2*x^2 - (1 - a*x)^2)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x]
```

```
[Out] (1 - x^2 + 2*a*x*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]])/(Sqrt[-1 + x]*x*Sqr
t[1 + x])
```

IntegrateAlgebraic [A] time = 0.06, size = 49, normalized size = 1.26

$$
4a\tan^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{x+1}}\right)-\frac{2\sqrt{x-1}}{\sqrt{x+1}\,\left(\frac{x-1}{x+1}+1\right)}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a^2*x^2 - (1 - a*x)^2)/(Sqrt[-1 + x]*x^2*Sqrt[1 + x]),x
]
```

```
[Out] (-2*Sqrt[-1 + x])/(Sqrt[1 + x]*(1 + (-1 + x)/(1 + x))) + 4*a*ArcTan[Sqrt[-1
 + x]/Sqrt[1 + x]]
```

fricas [A] time = 1.08, size = 40, normalized size = 1.03

$$
\frac{4\,ax\arctan\left(\sqrt{x+1}\,\sqrt{x-1}\,-x\right)-\sqrt{x+1}\,\sqrt{x-1}\,-x}{x}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-(-a*x+1)^2)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="f
ricas")
```

```
[Out] (4*a*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

giac [A] time = 1.29, size = 43, normalized size = 1.10

$$
-4\,a\arctan\left(\frac{1}{2}\left(\sqrt{x+1}\,-\sqrt{x-1}\right)^2\right)-\frac{8}{\left(\sqrt{x+1}\,-\sqrt{x-1}\right)^4+4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-(-a*x+1)^2)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="g
iac")
```

```
[Out] -4*a*arctan(1/2*(sqrt(x + 1) - sqrt(x - 1))^2) - 8/((sqrt(x + 1) - sqrt(x -
 1))^4 + 4)
```

maple [A] time = 0.00, size = 44, normalized size = 1.13

$$\frac{\left(-2ax\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)-\sqrt{x^2-1}\right)\sqrt{x-1}\,\sqrt{x+1}}{\sqrt{x^2-1}\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a^2*x^2-(-a*x+1)^2)/x^2/(x-1)^(1/2)/(x+1)^(1/2),x)
```

```
[Out] (-2*a*x*arctan(1/(x^2-1)^(1/2))-(x^2-1)^(1/2))*(x-1)^(1/2)*(x+1)^(1/2)/(x^2
-1)^(1/2)/x
```

maxima [A] time = 0.98, size = 21, normalized size = 0.54

$$-2\,a\arcsin\left(\frac{1}{|x|}\right)-\frac{\sqrt{x^2-1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a^2*x^2-(-a*x+1)^2)/x^2/(-1+x)^(1/2)/(1+x)^(1/2),x, algorithm="m
axima")
```

```
[Out] -2*a*arcsin(1/abs(x)) - sqrt(x^2 - 1)/x
```

mupad [B] time = 5.27, size = 444, normalized size = 11.38

$$a\ln\left(\frac{\sqrt{x-1}-i}{\sqrt{x+1}-1}\right)2i-a^2\operatorname{atan}\left(\frac{1024\,a^6}{1024\,a^5+1024\,a^7+\frac{a^6\left(\sqrt{x-1}-i\right)1024i}{\sqrt{x+1}-1}+\frac{a^8\left(\sqrt{x-1}-i\right)1024i}{\sqrt{x+1}-1}}+\frac{1024\,a^8}{1024\,a^5+1024\,a^7+\frac{a^6\left(\sqrt{x-1}-i\right)1024i}{\sqrt{x+1}-1}+\frac{a^8\left(\sqrt{x-1}-i\right)1024i}{\sqrt{x+1}-1}}-\frac{a^5\left(\sqrt{x-1}-i\right)1024i}{\left(\sqrt{x+1}-1\right)\left(1024\,a^5+1024\,a^7+\frac{a^6\left(\sqrt{x-1}-i\right)1024i}{\sqrt{x+1}-1}+\frac{a^8\left(\sqrt{x-1}-i\right)1024i}{\sqrt{x+1}-1}\right)}-\frac{a^7\left(\sqrt{x-1}-i\right)1024i}{\left(\sqrt{x+1}-1\right)\left(1024\,a^5+1024\,a^7+\frac{a^6\left(\sqrt{x-1}-i\right)1024i}{\sqrt{x+1}-1}+\frac{a^8\left(\sqrt{x-1}-i\right)1024i}{\sqrt{x+1}-1}\right)}\right)4i-a\ln\left(\frac{\left(\sqrt{x-1}-i\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1\right)2i-\frac{\sqrt{x-1}-i}{4\left(\sqrt{x+1}-1\right)}+a^2\operatorname{acosh}(x)-\frac{\frac{5\left(\sqrt{x-1}-i\right)^2}{4\left(\sqrt{x+1}-1\right)^2}+\frac{1}{4}}{\frac{\left(\sqrt{x-1}-i\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{\sqrt{x-1}-i}{\sqrt{x+1}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-((a*x - 1)^2 - a^2*x^2)/(x^2*(x - 1)^(1/2)*(x + 1)^(1/2)),x)
```

```
[Out] a*log(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1))*2i - a^2*atan((1024*a^6)/(1
024*a^5 + 1024*a^7 + (a^6*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1) +
 (a^8*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1)) + (1024*a^8)/(1024*a
^5 + 1024*a^7 + (a^6*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1) + (a^8
*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1)) - (a^5*((x - 1)^(1/2) - 1
i)*1024i)/(((x + 1)^(1/2) - 1)*(1024*a^5 + 1024*a^7 + (a^6*((x - 1)^(1/2) -
 1i)*1024i)/((x + 1)^(1/2) - 1) + (a^8*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)
^(1/2) - 1))) - (a^7*((x - 1)^(1/2) - 1i)*1024i)/(((x + 1)^(1/2) - 1)*(1024
*a^5 + 1024*a^7 + (a^6*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1) + (a
^8*((x - 1)^(1/2) - 1i)*1024i)/((x + 1)^(1/2) - 1))))*4i - a*log(((x - 1)^(
1/2) - 1i)^2/((x + 1)^(1/2) - 1)^2 + 1)*2i - ((x - 1)^(1/2) - 1i)/(4*((x +
1)^(1/2) - 1)) + a^2*acosh(x) - ((5*((x - 1)^(1/2) - 1i)^2)/(4*((x + 1)^(1/
2) - 1)^2) + 1/4)/(((x - 1)^(1/2) - 1i)^3/((x + 1)^(1/2) - 1)^3 + ((x - 1)^
(1/2) - 1i)/((x + 1)^(1/2) - 1))
```

sympy [C] time = 72.78, size = 117, normalized size = 3.00

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{x^2}\right)}{2\pi^{\frac{3}{2}}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{x^2}\right)}{2\pi^{\frac{3}{2}}}+\frac{G_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{2i\pi}}{x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a**2*x**2-(-a*x+1)**2)/x**2/(-1+x)**(1/2)/(1+x)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), x
**(-2))/(2*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4,
3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/x**2)/(2*pi**(3/2)) + meijerg(((
5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), x**(-2))/(4*pi
**(3/2)) + I*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1
, 1, 0)), exp_polar(2*I*pi)/x**2)/(4*pi**(3/2))
```

3.33 $\int (a+bx)^m (c+dx)(e+fx)(g+hx)\,dx$

Optimal. Leaf size=167

$$\frac{(a+bx)^{m+2}\left(3a^2dfh-2ab(cfh+deh+dfg)+b^2(ceh+cfg+deg)\right)}{b^4(m+2)}+\frac{(bc-ad)(be-af)(bg-ah)(a+bx)^{m+1}}{b^4(m+1)}$$

Rubi [A] time = 0.13, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {142}

$$\frac{(a+bx)^{m+2}\left(3a^2dfh-2ab(cfh+deh+dfg)+b^2(ceh+cfg+deg)\right)}{b^4(m+2)}+\frac{(bc-ad)(be-af)(bg-ah)(a+bx)^{m+1}}{b^4(m+1)}-\frac{(a+bx)^{m+3}(3adfh-b(cfh+deh+dfg))}{b^4(m+3)}+\frac{dfh(a+bx)^{m+4}}{b^4(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((b*c - a*d)*(b*e - a*f)*(b*g - a*h)*(a + b*x)^(1 + m))/(b^4*(1 + m)) + ((3
*a^2*d*f*h + b^2*(d*e*g + c*f*g + c*e*h) - 2*a*b*(d*f*g + d*e*h + c*f*h))*(
a + b*x)^(2 + m))/(b^4*(2 + m)) - ((3*a*d*f*h - b*(d*f*g + d*e*h + c*f*h))*
(a + b*x)^(3 + m))/(b^4*(3 + m)) + (d*f*h*(a + b*x)^(4 + m))/(b^4*(4 + m))
```

Rule 142

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c +
d*x)^n*(e + f*x)*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] &&
 (IGtQ[m, 0] || IntegersQ[m, n])
```

Rubi steps

$$\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx=\int \left(\frac{(bc-ad)(be-af)(bg-ah)(a+bx)^m}{b^3}+\frac{\left(3a^2dfh+b^2(deg+cf\right.}{}\right.$$

$$=\frac{(bc-ad)(be-af)(bg-ah)(a+bx)^{1+m}}{b^4(1+m)}+\frac{\left(3a^2dfh+b^2(deg+cfg\right.}{}$$

Mathematica [A] time = 0.23, size = 149, normalized size = 0.89

$$\frac{(a+bx)^{m+1}\left(\frac{(a+bx)\left(3a^2dfh-2ab(cfh+deh+dfg)+b^2(ceh+cfg+deg)\right)}{m+2}+\frac{(a+bx)^2(b(cfh+deh+dfg)-3adfh)}{m+3}+\frac{(bc-ad)(be-af)(bg-ah)}{m+1}+\frac{dfh(a+bx)^3}{m+4}\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(((b*c - a*d)*(b*e - a*f)*(b*g - a*h))/(1 + m) + ((3*a^2
*d*f*h + b^2*(d*e*g + c*f*g + c*e*h) - 2*a*b*(d*f*g + d*e*h + c*f*h))*(a +
b*x))/(2 + m) + ((-3*a*d*f*h + b*(d*f*g + d*e*h + c*f*h))*(a + b*x)^2)/(3 +
 m) + (d*f*h*(a + b*x)^3)/(4 + m)))/b^4
```

IntegrateAlgebraic [F] time = 0.11, size = 0, normalized size = 0.00

$$\int (a+bx)^m(c+dx)(e+fx)(g+hx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^m*(c + d*x)*(e + f*x)*(g + h*x),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^m*(c + d*x)*(e + f*x)*(g + h*x), x]
```

fricas [B] time = 0.86, size = 877, normalized size = 5.25

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] (a*b^3*c*e*g*m^3 + (b^4*d*f*h*m^3 + 6*b^4*d*f*h*m^2 + 11*b^4*d*f*h*m + 6*b^
4*d*f*h)*x^4 + (8*b^4*d*f*g + (b^4*d*f*g + (b^4*d*e + (b^4*c + a*b^3*d)*f)*
h)*m^3 + (7*b^4*d*f*g + (7*b^4*d*e + (7*b^4*c + 3*a*b^3*d)*f)*h)*m^2 + 8*(b
^4*d*e + b^4*c*f)*h + 2*(7*b^4*d*f*g + (7*b^4*d*e + (7*b^4*c + a*b^3*d)*f)*
h)*m)*x^3 - (a^2*b^2*c*e*h + (a^2*b^2*c*f - (9*a*b^3*c - a^2*b^2*d)*e)*g)*m
^2 + (12*b^4*c*e*h + ((b^4*d*e + (b^4*c + a*b^3*d)*f)*g + (a*b^3*c*f + (b^4
*c + a*b^3*d)*e)*h)*m^3 + ((8*b^4*d*e + (8*b^4*c + 5*a*b^3*d)*f)*g + ((8*b^
4*c + 5*a*b^3*d)*e + (5*a*b^3*c - 3*a^2*b^2*d)*f)*h)*m^2 + 12*(b^4*d*e + b^
4*c*f)*g + ((19*b^4*d*e + (19*b^4*c + 4*a*b^3*d)*f)*g + ((19*b^4*c + 4*a*b^
3*d)*e + (4*a*b^3*c - 3*a^2*b^2*d)*f)*h)*m)*x^2 + 4*(3*(2*a*b^3*c - a^2*b^2
*d)*e - (3*a^2*b^2*c - 2*a^3*b*d)*f)*g - 2*(2*(3*a^2*b^2*c - 2*a^3*b*d)*e -
 (4*a^3*b*c - 3*a^4*d)*f)*h + (((26*a*b^3*c - 7*a^2*b^2*d)*e - (7*a^2*b^2*c
 - 2*a^3*b*d)*f)*g + (2*a^3*b*c*f - (7*a^2*b^2*c - 2*a^3*b*d)*e)*h)*m + (24
*b^4*c*e*g + (a*b^3*c*e*h + (a*b^3*c*f + (b^4*c + a*b^3*d)*e)*g)*m^3 + (((9
*b^4*c + 7*a*b^3*d)*e + (7*a*b^3*c - 2*a^2*b^2*d)*f)*g - (2*a^2*b^2*c*f - (
7*a*b^3*c - 2*a^2*b^2*d)*e)*h)*m^2 + 2*(((13*b^4*c + 6*a*b^3*d)*e + 2*(3*a*
b^3*c - 2*a^2*b^2*d)*f)*g + (2*(3*a*b^3*c - 2*a^2*b^2*d)*e - (4*a^2*b^2*c -
 3*a^3*b*d)*f)*h)*m)*x)*(b*x + a)^m/(b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 + 50
*b^4*m + 24*b^4)
```

giac [B] time = 1.03, size = 1665, normalized size = 9.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] ((b*x + a)^m*b^4*d*f*h*m^3*x^4 + (b*x + a)^m*b^4*d*f*g*m^3*x^3 + (b*x + a)^
m*b^4*c*f*h*m^3*x^3 + (b*x + a)^m*a*b^3*d*f*h*m^3*x^3 + 6*(b*x + a)^m*b^4*d
*f*h*m^2*x^4 + (b*x + a)^m*b^4*d*h*m^3*x^3*e + (b*x + a)^m*b^4*c*f*g*m^3*x^
2 + (b*x + a)^m*a*b^3*d*f*g*m^3*x^2 + (b*x + a)^m*a*b^3*c*f*h*m^3*x^2 + 7*(
b*x + a)^m*b^4*d*f*g*m^2*x^3 + 7*(b*x + a)^m*b^4*c*f*h*m^2*x^3 + 3*(b*x + a
)^m*a*b^3*d*f*h*m^2*x^3 + 11*(b*x + a)^m*b^4*d*f*h*m*x^4 + (b*x + a)^m*b^4*
d*g*m^3*x^2*e + (b*x + a)^m*b^4*c*h*m^3*x^2*e + (b*x + a)^m*a*b^3*d*h*m^3*x
^2*e + 7*(b*x + a)^m*b^4*d*h*m^2*x^3*e + (b*x + a)^m*a*b^3*c*f*g*m^3*x + 8*
(b*x + a)^m*b^4*c*f*g*m^2*x^2 + 5*(b*x + a)^m*a*b^3*d*f*g*m^2*x^2 + 5*(b*x
+ a)^m*a*b^3*c*f*h*m^2*x^2 - 3*(b*x + a)^m*a^2*b^2*d*f*h*m^2*x^2 + 14*(b*x
+ a)^m*b^4*d*f*g*m*x^3 + 14*(b*x + a)^m*b^4*c*f*h*m*x^3 + 2*(b*x + a)^m*a*b
^3*d*f*h*m*x^3 + 6*(b*x + a)^m*b^4*d*f*h*x^4 + (b*x + a)^m*b^4*c*g*m^3*x*e
+ (b*x + a)^m*a*b^3*d*g*m^3*x*e + (b*x + a)^m*a*b^3*c*h*m^3*x*e + 8*(b*x +
a)^m*b^4*d*g*m^2*x^2*e + 8*(b*x + a)^m*b^4*c*h*m^2*x^2*e + 5*(b*x + a)^m*a*
b^3*d*h*m^2*x^2*e + 14*(b*x + a)^m*b^4*d*h*m*x^3*e + 7*(b*x + a)^m*a*b^3*c*
f*g*m^2*x - 2*(b*x + a)^m*a^2*b^2*d*f*g*m^2*x - 2*(b*x + a)^m*a^2*b^2*c*f*h
*m^2*x + 19*(b*x + a)^m*b^4*c*f*g*m*x^2 + 4*(b*x + a)^m*a*b^3*d*f*g*m*x^2 +
 4*(b*x + a)^m*a*b^3*c*f*h*m*x^2 - 3*(b*x + a)^m*a^2*b^2*d*f*h*m*x^2 + 8*(b
*x + a)^m*b^4*d*f*g*x^3 + 8*(b*x + a)^m*b^4*c*f*h*x^3 + (b*x + a)^m*a*b^3*c
*g*m^3*e + 9*(b*x + a)^m*b^4*c*g*m^2*x*e + 7*(b*x + a)^m*a*b^3*d*g*m^2*x*e
+ 7*(b*x + a)^m*a*b^3*c*h*m^2*x*e - 2*(b*x + a)^m*a^2*b^2*d*h*m^2*x*e + 19*
(b*x + a)^m*b^4*d*g*m*x^2*e + 19*(b*x + a)^m*b^4*c*h*m*x^2*e + 4*(b*x + a)^
m*a*b^3*d*h*m*x^2*e + 8*(b*x + a)^m*b^4*d*h*x^3*e - (b*x + a)^m*a^2*b^2*c*f
```

```
*g*m^2 + 12*(b*x + a)^m*a*b^3*c*f*g*m*x - 8*(b*x + a)^m*a^2*b^2*d*f*g*m*x -
 8*(b*x + a)^m*a^2*b^2*c*f*h*m*x + 6*(b*x + a)^m*a^3*b*d*f*h*m*x + 12*(b*x
+ a)^m*b^4*c*f*g*x^2 + 9*(b*x + a)^m*a*b^3*c*g*m^2*e - (b*x + a)^m*a^2*b^2*
d*g*m^2*e - (b*x + a)^m*a^2*b^2*c*h*m^2*e + 26*(b*x + a)^m*b^4*c*g*m*x*e +
12*(b*x + a)^m*a*b^3*d*g*m*x*e + 12*(b*x + a)^m*a*b^3*c*h*m*x*e - 8*(b*x +
a)^m*a^2*b^2*d*h*m*x*e + 12*(b*x + a)^m*b^4*d*g*x^2*e + 12*(b*x + a)^m*b^4*
c*h*x^2*e - 7*(b*x + a)^m*a^2*b^2*c*f*g*m + 2*(b*x + a)^m*a^3*b*d*f*g*m + 2
*(b*x + a)^m*a^3*b*c*f*h*m + 26*(b*x + a)^m*a*b^3*c*g*m*e - 7*(b*x + a)^m*a
^2*b^2*d*g*m*e - 7*(b*x + a)^m*a^2*b^2*c*h*m*e + 2*(b*x + a)^m*a^3*b*d*h*m*
e + 24*(b*x + a)^m*b^4*c*g*x*e - 12*(b*x + a)^m*a^2*b^2*c*f*g + 8*(b*x + a)
^m*a^3*b*d*f*g + 8*(b*x + a)^m*a^3*b*c*f*h - 6*(b*x + a)^m*a^4*d*f*h + 24*(
b*x + a)^m*a*b^3*c*g*e - 12*(b*x + a)^m*a^2*b^2*d*g*e - 12*(b*x + a)^m*a^2*
b^2*c*h*e + 8*(b*x + a)^m*a^3*b*d*h*e)/(b^4*m^4 + 10*b^4*m^3 + 35*b^4*m^2 +
 50*b^4*m + 24*b^4)
```

maple [B] time = 0.01, size = 726, normalized size = 4.35

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x)
```

```
[Out] -(b*x+a)^(m+1)*(-b^3*d*f*h*m^3*x^3-b^3*c*f*h*m^3*x^2-b^3*d*e*h*m^3*x^2-b^3*
d*f*g*m^3*x^2-6*b^3*d*f*h*m^2*x^3+3*a*b^2*d*f*h*m^2*x^2-b^3*c*e*h*m^3*x-b^3
*c*f*g*m^3*x-7*b^3*c*f*h*m^2*x^2-b^3*d*e*g*m^3*x-7*b^3*d*e*h*m^2*x^2-7*b^3*
d*f*g*m^2*x^2-11*b^3*d*f*h*m*x^3+2*a*b^2*c*f*h*m^2*x+2*a*b^2*d*e*h*m^2*x+2*
a*b^2*d*f*g*m^2*x+9*a*b^2*d*f*h*m*x^2-b^3*c*e*g*m^3-8*b^3*c*e*h*m^2*x-8*b^3
*c*f*g*m^2*x-14*b^3*c*f*h*m*x^2-8*b^3*d*e*g*m^2*x-14*b^3*d*e*h*m*x^2-14*b^3
*d*f*g*m*x^2-6*b^3*d*f*h*x^3-6*a^2*b*d*f*h*m*x+a*b^2*c*e*h*m^2+a*b^2*c*f*g*
m^2+10*a*b^2*c*f*h*m*x+a*b^2*d*e*g*m^2+10*a*b^2*d*e*h*m*x+10*a*b^2*d*f*g*m*
x+6*a*b^2*d*f*h*x^2-9*b^3*c*e*g*m^2-19*b^3*c*e*h*m*x-19*b^3*c*f*g*m*x-8*b^3
*c*f*h*x^2-19*b^3*d*e*g*m*x-8*b^3*d*e*h*x^2-8*b^3*d*f*g*x^2-2*a^2*b*c*f*h*m
-2*a^2*b*d*e*h*m-2*a^2*b*d*f*g*m-6*a^2*b*d*f*h*x+7*a*b^2*c*e*h*m+7*a*b^2*c*
f*g*m+8*a*b^2*c*f*h*x+7*a*b^2*d*e*g*m+8*a*b^2*d*e*h*x+8*a*b^2*d*f*g*x-26*b^
3*c*e*g*m-12*b^3*c*e*h*x-12*b^3*c*f*g*x-12*b^3*d*e*g*x+6*a^3*d*f*h-8*a^2*b*
c*f*h-8*a^2*b*d*e*h-8*a^2*b*d*f*g+12*a*b^2*c*e*h+12*a*b^2*c*f*g+12*a*b^2*d*
e*g-24*b^3*c*e*g)/b^4/(m^4+10*m^3+35*m^2+50*m+24)
```

maxima [B] time = 0.54, size = 474, normalized size = 2.84

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*d*e*g/((m^2 + 3*m + 2)*b^2) +
 (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*c*f*g/((m^2 + 3*m + 2)*b^2)
+ (b^2*(m + 1)*x^2 + a*b*m*x - a^2)*(b*x + a)^m*c*e*h/((m^2 + 3*m + 2)*b^2)
 + (b*x + a)^(m + 1)*c*e*g/(b*(m + 1)) + ((m^2 + 3*m + 2)*b^3*x^3 + (m^2 +
m)*a*b^2*x^2 - 2*a^2*b*m*x + 2*a^3)*(b*x + a)^m*d*f*g/((m^3 + 6*m^2 + 11*m
+ 6)*b^3) + ((m^2 + 3*m + 2)*b^3*x^3 + (m^2 + m)*a*b^2*x^2 - 2*a^2*b*m*x +
2*a^3)*(b*x + a)^m*d*e*h/((m^3 + 6*m^2 + 11*m + 6)*b^3) + ((m^2 + 3*m + 2)*
b^3*x^3 + (m^2 + m)*a*b^2*x^2 - 2*a^2*b*m*x + 2*a^3)*(b*x + a)^m*c*f*h/((m^
3 + 6*m^2 + 11*m + 6)*b^3) + ((m^3 + 6*m^2 + 11*m + 6)*b^4*x^4 + (m^3 + 3*m
^2 + 2*m)*a*b^3*x^3 - 3*(m^2 + m)*a^2*b^2*x^2 + 6*a^3*b*m*x - 6*a^4)*(b*x +
 a)^m*d*f*h/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*b^4)
```

mupad [B] time = 2.95, size = 819, normalized size = 4.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e + f*x)*(g + h*x)*(a + b*x)^m*(c + d*x),x)
```

```
[Out] (x*(a + b*x)^m*(24*b^4*c*e*g + 9*b^4*c*e*g*m^2 + b^4*c*e*g*m^3 + 26*b^4*c*e
*g*m + 12*a*b^3*c*e*h*m + 12*a*b^3*c*f*g*m + 12*a*b^3*d*e*g*m + 6*a^3*b*d*f
*h*m + 7*a*b^3*c*e*h*m^2 + 7*a*b^3*c*f*g*m^2 + 7*a*b^3*d*e*g*m^2 + a*b^3*c*
e*h*m^3 + a*b^3*c*f*g*m^3 + a*b^3*d*e*g*m^3 - 8*a^2*b^2*c*f*h*m - 8*a^2*b^2
*d*e*h*m - 8*a^2*b^2*d*f*g*m - 2*a^2*b^2*c*f*h*m^2 - 2*a^2*b^2*d*e*h*m^2 -
2*a^2*b^2*d*f*g*m^2))/(b^4*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) - ((a + b*x
)^m*(6*a^4*d*f*h + 12*a^2*b^2*c*e*h + 12*a^2*b^2*c*f*g + 12*a^2*b^2*d*e*g -
 24*a*b^3*c*e*g - 8*a^3*b*c*f*h - 8*a^3*b*d*e*h - 8*a^3*b*d*f*g - 26*a*b^3*
c*e*g*m - 2*a^3*b*c*f*h*m - 2*a^3*b*d*e*h*m - 2*a^3*b*d*f*g*m - 9*a*b^3*c*e
*g*m^2 - a*b^3*c*e*g*m^3 + 7*a^2*b^2*c*e*h*m + 7*a^2*b^2*c*f*g*m + 7*a^2*b^
2*d*e*g*m + a^2*b^2*c*e*h*m^2 + a^2*b^2*c*f*g*m^2 + a^2*b^2*d*e*g*m^2))/(b^
4*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x^3*(a + b*x)^m*(3*m + m^2 + 2)*(
4*b*c*f*h + 4*b*d*e*h + 4*b*d*f*g + a*d*f*h*m + b*c*f*h*m + b*d*e*h*m + b*d
*f*g*m))/(b*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x^2*(m + 1)*(a + b*x)^m
*(12*b^2*c*e*h + 12*b^2*c*f*g + 12*b^2*d*e*g + b^2*c*e*h*m^2 + b^2*c*f*g*m^
2 + b^2*d*e*g*m^2 + 7*b^2*c*e*h*m + 7*b^2*c*f*g*m + 7*b^2*d*e*g*m - 3*a^2*d
*f*h*m + a*b*c*f*h*m^2 + a*b*d*e*h*m^2 + a*b*d*f*g*m^2 + 4*a*b*c*f*h*m + 4*
a*b*d*e*h*m + 4*a*b*d*f*g*m))/(b^2*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (
d*f*h*x^4*(a + b*x)^m*(11*m + 6*m^2 + m^3 + 6))/(50*m + 35*m^2 + 10*m^3 + m
^4 + 24)
```

sympy [A] time = 8.12, size = 8221, normalized size = 49.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)*(f*x+e)*(h*x+g),x)
```

```
[Out] Piecewise((a**m*(c*e*g*x + c*e*h*x**2/2 + c*f*g*x**2/2 + c*f*h*x**3/3 + d*e
*g*x**2/2 + d*e*h*x**3/3 + d*f*g*x**3/3 + d*f*h*x**4/4), Eq(b, 0)), (6*a**3
*d*f*h*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7
*x**3) + 11*a**3*d*f*h/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b
**7*x**3) - 2*a**2*b*c*f*h/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 +
 6*b**7*x**3) - 2*a**2*b*d*e*h/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x*
*2 + 6*b**7*x**3) - 2*a**2*b*d*f*g/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**
6*x**2 + 6*b**7*x**3) + 18*a**2*b*d*f*h*x*log(a/b + x)/(6*a**3*b**4 + 18*a*
*2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*d*f*h*x/(6*a**3*b**4
+ 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - a*b**2*c*e*h/(6*a**3*b**
4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - a*b**2*c*f*g/(6*a**3*b
**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*a*b**2*c*f*h*x/(6*
a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - a*b**2*d*e*g/(
6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*a*b**2*d*e
*h*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*a*b*
*2*d*f*g*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) +
18*a*b**2*d*f*h*x**2*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6
*x**2 + 6*b**7*x**3) + 18*a*b**2*d*f*h*x**2/(6*a**3*b**4 + 18*a**2*b**5*x +
 18*a*b**6*x**2 + 6*b**7*x**3) - 2*b**3*c*e*g/(6*a**3*b**4 + 18*a**2*b**5*x
 + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*b**3*c*e*h*x/(6*a**3*b**4 + 18*a**2*b*
*5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*b**3*c*f*g*x/(6*a**3*b**4 + 18*a**
2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*b**3*c*f*h*x**2/(6*a**3*b**4 +
 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 3*b**3*d*e*g*x/(6*a**3*b*
*4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*b**3*d*e*h*x**2/(6*
a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 6*b**3*d*f*g*x
**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3*
d*f*h*x**3*log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*
b**7*x**3), Eq(m, -4)), (-6*a**3*d*f*h*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5
*x + 2*b**6*x**2) - 9*a**3*d*f*h/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) +
```

```
2*a**2*b*c*f*h*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 3*a
**2*b*c*f*h/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*a**2*b*d*e*h*log(a
/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 3*a**2*b*d*e*h/(2*a**2*b
**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*a**2*b*d*f*g*log(a/b + x)/(2*a**2*b**4
+ 4*a*b**5*x + 2*b**6*x**2) + 3*a**2*b*d*f*g/(2*a**2*b**4 + 4*a*b**5*x + 2*
b**6*x**2) - 12*a**2*b*d*f*h*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b
**6*x**2) - 12*a**2*b*d*f*h*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - a*
b**2*c*e*h/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - a*b**2*c*f*g/(2*a**2*
b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a*b**2*c*f*h*x*log(a/b + x)/(2*a**2*b*
*4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a*b**2*c*f*h*x/(2*a**2*b**4 + 4*a*b**5*x
 + 2*b**6*x**2) - a*b**2*d*e*g/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4
*a*b**2*d*e*h*x*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a
*b**2*d*e*h*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a*b**2*d*f*g*x*l
og(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 4*a*b**2*d*f*g*x/(2*
a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*d*f*h*x**2*log(a/b + x)/(2
*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - b**3*c*e*g/(2*a**2*b**4 + 4*a*b**5
*x + 2*b**6*x**2) - 2*b**3*c*e*h*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2)
 - 2*b**3*c*f*g*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*c*f*h*x
**2*log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 2*b**3*d*e*g*x/
(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*e*h*x**2*log(a/b + x)/(
2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*f*g*x**2*log(a/b + x)/(2
*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*f*h*x**3/(2*a**2*b**4 + 4
*a*b**5*x + 2*b**6*x**2), Eq(m, -3)), (6*a**3*d*f*h*log(a/b + x)/(2*a*b**4
+ 2*b**5*x) + 6*a**3*d*f*h/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*c*f*h*log(a/b +
 x)/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*c*f*h/(2*a*b**4 + 2*b**5*x) - 4*a**2*b
*d*e*h*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*d*e*h/(2*a*b**4 + 2*b*
*5*x) - 4*a**2*b*d*f*g*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 4*a**2*b*d*f*g/
(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d*f*h*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x)
 + 2*a*b**2*c*e*h*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*c*e*h/(2*a*
b**4 + 2*b**5*x) + 2*a*b**2*c*f*g*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*
b**2*c*f*g/(2*a*b**4 + 2*b**5*x) - 4*a*b**2*c*f*h*x*log(a/b + x)/(2*a*b**4
+ 2*b**5*x) + 2*a*b**2*d*e*g*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*a*b**2*
d*e*g/(2*a*b**4 + 2*b**5*x) - 4*a*b**2*d*e*h*x*log(a/b + x)/(2*a*b**4 + 2*b
**5*x) - 4*a*b**2*d*f*g*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 3*a*b**2*d*f
*h*x**2/(2*a*b**4 + 2*b**5*x) - 2*b**3*c*e*g/(2*a*b**4 + 2*b**5*x) + 2*b**3
*c*e*h*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*b**3*c*f*g*x*log(a/b + x)/(
2*a*b**4 + 2*b**5*x) + 2*b**3*c*f*h*x**2/(2*a*b**4 + 2*b**5*x) + 2*b**3*d*e
*g*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 2*b**3*d*e*h*x**2/(2*a*b**4 + 2*b
**5*x) + 2*b**3*d*f*g*x**2/(2*a*b**4 + 2*b**5*x) + b**3*d*f*h*x**3/(2*a*b**
4 + 2*b**5*x), Eq(m, -2)), (-a**3*d*f*h*log(a/b + x)/b**4 + a**2*c*f*h*log(
a/b + x)/b**3 + a**2*d*e*h*log(a/b + x)/b**3 + a**2*d*f*g*log(a/b + x)/b**3
 + a**2*d*f*h*x/b**3 - a*c*e*h*log(a/b + x)/b**2 - a*c*f*g*log(a/b + x)/b**
2 - a*c*f*h*x/b**2 - a*d*e*g*log(a/b + x)/b**2 - a*d*e*h*x/b**2 - a*d*f*g*x
/b**2 - a*d*f*h*x**2/(2*b**2) + c*e*g*log(a/b + x)/b + c*e*h*x/b + c*f*g*x/
b + c*f*h*x**2/(2*b) + d*e*g*x/b + d*e*h*x**2/(2*b) + d*f*g*x**2/(2*b) + d*
f*h*x**3/(3*b), Eq(m, -1)), (-6*a**4*d*f*h*(a + b*x)**m/(b**4*m**4 + 10*b**
4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 2*a**3*b*c*f*h*m*(a + b*x)**
m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*a**3*
b*c*f*h*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m +
 24*b**4) + 2*a**3*b*d*e*h*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b*
*4*m**2 + 50*b**4*m + 24*b**4) + 8*a**3*b*d*e*h*(a + b*x)**m/(b**4*m**4 + 1
0*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 2*a**3*b*d*f*g*m*(a + b
*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*
a**3*b*d*f*g*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**
4*m + 24*b**4) + 6*a**3*b*d*f*h*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3
+ 35*b**4*m**2 + 50*b**4*m + 24*b**4) - a**2*b**2*c*e*h*m**2*(a + b*x)**m/(
b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 7*a**2*b**
2*c*e*h*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m
 + 24*b**4) - 12*a**2*b**2*c*e*h*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 3
```

```
5*b**4*m**2 + 50*b**4*m + 24*b**4) - a**2*b**2*c*f*g*m**2*(a + b*x)**m/(b**
4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 7*a**2*b**2*c
*f*g*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m +
24*b**4) - 12*a**2*b**2*c*f*g*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b
**4*m**2 + 50*b**4*m + 24*b**4) - 2*a**2*b**2*c*f*h*m**2*x*(a + b*x)**m/(b*
*4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 8*a**2*b**2*
c*f*h*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m
 + 24*b**4) - a**2*b**2*d*e*g*m**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 +
 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 7*a**2*b**2*d*e*g*m*(a + b*x)**m/(b*
*4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 12*a**2*b**2
*d*e*g*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m +
24*b**4) - 2*a**2*b**2*d*e*h*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3
+ 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 8*a**2*b**2*d*e*h*m*x*(a + b*x)**m/
(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 2*a**2*b*
*2*d*f*g*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*
b**4*m + 24*b**4) - 8*a**2*b**2*d*f*g*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4
*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) - 3*a**2*b**2*d*f*h*m**2*x**2*(
a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4)
 - 3*a**2*b**2*d*f*h*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**
4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*c*e*g*m**3*(a + b*x)**m/(b**4*m**4 +
 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 9*a*b**3*c*e*g*m**2*(
a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4)
 + 26*a*b**3*c*e*g*m*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2
+ 50*b**4*m + 24*b**4) + 24*a*b**3*c*e*g*(a + b*x)**m/(b**4*m**4 + 10*b**4*
m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*c*e*h*m**3*x*(a + b*x)*
*m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 7*a*b*
*3*c*e*h*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*
b**4*m + 24*b**4) + 12*a*b**3*c*e*h*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m
**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*c*f*g*m**3*x*(a + b*x)**
m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 7*a*b**
3*c*f*g*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b
**4*m + 24*b**4) + 12*a*b**3*c*f*g*m*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m*
*3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*c*f*h*m**3*x**2*(a + b*x)
**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 5*a*b
**3*c*f*h*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 +
 50*b**4*m + 24*b**4) + 4*a*b**3*c*f*h*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*
b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*d*e*g*m**3*x*(a +
b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 7
*a*b**3*d*e*g*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2
+ 50*b**4*m + 24*b**4) + 12*a*b**3*d*e*g*m*x*(a + b*x)**m/(b**4*m**4 + 10*b
**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*d*e*h*m**3*x**2*(a
+ b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) +
 5*a*b**3*d*e*h*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*
m**2 + 50*b**4*m + 24*b**4) + 4*a*b**3*d*e*h*m*x**2*(a + b*x)**m/(b**4*m**4
 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*d*f*g*m**3*x
**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*
b**4) + 5*a*b**3*d*f*g*m**2*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 3
5*b**4*m**2 + 50*b**4*m + 24*b**4) + 4*a*b**3*d*f*g*m*x**2*(a + b*x)**m/(b*
*4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + a*b**3*d*f*h
*m**3*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*
m + 24*b**4) + 3*a*b**3*d*f*h*m**2*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m
**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 2*a*b**3*d*f*h*m*x**3*(a + b*x)
**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*
c*e*g*m**3*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**
4*m + 24*b**4) + 9*b**4*c*e*g*m**2*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3
 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 26*b**4*c*e*g*m*x*(a + b*x)**m/(b*
*4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 24*b**4*c*e*
g*x*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*
b**4) + b**4*c*e*h*m**3*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b*
```

```
*4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*c*e*h*m**2*x**2*(a + b*x)**m/(b**4*
m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 19*b**4*c*e*h*m
*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 2
4*b**4) + 12*b**4*c*e*h*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b*
*4*m**2 + 50*b**4*m + 24*b**4) + b**4*c*f*g*m**3*x**2*(a + b*x)**m/(b**4*m*
*4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*c*f*g*m**2
*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 2
4*b**4) + 19*b**4*c*f*g*m*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*
b**4*m**2 + 50*b**4*m + 24*b**4) + 12*b**4*c*f*g*x**2*(a + b*x)**m/(b**4*m*
*4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*c*f*h*m**3*x
**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*
b**4) + 7*b**4*c*f*h*m**2*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*
b**4*m**2 + 50*b**4*m + 24*b**4) + 14*b**4*c*f*h*m*x**3*(a + b*x)**m/(b**4*
m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*c*f*h*x*
*3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b
**4) + b**4*d*e*g*m**3*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**
4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*d*e*g*m**2*x**2*(a + b*x)**m/(b**4*m
**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 19*b**4*d*e*g*m*
x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24
*b**4) + 12*b**4*d*e*g*x**2*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**
4*m**2 + 50*b**4*m + 24*b**4) + b**4*d*e*h*m**3*x**3*(a + b*x)**m/(b**4*m**
4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 7*b**4*d*e*h*m**2*
x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24
*b**4) + 14*b**4*d*e*h*m*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b
**4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*d*e*h*x**3*(a + b*x)**m/(b**4*m**4
 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + b**4*d*f*g*m**3*x**
3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b*
*4) + 7*b**4*d*f*g*m**2*x**3*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b*
*4*m**2 + 50*b**4*m + 24*b**4) + 14*b**4*d*f*g*m*x**3*(a + b*x)**m/(b**4*m*
*4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 8*b**4*d*f*g*x**3
*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**
4) + b**4*d*f*h*m**3*x**4*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*
m**2 + 50*b**4*m + 24*b**4) + 6*b**4*d*f*h*m**2*x**4*(a + b*x)**m/(b**4*m**
4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b**4) + 11*b**4*d*f*h*m*x*
*4*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m**2 + 50*b**4*m + 24*b
**4) + 6*b**4*d*f*h*x**4*(a + b*x)**m/(b**4*m**4 + 10*b**4*m**3 + 35*b**4*m
**2 + 50*b**4*m + 24*b**4), True))
```

3.34 $\int (a+bx)^m (c+dx)^{-4-m} (e+fx)(g+hx)\,dx$

Optimal. Leaf size=362

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-2}\left(a^2d^2fh\left(m^2+5m+6\right)-abd(m+3)(2cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3m+\right.\right.\right.}{bd^2(m+2)(m+3)(bc-ad)^2}$$

Rubi [A] time = 0.36, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {146, 45, 37}

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-2}\left(a^2d^2fh\left(m^2+5m+6\right)-abd(m+3)(2cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3m+2\right)+cd(m+1)(eh+fg)+2d^2eg\right)\right)}{bd^2(m+2)(m+3)(bc-ad)^2}+\frac{(a+bx)^{m+1}(c+dx)^{-m-1}\left(a^2d^2fh\left(m^2+5m+6\right)-abd(m+3)(2cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3m+2\right)+cd(m+1)(eh+fg)+2d^2eg\right)\right)}{d^2(m+1)(m+2)(m+3)(bc-ad)^3}+\frac{(a+bx)^{m+1}(c+dx)^{-m-3}\left(-dfh(m+3)x(bc-ad)+acdfh(m+3)+b\left(c^2(-f)h(m+2)-cd(eh+fg)+d^2eg\right)\right)}{bd^2(m+3)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(-4 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a^2*d^2*f*h*(6 + 5*m + m^2) - a*b*d*(3 + m)*(d*(f*g + e*h) + 2*c*f*h*(1 +
 m)) + b^2*(2*d^2*e*g + c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))
*(a + b*x)^(1 + m)*(c + d*x)^(-2 - m))/(b*d^2*(b*c - a*d)^2*(2 + m)*(3 + m)
) + ((a^2*d^2*f*h*(6 + 5*m + m^2) - a*b*d*(3 + m)*(d*(f*g + e*h) + 2*c*f*h*
(1 + m)) + b^2*(2*d^2*e*g + c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^
2)))*(a + b*x)^(1 + m)*(c + d*x)^(-1 - m))/(d^2*(b*c - a*d)^3*(1 + m)*(2 +
m)*(3 + m)) + ((a + b*x)^(1 + m)*(c + d*x)^(-3 - m)*(a*c*d*f*h*(3 + m) + b*
(d^2*e*g - c*d*(f*g + e*h) - c^2*f*h*(2 + m)) - d*(b*c - a*d)*f*h*(3 + m)*x
))/(b*d^2*(b*c - a*d)*(3 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int (a+bx)^m (c+dx)^{-4-m}(e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{-3-m}\left(acdfh(3+m)+b\left(d^2eg-cd(fg+eh\right.\right.}{bd^2(bc-ad)(3+m)}$$

$$= \frac{\left(a^2d^2fh\left(6+5m+m^2\right)-abd(3+m)(d(fg+eh)+2cfh(1+m\right.}{bd^2}$$

$$= \frac{\left(a^2d^2fh\left(6+5m+m^2\right)-abd(3+m)(d(fg+eh)+2cfh(1+m\right.}{bd^2}$$

Mathematica [A] time = 0.48, size = 220, normalized size = 0.61

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-3}\left(\frac{(c+dx)(-ad(m+1)+bc(m+2)+bdx)\left(a^2d^2fh\left(m^2+5m+6\right)-abd(m+3)(2cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3m+2\right)+cd(m+1)(eh+fg)+2d^2eg\right)\right)}{(m+1)(m+2)(bc-ad)^2}+adfh(m+3)(c+dx)+b\left(c^2(-f)h(m+2)-cd(eh+f(g+h(m+3)x))+d^2eg\right)\right)}{bd^2(m+3)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-4 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(-3 - m)*(a*d*f*h*(3 + m)*(c + d*x) + ((a^2*d^
2*f*h*(6 + 5*m + m^2) - a*b*d*(3 + m)*(d*(f*g + e*h) + 2*c*f*h*(1 + m)) + b
^2*(2*d^2*e*g + c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(c + d*
x)*(-(a*d*(1 + m)) + b*c*(2 + m) + b*d*x))/((b*c - a*d)^2*(1 + m)*(2 + m))
+ b*(d^2*e*g - c^2*f*h*(2 + m) - c*d*(e*h + f*(g + h*(3 + m)*x)))))/(b*d^2*
(b*c - a*d)*(3 + m))
```

IntegrateAlgebraic [F] time = 0.12, size = 0, normalized size = 0.00

$$\int (a+bx)^m (c+dx)^{-4-m}(e+fx)(g+hx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^m*(c + d*x)^(-4 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^m*(c + d*x)^(-4 - m)*(e + f*x)*(g + h*x
), x]
```

fricas [B] time = 0.93, size = 1659, normalized size = 4.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] ((a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*e*g*m^2 + ((b^3*c^2*d - 2*a*b^2*c*
d^2 + a^2*b*d^3)*f*h*m^2 + (2*b^3*d^3*e + (b^3*c*d^2 - 3*a*b^2*d^3)*f)*g +
((b^3*c*d^2 - 3*a*b^2*d^3)*e + 2*(b^3*c^2*d - 3*a*b^2*c*d^2 + 3*a^2*b*d^3)*
f)*h + ((b^3*c*d^2 - a*b^2*d^3)*f*g + ((b^3*c*d^2 - a*b^2*d^3)*e + (3*b^3*c
^2*d - 8*a*b^2*c*d^2 + 5*a^2*b*d^3)*f)*h)*m)*x^4 + (((b^3*c^2*d - 2*a*b^2*c
*d^2 + a^2*b*d^3)*f*g + ((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*e + (b^3*c
^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*f)*h)*m^2 + 4*(2*b^3*c*d^2*e + (b
^3*c^2*d - 3*a*b^2*c*d^2)*f)*g + 2*(2*(b^3*c^2*d - 3*a*b^2*c*d^2)*e + (b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 + 3*a^3*d^3)*f)*h + ((2*(b^3*c*d^2 - a*
b^2*d^3)*e + (5*b^3*c^2*d - 8*a*b^2*c*d^2 + 3*a^2*b*d^3)*f)*g + ((5*b^3*c^2
*d - 8*a*b^2*c*d^2 + 3*a^2*b*d^3)*e + (3*b^3*c^3 - 7*a*b^2*c^2*d - a^2*b*c*
d^2 + 5*a^3*d^3)*f)*h)*m)*x^3 + ((((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*
e + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*f)*g + ((b^3*c^3 - a*b^
2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*e + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2
)*f)*h)*m^2 + 3*(4*b^3*c^2*d*e + (b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
 a^3*d^3)*f)*g + 3*(4*a^3*c*d^2*f + (b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^
```

```
2 + a^3*d^3)*e)*h + (((7*b^3*c^2*d - 8*a*b^2*c*d^2 + a^2*b*d^3)*e + 4*(b^3*
c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*f)*g + (4*(b^3*c^3 - a*b^2*c^2*d
 - a^2*b*c*d^2 + a^3*d^3)*e + (a*b^2*c^3 - 8*a^2*b*c^2*d + 7*a^3*c*d^2)*f)*
h)*m)*x^2 + (2*(3*a*b^2*c^3 - 3*a^2*b*c^2*d + a^3*c*d^2)*e - (3*a^2*b*c^3 -
 a^3*c^2*d)*f)*g + (2*a^3*c^3*f - (3*a^2*b*c^3 - a^3*c^2*d)*e)*h - ((a^2*b*
c^3 - a^3*c^2*d)*e*h - ((5*a*b^2*c^3 - 8*a^2*b*c^2*d + 3*a^3*c*d^2)*e - (a^
2*b*c^3 - a^3*c^2*d)*f)*g)*m + (((a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*e*
h + ((b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*e + (a*b^2*c^3 - 2*a^2
*b*c^2*d + a^3*c*d^2)*f)*g)*m^2 + 2*((3*b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2 + a^3*d^3)*e - 2*(3*a^2*b*c^2*d - a^3*c*d^2)*f)*g + 4*(2*a^3*c^2*d*f -
 (3*a^2*b*c^2*d - a^3*c*d^2)*e)*h + (((5*b^3*c^3 - a*b^2*c^2*d - 7*a^2*b*c*
d^2 + 3*a^3*d^3)*e + (3*a*b^2*c^3 - 8*a^2*b*c^2*d + 5*a^3*c*d^2)*f)*g + ((3
*a*b^2*c^3 - 8*a^2*b*c^2*d + 5*a^3*c*d^2)*e - 2*(a^2*b*c^3 - a^3*c^2*d)*f)*
h)*m)*x)*(b*x + a)^m*(d*x + c)^(-m - 4)/(6*b^3*c^3 - 18*a*b^2*c^2*d + 18*a^
2*b*c*d^2 - 6*a^3*d^3 + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*m^3 + 6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*m^2 + 11*(b^3*
c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 4), x)
```

maple [B] time = 0.01, size = 894, normalized size = 2.47

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(-m-4)*(f*x+e)*(h*x+g),x)
```

```
[Out] -(b*x+a)^(m+1)*(d*x+c)^(-m-3)*(a^2*d^2*f*h*m^2*x^2-2*a*b*c*d*f*h*m^2*x^2+b^
2*c^2*f*h*m^2*x^2+a^2*d^2*e*h*m^2*x+a^2*d^2*f*g*m^2*x+5*a^2*d^2*f*h*m*x^2-2
*a*b*c*d*e*h*m^2*x-2*a*b*c*d*f*g*m^2*x-8*a*b*c*d*f*h*m*x^2-a*b*d^2*e*h*m*x^
2-a*b*d^2*f*g*m*x^2+b^2*c^2*e*h*m^2*x+b^2*c^2*f*g*m^2*x+3*b^2*c^2*f*h*m*x^2
+b^2*c*d*e*h*m*x^2+b^2*c*d*f*g*m*x^2+2*a^2*c*d*f*h*m*x+a^2*d^2*e*g*m^2+4*a^
2*d^2*e*h*m*x+4*a^2*d^2*f*g*m*x+6*a^2*d^2*f*h*x^2-2*a*b*c^2*f*h*m*x-2*a*b*c
*d*e*g*m^2-8*a*b*c*d*e*h*m*x-8*a*b*c*d*f*g*m*x-6*a*b*c*d*f*h*x^2-2*a*b*d^2*
e*g*m*x-3*a*b*d^2*e*h*x^2-3*a*b*d^2*f*g*x^2+b^2*c^2*e*g*m^2+4*b^2*c^2*e*h*m
*x+4*b^2*c^2*f*g*m*x+2*b^2*c^2*f*h*x^2+2*b^2*c*d*e*g*m*x+b^2*c*d*e*h*x^2+b^
2*c*d*f*g*x^2+2*b^2*d^2*e*g*x^2+a^2*c*d*e*h*m+a^2*c*d*f*g*m+6*a^2*c*d*f*h*x
+3*a^2*d^2*e*g*m+3*a^2*d^2*e*h*x+3*a^2*d^2*f*g*x-a*b*c^2*e*h*m-a*b*c^2*f*g*
m-2*a*b*c^2*f*h*x-8*a*b*c*d*e*g*m-10*a*b*c*d*e*h*x-10*a*b*c*d*f*g*x-2*a*b*d
^2*e*g*x+5*b^2*c^2*e*g*m+3*b^2*c^2*e*h*x+3*b^2*c^2*f*g*x+6*b^2*c*d*e*g*x+2*
a^2*c^2*f*h+a^2*c*d*e*h+a^2*c*d*f*g+2*a^2*d^2*e*g-3*a*b*c^2*e*h-3*a*b*c^2*f
*g-6*a*b*c*d*e*g+6*b^2*c^2*e*g)/(a^3*d^3*m^3-3*a^2*b*c*d^2*m^3+3*a*b^2*c^2*
d*m^3-b^3*c^3*m^3+6*a^3*d^3*m^2-18*a^2*b*c*d^2*m^2+18*a*b^2*c^2*d*m^2-6*b^3
*c^3*m^2+11*a^3*d^3*m-33*a^2*b*c*d^2*m+33*a*b^2*c^2*d*m-11*b^3*c^3*m+6*a^3*
d^3-18*a^2*b*c*d^2+18*a*b^2*c^2*d-6*b^3*c^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-4-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 4), x)
```

mupad [B] time = 4.49, size = 1895, normalized size = 5.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 4),x)
[Out] - ((a + b*x)^m*(2*a^3*c^3*f*h + 6*a*b^2*c^3*e*g - 3*a^2*b*c^3*e*h - 3*a^2*b
*c^3*f*g + 2*a^3*c*d^2*e*g + a^3*c^2*d*e*h + a^3*c^2*d*f*g - 6*a^2*b*c^2*d*
e*g + 5*a*b^2*c^3*e*g*m - a^2*b*c^3*e*h*m - a^2*b*c^3*f*g*m + 3*a^3*c*d^2*e
*g*m + a^3*c^2*d*e*h*m + a^3*c^2*d*f*g*m + a*b^2*c^3*e*g*m^2 + a^3*c*d^2*e*
g*m^2 - 2*a^2*b*c^2*d*e*g*m^2 - 8*a^2*b*c^2*d*e*g*m))/((a*d - b*c)^3*(c + d
*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) - (x^3*(a + b*x)^m*(6*a^3*d^3*f*h + 2
*b^3*c^3*f*h + 8*b^3*c*d^2*e*g + 4*b^3*c^2*d*e*h + 4*b^3*c^2*d*f*g + 5*a^3*
d^3*f*h*m + 3*b^3*c^3*f*h*m + a^3*d^3*f*h*m^2 + b^3*c^3*f*h*m^2 - 12*a*b^2*
c*d^2*e*h - 12*a*b^2*c*d^2*f*g - 6*a*b^2*c^2*d*f*h + 6*a^2*b*c*d^2*f*h - 2*
a*b^2*d^3*e*g*m + 3*a^2*b*d^3*e*h*m + 3*a^2*b*d^3*f*g*m + 2*b^3*c*d^2*e*g*m
 + 5*b^3*c^2*d*e*h*m + 5*b^3*c^2*d*f*g*m + a^2*b*d^3*e*h*m^2 + a^2*b*d^3*f*
g*m^2 + b^3*c^2*d*e*h*m^2 + b^3*c^2*d*f*g*m^2 - 2*a*b^2*c*d^2*e*h*m^2 - 2*a
*b^2*c*d^2*f*g*m^2 - a*b^2*c^2*d*f*h*m^2 - a^2*b*c*d^2*f*h*m^2 - 8*a*b^2*c*
d^2*e*h*m - 8*a*b^2*c*d^2*f*g*m - 7*a*b^2*c^2*d*f*h*m - a^2*b*c*d^2*f*h*m))
/((a*d - b*c)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) - (x*(a + b*x)^
m*(2*a^3*d^3*e*g + 6*b^3*c^3*e*g + 4*a^3*c*d^2*e*h + 4*a^3*c*d^2*f*g + 8*a^
3*c^2*d*f*h + 3*a^3*d^3*e*g*m + 5*b^3*c^3*e*g*m + a^3*d^3*e*g*m^2 + b^3*c^3
*e*g*m^2 + 6*a*b^2*c^2*d*e*g - 6*a^2*b*c*d^2*e*g - 12*a^2*b*c^2*d*e*h - 12*
a^2*b*c^2*d*f*g + 3*a*b^2*c^3*e*h*m + 3*a*b^2*c^3*f*g*m - 2*a^2*b*c^3*f*h*m
 + 5*a^3*c*d^2*e*h*m + 5*a^3*c*d^2*f*g*m + 2*a^3*c^2*d*f*h*m + a*b^2*c^3*e*
h*m^2 + a*b^2*c^3*f*g*m^2 + a^3*c*d^2*e*h*m^2 + a^3*c*d^2*f*g*m^2 - a*b^2*c
^2*d*e*g*m^2 - a^2*b*c*d^2*e*g*m^2 - 2*a^2*b*c^2*d*e*h*m^2 - 2*a^2*b*c^2*d*
f*g*m^2 - a*b^2*c^2*d*e*g*m - 7*a^2*b*c*d^2*e*g*m - 8*a^2*b*c^2*d*e*h*m - 8
*a^2*b*c^2*d*f*g*m))/((a*d - b*c)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 +
 6)) - (x^2*(a + b*x)^m*(3*a^3*d^3*e*h + 3*a^3*d^3*f*g + 3*b^3*c^3*e*h + 3*
b^3*c^3*f*g + 12*b^3*c^2*d*e*g + 12*a^3*c*d^2*f*h + 4*a^3*d^3*e*h*m + 4*a^3
*d^3*f*g*m + 4*b^3*c^3*e*h*m + 4*b^3*c^3*f*g*m + a^3*d^3*e*h*m^2 + a^3*d^3*
f*g*m^2 + b^3*c^3*e*h*m^2 + b^3*c^3*f*g*m^2 - 9*a*b^2*c^2*d*e*h - 9*a*b^2*c
^2*d*f*g - 9*a^2*b*c*d^2*e*h - 9*a^2*b*c*d^2*f*g + a^2*b*d^3*e*g*m + a*b^2*
c^3*f*h*m + 7*b^3*c^2*d*e*g*m + 7*a^3*c*d^2*f*h*m + a^2*b*d^3*e*g*m^2 + a*b
^2*c^3*f*h*m^2 + b^3*c^2*d*e*g*m^2 + a^3*c*d^2*f*h*m^2 - 2*a*b^2*c*d^2*e*g*
m^2 - a*b^2*c^2*d*e*h*m^2 - a*b^2*c^2*d*f*g*m^2 - a^2*b*c*d^2*e*h*m^2 - a^2
*b*c*d^2*f*g*m^2 - 2*a^2*b*c^2*d*f*h*m^2 - 8*a*b^2*c*d^2*e*g*m - 4*a*b^2*c^
2*d*e*h*m - 4*a*b^2*c^2*d*f*g*m - 4*a^2*b*c*d^2*e*h*m - 4*a^2*b*c*d^2*f*g*m
 - 8*a^2*b*c^2*d*f*h*m))/((a*d - b*c)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m
^3 + 6)) - (x^4*(a + b*x)^m*(2*b^3*d^3*e*g - 3*a*b^2*d^3*e*h - 3*a*b^2*d^3*
f*g + 6*a^2*b*d^3*f*h + b^3*c*d^2*e*h + b^3*c*d^2*f*g + 2*b^3*c^2*d*f*h - 6
*a*b^2*c*d^2*f*h - a*b^2*d^3*e*h*m - a*b^2*d^3*f*g*m + 5*a^2*b*d^3*f*h*m +
b^3*c*d^2*e*h*m + b^3*c*d^2*f*g*m + 3*b^3*c^2*d*f*h*m + a^2*b*d^3*f*h*m^2 +
 b^3*c^2*d*f*h*m^2 - 2*a*b^2*c*d^2*f*h*m^2 - 8*a*b^2*c*d^2*f*h*m))/((a*d -
b*c)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(-4-m)*(f*x+e)*(h*x+g),x)
[Out] Timed out
```

3.35 $\int (a+bx)^m (c+dx)^{-5-m} (e+fx)(g+hx)\,dx$

Optimal. Leaf size=507

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-3}\left(a^2d^2fh\left(m^2+7m+12\right)-2abd(m+4)(cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3m\right.\right.\right.}{2bd^2(m+3)(m+4)(bc-ad)^2}$$

Rubi [A] time = 0.59, antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {146, 45, 37}

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-3}\left(a^2d^2fh\left(m^2+7m+12\right)-2abd(m+4)(cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3m+2\right)+2cd(m+1)(eh+fg)+6d^2eg\right)\right)}{2bd^2(m+3)(m+4)(bc-ad)^2}+\frac{(a+bx)^{m+1}(c+dx)^{-m-2}\left(a^2d^2fh\left(m^2+7m+12\right)-2abd(m+4)(cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3m+2\right)+2cd(m+1)(eh+fg)+6d^2eg\right)\right)}{d^2(m+2)(m+3)(m+4)(bc-ad)^3}+\frac{b(a+bx)^{m+1}(c+dx)^{-m-1}\left(a^2d^2fh\left(m^2+7m+12\right)-2abd(m+4)(cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3m+2\right)+2cd(m+1)(eh+fg)+6d^2eg\right)\right)}{d^2(m+1)(m+2)(m+3)(m+4)(bc-ad)^4}+\frac{(a+bx)^{m+1}(c+dx)^{-m-4}\left(-dfh(m+4)x(bc-ad)+acdfh(m+4)+b\left(c^2(-f)h(m+2)-2cd(eh+fg)+2d^2eg\right)\right)}{2bd^2(m+4)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)^m*(c + d*x)^(-5 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a^2*d^2*f*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d*(f*g + e*h) + c*f*h*(1
+ m)) + b^2*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2
)))*(a + b*x)^(1 + m)*(c + d*x)^(-3 - m))/(2*b*d^2*(b*c - a*d)^2*(3 + m)*(4
 + m)) + ((a^2*d^2*f*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d*(f*g + e*h) +
c*f*h*(1 + m)) + b^2*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 +
3*m + m^2)))*(a + b*x)^(1 + m)*(c + d*x)^(-2 - m))/(d^2*(b*c - a*d)^3*(2 +
m)*(3 + m)*(4 + m)) + (b*(a^2*d^2*f*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d
*(f*g + e*h) + c*f*h*(1 + m)) + b^2*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m)
+ c^2*f*h*(2 + 3*m + m^2)))*(a + b*x)^(1 + m)*(c + d*x)^(-1 - m))/(d^2*(b*c
 - a*d)^4*(1 + m)*(2 + m)*(3 + m)*(4 + m)) + ((a + b*x)^(1 + m)*(c + d*x)^(
-4 - m)*(a*c*d*f*h*(4 + m) + b*(2*d^2*e*g - 2*c*d*(f*g + e*h) - c^2*f*h*(2
+ m)) - d*(b*c - a*d)*f*h*(4 + m)*x))/(2*b*d^2*(b*c - a*d)*(4 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int (a+bx)^m (c+dx)^{-5-m} (e+fx)(g+hx)\,dx = \frac{(a+bx)^{1+m}(c+dx)^{-4-m}\left(acdfh(4+m)+b\left(2d^2eg-2cd(fg+\right.\right.}{2bd^2(bc-ad)(4+m}$$

$$= \frac{\left(a^2d^2fh\left(12+7m+m^2\right)-2abd(4+m)(d(fg+eh)+cfh(1+\right.}{2b}$$

$$= \frac{\left(a^2d^2fh\left(12+7m+m^2\right)-2abd(4+m)(d(fg+eh)+cfh(1+\right.}{2b}$$

$$= \frac{\left(a^2d^2fh\left(12+7m+m^2\right)-2abd(4+m)(d(fg+eh)+cfh(1+\right.}{2b}$$

Mathematica [A] time = 0.70, size = 279, normalized size = 0.55

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-4}\left(\frac{(c+dx)\left(a^2d^2\left(m^2+3m+2\right)-2abd(m+1)(c(m+3)+dx)+b^2\left(c^2\left(m^2+5m+6\right)+2cd(m+3)x+2d^2x^2\right)\right)\left(a^2d^2fh\left(m^2+7m+12\right)-2abd(m+4)(cfh(m+1)+d(eh+fg))+b^2\left(c^2fh\left(m^2+3m+2\right)+2cd(m+1)(eh+fg)+6d^2eg\right)\right)}{(m+1)(m+2)(m+3)(bc-ad)^3}+adfh(m+4)(c+dx)+b\left(c^2(-f)h(m+2)-cd(2eh+2fg+fh(m+4)x)+2d^2eg\right)\right)}{2bd^2(m+4)(bc-ad)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)^m*(c + d*x)^(-5 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] ((a + b*x)^(1 + m)*(c + d*x)^(-4 - m)*(a*d*f*h*(4 + m)*(c + d*x) + b*(2*d^2
*e*g - c^2*f*h*(2 + m) - c*d*(2*f*g + 2*e*h + f*h*(4 + m)*x)) + ((a^2*d^2*f
*h*(12 + 7*m + m^2) - 2*a*b*d*(4 + m)*(d*(f*g + e*h) + c*f*h*(1 + m)) + b^2
*(6*d^2*e*g + 2*c*d*(f*g + e*h)*(1 + m) + c^2*f*h*(2 + 3*m + m^2)))*(c + d*
x)*(a^2*d^2*(2 + 3*m + m^2) - 2*a*b*d*(1 + m)*(c*(3 + m) + d*x) + b^2*(c^2*
(6 + 5*m + m^2) + 2*c*d*(3 + m)*x + 2*d^2*x^2)))/((b*c - a*d)^3*(1 + m)*(2
+ m)*(3 + m))))/(2*b*d^2*(b*c - a*d)*(4 + m))
```

IntegrateAlgebraic [F] time = 0.12, size = 0, normalized size = 0.00

$$\int (a+bx)^m (c+dx)^{-5-m} (e+fx)(g+hx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^m*(c + d*x)^(-5 - m)*(e + f*x)*(g + h*x),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)^m*(c + d*x)^(-5 - m)*(e + f*x)*(g + h*x
), x]
```

fricas [B] time = 1.18, size = 3441, normalized size = 6.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-5-m)*(f*x+e)*(h*x+g),x, algorithm="fricas")
```

```
[Out] ((a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*e*g*m^3 + ((b^
4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*f*h*m^2 + 2*(3*b^4*d^4*e + (b^4*c*
d^3 - 4*a*b^3*d^4)*f)*g + 2*((b^4*c*d^3 - 4*a*b^3*d^4)*e + (b^4*c^2*d^2 - 4
*a*b^3*c*d^3 + 6*a^2*b^2*d^4)*f)*h + (2*(b^4*c*d^3 - a*b^3*d^4)*f*g + (2*(b
^4*c*d^3 - a*b^3*d^4)*e + (3*b^4*c^2*d^2 - 10*a*b^3*c*d^3 + 7*a^2*b^2*d^4)*
f)*h)*m)*x^5 + ((b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)
*f*h*m^3 + (2*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*f*g + (2*(b^4*c^2
*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*e + (8*b^4*c^3*d - 23*a*b^3*c^2*d^2 + 2
2*a^2*b^2*c*d^3 - 7*a^3*b*d^4)*f)*h)*m^2 + 10*(3*b^4*c*d^3*e + (b^4*c^2*d^2
 - 4*a*b^3*c*d^3)*f)*g + 10*((b^4*c^2*d^2 - 4*a*b^3*c*d^3)*e + (b^4*c^3*d -
 4*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3)*f)*h + (2*(3*(b^4*c*d^3 - a*b^3*d^4)*e
```

```
+ 2*(3*b^4*c^2*d^2 - 5*a*b^3*c*d^3 + 2*a^2*b^2*d^4)*f)*g + (4*(3*b^4*c^2*d^
2 - 5*a*b^3*c*d^3 + 2*a^2*b^2*d^4)*e + (17*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 5
5*a^2*b^2*c*d^3 - 12*a^3*b*d^4)*f)*h)*m)*x^4 + (((b^4*c^3*d - 3*a*b^3*c^2*d
^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*f*g + ((b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a
^2*b^2*c*d^3 - a^3*b*d^4)*e + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^
4*d^4)*f)*h)*m^3 + ((3*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*e + 5*(2
*b^4*c^3*d - 5*a*b^3*c^2*d^2 + 4*a^2*b^2*c*d^3 - a^3*b*d^4)*f)*g + (5*(2*b^
4*c^3*d - 5*a*b^3*c^2*d^2 + 4*a^2*b^2*c*d^3 - a^3*b*d^4)*e + (7*b^4*c^4 - 1
6*a*b^3*c^3*d + 3*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 8*a^4*d^4)*f)*h)*m^2 +
 20*(3*b^4*c^2*d^2*e + (b^4*c^3*d - 4*a*b^3*c^2*d^2)*f)*g + 4*(5*(b^4*c^3*d
 - 4*a*b^3*c^2*d^2)*e + (2*b^4*c^4 - 8*a*b^3*c^3*d + 12*a^2*b^2*c^2*d^2 + 1
2*a^3*b*c*d^3 - 3*a^4*d^4)*f)*h + ((3*(9*b^4*c^2*d^2 - 10*a*b^3*c*d^3 + a^2
*b^2*d^4)*e + (29*b^4*c^3*d - 66*a*b^3*c^2*d^2 + 41*a^2*b^2*c*d^3 - 4*a^3*b
*d^4)*f)*g + ((29*b^4*c^3*d - 66*a*b^3*c^2*d^2 + 41*a^2*b^2*c*d^3 - 4*a^3*b
*d^4)*e + (14*b^4*c^4 - 46*a*b^3*c^3*d + 15*a^2*b^2*c^2*d^2 + 36*a^3*b*c*d^
3 - 19*a^4*d^4)*f)*h)*m)*x^3 - ((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)
*e*h - (3*(3*a*b^3*c^4 - 8*a^2*b^2*c^3*d + 7*a^3*b*c^2*d^2 - 2*a^4*c*d^3)*e
 - (a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*f)*g)*m^2 + ((((b^4*c^3*d -
3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*e + (b^4*c^4 - 2*a*b^3*c^3*d
 + 2*a^3*b*c*d^3 - a^4*d^4)*f)*g + ((b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^
3 - a^4*d^4)*e + (a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3
)*f)*h)*m^3 + ((3*(4*b^4*c^3*d - 9*a*b^3*c^2*d^2 + 6*a^2*b^2*c*d^3 - a^3*b*
d^4)*e + (8*b^4*c^4 - 14*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 16*a^3*b*c*d^3 -
 7*a^4*d^4)*f)*g + ((8*b^4*c^4 - 14*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 16*a^
3*b*c*d^3 - 7*a^4*d^4)*e + 5*(a*b^3*c^4 - 4*a^2*b^2*c^3*d + 5*a^3*b*c^2*d^2
 - 2*a^4*c*d^3)*f)*h)*m^2 + 4*(15*b^4*c^3*d*e + (3*b^4*c^4 - 12*a*b^3*c^3*d
 - 12*a^2*b^2*c^2*d^2 + 8*a^3*b*c*d^3 - 2*a^4*d^4)*f)*g + 4*((3*b^4*c^4 - 1
2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 8*a^3*b*c*d^3 - 2*a^4*d^4)*e + 5*(4*a^
3*b*c^2*d^2 - a^4*c*d^3)*f)*h + (((47*b^4*c^3*d - 60*a*b^3*c^2*d^2 + 15*a^2
*b^2*c*d^3 - 2*a^3*b*d^4)*e + (19*b^4*c^4 - 36*a*b^3*c^3*d - 15*a^2*b^2*c^2
*d^2 + 46*a^3*b*c*d^3 - 14*a^4*d^4)*f)*g + ((19*b^4*c^4 - 36*a*b^3*c^3*d -
15*a^2*b^2*c^2*d^2 + 46*a^3*b*c*d^3 - 14*a^4*d^4)*e + (4*a*b^3*c^4 - 41*a^2
*b^2*c^3*d + 66*a^3*b*c^2*d^2 - 29*a^4*c*d^3)*f)*h)*m)*x^2 + 2*(3*(4*a*b^3*
c^4 - 6*a^2*b^2*c^3*d + 4*a^3*b*c^2*d^2 - a^4*c*d^3)*e - (6*a^2*b^2*c^4 - 4
*a^3*b*c^3*d + a^4*c^2*d^2)*f)*g - 2*((6*a^2*b^2*c^4 - 4*a^3*b*c^3*d + a^4*
c^2*d^2)*e - (4*a^3*b*c^4 - a^4*c^3*d)*f)*h + (((26*a*b^3*c^4 - 57*a^2*b^2*
c^3*d + 42*a^3*b*c^2*d^2 - 11*a^4*c*d^3)*e - (7*a^2*b^2*c^4 - 10*a^3*b*c^3*
d + 3*a^4*c^2*d^2)*f)*g - ((7*a^2*b^2*c^4 - 10*a^3*b*c^3*d + 3*a^4*c^2*d^2)
*e - 2*(a^3*b*c^4 - a^4*c^3*d)*f)*h)*m + (((a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3
*a^3*b*c^2*d^2 - a^4*c*d^3)*e*h + ((b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3
 - a^4*d^4)*e + (a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)
*f)*g)*m^3 + ((3*(3*b^4*c^4 - 4*a*b^3*c^3*d - 3*a^2*b^2*c^2*d^2 + 6*a^3*b*c
*d^3 - 2*a^4*d^4)*e + (7*a*b^3*c^4 - 22*a^2*b^2*c^3*d + 23*a^3*b*c^2*d^2 -
8*a^4*c*d^3)*f)*g + ((7*a*b^3*c^4 - 22*a^2*b^2*c^3*d + 23*a^3*b*c^2*d^2 - 8
*a^4*c*d^3)*e - 2*(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*f)*h)*m^2 + 2
*(3*(4*b^4*c^4 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^
4)*e - 5*(6*a^2*b^2*c^3*d - 4*a^3*b*c^2*d^2 + a^4*c*d^3)*f)*g - 10*((6*a^2*
b^2*c^3*d - 4*a^3*b*c^2*d^2 + a^4*c*d^3)*e - (4*a^3*b*c^3*d - a^4*c^2*d^2)*
f)*h + (((26*b^4*c^4 - 10*a*b^3*c^3*d - 45*a^2*b^2*c^2*d^2 + 40*a^3*b*c*d^3
 - 11*a^4*d^4)*e + (12*a*b^3*c^4 - 55*a^2*b^2*c^3*d + 60*a^3*b*c^2*d^2 - 17
*a^4*c*d^3)*f)*g + ((12*a*b^3*c^4 - 55*a^2*b^2*c^3*d + 60*a^3*b*c^2*d^2 - 1
7*a^4*c*d^3)*e - 4*(2*a^2*b^2*c^4 - 5*a^3*b*c^3*d + 3*a^4*c^2*d^2)*f)*h)*m)
*x)*(b*x + a)^m*(d*x + c)^(-m - 5)/(24*b^4*c^4 - 96*a*b^3*c^3*d + 144*a^2*b
^2*c^2*d^2 - 96*a^3*b*c*d^3 + 24*a^4*d^4 + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2
*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*m^4 + 10*(b^4*c^4 - 4*a*b^3*c^3*d +
 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*m^3 + 35*(b^4*c^4 - 4*a*b^3*c
^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*m^2 + 50*(b^4*c^4 - 4*a
*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (fx+e)(hx+g)(bx+a)^m(dx+c)^{-m-5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-5-m)*(f*x+e)*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 5), x)
```

maple [B] time = 0.02, size = 2343, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)^m*(d*x+c)^(-m-5)*(f*x+e)*(h*x+g),x)
```

```
[Out] -(b*x+a)^(m+1)*(d*x+c)^(-m-4)*(a^3*d^3*f*h*m^3*x^2-3*a^2*b*c*d^2*f*h*m^3*x^
2-a^2*b*d^3*f*h*m^2*x^3+3*a*b^2*c^2*d*f*h*m^3*x^2+2*a*b^2*c*d^2*f*h*m^2*x^3
-b^3*c^3*f*h*m^3*x^2-b^3*c^2*d*f*h*m^2*x^3+a^3*d^3*e*h*m^3*x+a^3*d^3*f*g*m^
3*x+8*a^3*d^3*f*h*m^2*x^2-3*a^2*b*c*d^2*e*h*m^3*x-3*a^2*b*c*d^2*f*g*m^3*x-2
3*a^2*b*c*d^2*f*h*m^2*x^2-2*a^2*b*d^3*e*h*m^2*x^2-2*a^2*b*d^3*f*g*m^2*x^2-7
*a^2*b*d^3*f*h*m*x^3+3*a*b^2*c^2*d*e*h*m^3*x+3*a*b^2*c^2*d*f*g*m^3*x+22*a*b
^2*c^2*d*f*h*m^2*x^2+4*a*b^2*c*d^2*e*h*m^2*x^2+4*a*b^2*c*d^2*f*g*m^2*x^2+10
*a*b^2*c*d^2*f*h*m*x^3+2*a*b^2*d^3*e*h*m*x^3+2*a*b^2*d^3*f*g*m*x^3-b^3*c^3*
e*h*m^3*x-b^3*c^3*f*g*m^3*x-7*b^3*c^3*f*h*m^2*x^2-2*b^3*c^2*d*e*h*m^2*x^2-2
*b^3*c^2*d*f*g*m^2*x^2-3*b^3*c^2*d*f*h*m*x^3-2*b^3*c*d^2*e*h*m*x^3-2*b^3*c*
d^2*f*g*m*x^3+2*a^3*c*d^2*f*h*m^2*x+a^3*d^3*e*g*m^3+7*a^3*d^3*e*h*m^2*x+7*a
^3*d^3*f*g*m^2*x+19*a^3*d^3*f*h*m*x^2-4*a^2*b*c^2*d*f*h*m^2*x-3*a^2*b*c*d^2
*e*g*m^3-22*a^2*b*c*d^2*e*h*m^2*x-22*a^2*b*c*d^2*f*g*m^2*x-58*a^2*b*c*d^2*f
*h*m*x^2-3*a^2*b*d^3*e*g*m^2*x-10*a^2*b*d^3*e*h*m*x^2-10*a^2*b*d^3*f*g*m*x^
2-12*a^2*b*d^3*f*h*x^3+2*a*b^2*c^3*f*h*m^2*x+3*a*b^2*c^2*d*e*g*m^3+23*a*b^2
*c^2*d*e*h*m^2*x+23*a*b^2*c^2*d*f*g*m^2*x+53*a*b^2*c^2*d*f*h*m*x^2+6*a*b^2*
c*d^2*e*g*m^2*x+20*a*b^2*c*d^2*e*h*m*x^2+20*a*b^2*c*d^2*f*g*m*x^2+8*a*b^2*c
*d^2*f*h*x^3+6*a*b^2*d^3*e*g*m*x^2+8*a*b^2*d^3*e*h*x^3+8*a*b^2*d^3*f*g*x^3-
b^3*c^3*e*g*m^3-8*b^3*c^3*e*h*m^2*x-8*b^3*c^3*f*g*m^2*x-14*b^3*c^3*f*h*m*x^
2-3*b^3*c^2*d*e*g*m^2*x-10*b^3*c^2*d*e*h*m*x^2-10*b^3*c^2*d*f*g*m*x^2-2*b^3
*c^2*d*f*h*x^3-6*b^3*c*d^2*e*g*m*x^2-2*b^3*c*d^2*e*h*x^3-2*b^3*c*d^2*f*g*x^
3-6*b^3*d^3*e*g*x^3+a^3*c*d^2*e*h*m^2+a^3*c*d^2*f*g*m^2+10*a^3*c*d^2*f*h*m*
x+6*a^3*d^3*e*g*m^2+14*a^3*d^3*e*h*m*x+14*a^3*d^3*f*g*m*x+12*a^3*d^3*f*h*x^
2-2*a^2*b*c^2*d*e*h*m^2-2*a^2*b*c^2*d*f*g*m^2-20*a^2*b*c^2*d*f*h*m*x-21*a^2
*b*c*d^2*e*g*m^2-53*a^2*b*c*d^2*e*h*m*x-53*a^2*b*c*d^2*f*g*m*x-56*a^2*b*c*d
^2*f*h*x^2-9*a^2*b*d^3*e*g*m*x-8*a^2*b*d^3*e*h*x^2-8*a^2*b*d^3*f*g*x^2+a*b^
2*c^3*e*h*m^2+a*b^2*c^3*f*g*m^2+10*a*b^2*c^3*f*h*m*x+24*a*b^2*c^2*d*e*g*m^2
+58*a*b^2*c^2*d*e*h*m*x+58*a*b^2*c^2*d*f*g*m*x+34*a*b^2*c^2*d*f*h*x^2+30*a*
b^2*c*d^2*e*g*m*x+34*a*b^2*c*d^2*e*h*x^2+34*a*b^2*c*d^2*f*g*x^2+6*a*b^2*d^3
*e*g*x^2-9*b^3*c^3*e*g*m^2-19*b^3*c^3*e*h*m*x-19*b^3*c^3*f*g*m*x-8*b^3*c^3*
f*h*x^2-21*b^3*c^2*d*e*g*m*x-8*b^3*c^2*d*e*h*x^2-8*b^3*c^2*d*f*g*x^2-24*b^3
*c*d^2*e*g*x^2+2*a^3*c^2*d*f*h*m+3*a^3*c*d^2*e*h*m+3*a^3*c*d^2*f*g*m+8*a^3*
c*d^2*f*h*x+11*a^3*d^3*e*g*m+8*a^3*d^3*e*h*x+8*a^3*d^3*f*g*x-2*a^2*b*c^3*f*
h*m-10*a^2*b*c^2*d*e*h*m-10*a^2*b*c^2*d*f*g*m-34*a^2*b*c^2*d*f*h*x-42*a^2*b
*c*d^2*e*g*m-34*a^2*b*c*d^2*e*h*x-34*a^2*b*c*d^2*f*g*x-6*a^2*b*d^3*e*g*x+7*
a*b^2*c^3*e*h*m+7*a*b^2*c^3*f*g*m+8*a*b^2*c^3*f*h*x+57*a*b^2*c^2*d*e*g*m+56
*a*b^2*c^2*d*e*h*x+56*a*b^2*c^2*d*f*g*x+24*a*b^2*c*d^2*e*g*x-26*b^3*c^3*e*g
*m-12*b^3*c^3*e*h*x-12*b^3*c^3*f*g*x-36*b^3*c^2*d*e*g*x+2*a^3*c^2*d*f*h+2*a
^3*c*d^2*e*h+2*a^3*c*d^2*f*g+6*a^3*d^3*e*g-8*a^2*b*c^3*f*h-8*a^2*b*c^2*d*e*
h-8*a^2*b*c^2*d*f*g-24*a^2*b*c*d^2*e*g+12*a*b^2*c^3*e*h+12*a*b^2*c^3*f*g+36
*a*b^2*c^2*d*e*g-24*b^3*c^3*e*g)/(a^4*d^4*m^4-4*a^3*b*c*d^3*m^4+6*a^2*b^2*c
^2*d^2*m^4-4*a*b^3*c^3*d*m^4+b^4*c^4*m^4+10*a^4*d^4*m^3-40*a^3*b*c*d^3*m^3+
60*a^2*b^2*c^2*d^2*m^3-40*a*b^3*c^3*d*m^3+10*b^4*c^4*m^3+35*a^4*d^4*m^2-140
```

```
*a^3*b*c*d^3*m^2+210*a^2*b^2*c^2*d^2*m^2-140*a*b^3*c^3*d*m^2+35*b^4*c^4*m^2
+50*a^4*d^4*m-200*a^3*b*c*d^3*m+300*a^2*b^2*c^2*d^2*m-200*a*b^3*c^3*d*m+50*
b^4*c^4*m+24*a^4*d^4-96*a^3*b*c*d^3+144*a^2*b^2*c^2*d^2-96*a*b^3*c^3*d+24*b
^4*c^4)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx+e\right)\left(hx+g\right)(bx+a)^m (dx+c)^{-m-5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^m*(d*x+c)^(-5-m)*(f*x+e)*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((f*x + e)*(h*x + g)*(b*x + a)^m*(d*x + c)^(-m - 5), x)
```

mupad [B] time = 6.75, size = 3720, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)*(g + h*x)*(a + b*x)^m)/(c + d*x)^(m + 5),x)
```

```
[Out] (x^5*(a + b*x)^m*(6*b^4*d^4*e*g - 8*a*b^3*d^4*e*h - 8*a*b^3*d^4*f*g + 2*b^4
*c*d^3*e*h + 2*b^4*c*d^3*f*g + 12*a^2*b^2*d^4*f*h + 2*b^4*c^2*d^2*f*h + a^2
*b^2*d^4*f*h*m^2 + b^4*c^2*d^2*f*h*m^2 - 8*a*b^3*c*d^3*f*h - 2*a*b^3*d^4*e*
h*m - 2*a*b^3*d^4*f*g*m + 2*b^4*c*d^3*e*h*m + 2*b^4*c*d^3*f*g*m + 7*a^2*b^2
*d^4*f*h*m + 3*b^4*c^2*d^2*f*h*m - 2*a*b^3*c*d^3*f*h*m^2 - 10*a*b^3*c*d^3*f
*h*m))/((a*d - b*c)^4*(c + d*x)^(m + 5)*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)
) - (x*(a + b*x)^m*(6*a^4*d^4*e*g - 24*b^4*c^4*e*g + 10*a^4*c*d^3*e*h + 10*
a^4*c*d^3*f*g + 11*a^4*d^4*e*g*m - 26*b^4*c^4*e*g*m + 10*a^4*c^2*d^2*f*h +
6*a^4*d^4*e*g*m^2 - 9*b^4*c^4*e*g*m^2 + a^4*d^4*e*g*m^3 - b^4*c^4*e*g*m^3 +
 36*a^2*b^2*c^2*d^2*e*g + 2*a^2*b^2*c^4*f*h*m^2 + 2*a^4*c^2*d^2*f*h*m^2 - 2
4*a*b^3*c^3*d*e*g - 24*a^3*b*c*d^3*e*g - 40*a^3*b*c^3*d*f*h - 12*a*b^3*c^4*
e*h*m - 12*a*b^3*c^4*f*g*m + 17*a^4*c*d^3*e*h*m + 17*a^4*c*d^3*f*g*m + 60*a
^2*b^2*c^3*d*e*h + 60*a^2*b^2*c^3*d*f*g - 40*a^3*b*c^2*d^2*e*h - 40*a^3*b*c
^2*d^2*f*g - 7*a*b^3*c^4*e*h*m^2 - 7*a*b^3*c^4*f*g*m^2 - a*b^3*c^4*e*h*m^3
- a*b^3*c^4*f*g*m^3 + 8*a^2*b^2*c^4*f*h*m + 8*a^4*c*d^3*e*h*m^2 + 8*a^4*c*d
^3*f*g*m^2 + a^4*c*d^3*e*h*m^3 + a^4*c*d^3*f*g*m^3 + 12*a^4*c^2*d^2*f*h*m +
 12*a*b^3*c^3*d*e*g*m^2 - 18*a^3*b*c*d^3*e*g*m^2 + 2*a*b^3*c^3*d*e*g*m^3 -
2*a^3*b*c*d^3*e*g*m^3 + 55*a^2*b^2*c^3*d*e*h*m + 55*a^2*b^2*c^3*d*f*g*m - 6
0*a^3*b*c^2*d^2*e*h*m - 60*a^3*b*c^2*d^2*f*g*m - 4*a^3*b*c^3*d*f*h*m^2 + 45
*a^2*b^2*c^2*d^2*e*g*m + 22*a^2*b^2*c^3*d*e*h*m^2 + 22*a^2*b^2*c^3*d*f*g*m^
2 - 23*a^3*b*c^2*d^2*e*h*m^2 - 23*a^3*b*c^2*d^2*f*g*m^2 + 3*a^2*b^2*c^3*d*e
*h*m^3 + 3*a^2*b^2*c^3*d*f*g*m^3 - 3*a^3*b*c^2*d^2*e*h*m^3 - 3*a^3*b*c^2*d^
2*f*g*m^3 + 10*a*b^3*c^3*d*e*g*m - 40*a^3*b*c*d^3*e*g*m - 20*a^3*b*c^3*d*f*
h*m + 9*a^2*b^2*c^2*d^2*e*g*m^2))/((a*d - b*c)^4*(c + d*x)^(m + 5)*(50*m +
35*m^2 + 10*m^3 + m^4 + 24)) - ((a + b*x)^m*(6*a^4*c*d^3*e*g - 8*a^3*b*c^4*
f*h - 24*a*b^3*c^4*e*g + 2*a^4*c^3*d*f*h + 12*a^2*b^2*c^4*e*h + 12*a^2*b^2*
c^4*f*g + 2*a^4*c^2*d^2*e*h + 2*a^4*c^2*d^2*f*g + a^2*b^2*c^4*e*h*m^2 + a^2
*b^2*c^4*f*g*m^2 + a^4*c^2*d^2*e*h*m^2 + a^4*c^2*d^2*f*g*m^2 - 8*a^3*b*c^3*
d*e*h - 8*a^3*b*c^3*d*f*g - 26*a*b^3*c^4*e*g*m - 2*a^3*b*c^4*f*h*m + 11*a^4
*c*d^3*e*g*m + 2*a^4*c^3*d*f*h*m + 36*a^2*b^2*c^3*d*e*g - 24*a^3*b*c^2*d^2*
e*g - 9*a*b^3*c^4*e*g*m^2 - a*b^3*c^4*e*g*m^3 + 7*a^2*b^2*c^4*e*h*m + 7*a^2
*b^2*c^4*f*g*m + 6*a^4*c*d^3*e*g*m^2 + a^4*c*d^3*e*g*m^3 + 3*a^4*c^2*d^2*e*
h*m + 3*a^4*c^2*d^2*f*g*m + 57*a^2*b^2*c^3*d*e*g*m - 42*a^3*b*c^2*d^2*e*g*m
 - 2*a^3*b*c^3*d*e*h*m^2 - 2*a^3*b*c^3*d*f*g*m^2 + 24*a^2*b^2*c^3*d*e*g*m^2
 - 21*a^3*b*c^2*d^2*e*g*m^2 + 3*a^2*b^2*c^3*d*e*g*m^3 - 3*a^3*b*c^2*d^2*e*g
*m^3 - 10*a^3*b*c^3*d*e*h*m - 10*a^3*b*c^3*d*f*g*m))/((a*d - b*c)^4*(c + d*
x)^(m + 5)*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (x^3*(a + b*x)^m*(8*b^4*c
^4*f*h - 12*a^4*d^4*f*h + 20*b^4*c^3*d*e*h + 20*b^4*c^3*d*f*g - 19*a^4*d^4*
f*h*m + 14*b^4*c^4*f*h*m + 60*b^4*c^2*d^2*e*g - 8*a^4*d^4*f*h*m^2 + 7*b^4*c
```

```
^4*f*h*m^2 - a^4*d^4*f*h*m^3 + b^4*c^4*f*h*m^3 + 48*a^2*b^2*c^2*d^2*f*h + 3
*a^2*b^2*d^4*e*g*m^2 + 3*b^4*c^2*d^2*e*g*m^2 - 32*a*b^3*c^3*d*f*h + 48*a^3*
b*c*d^3*f*h - 4*a^3*b*d^4*e*h*m - 4*a^3*b*d^4*f*g*m + 29*b^4*c^3*d*e*h*m +
29*b^4*c^3*d*f*g*m - 80*a*b^3*c^2*d^2*e*h - 80*a*b^3*c^2*d^2*f*g + 3*a^2*b^
2*d^4*e*g*m - 5*a^3*b*d^4*e*h*m^2 - 5*a^3*b*d^4*f*g*m^2 - a^3*b*d^4*e*h*m^3
 - a^3*b*d^4*f*g*m^3 + 27*b^4*c^2*d^2*e*g*m + 10*b^4*c^3*d*e*h*m^2 + 10*b^4
*c^3*d*f*g*m^2 + b^4*c^3*d*e*h*m^3 + b^4*c^3*d*f*g*m^3 + 3*a^2*b^2*c^2*d^2*
f*h*m^2 - 6*a*b^3*c*d^3*e*g*m^2 - 66*a*b^3*c^2*d^2*e*h*m - 66*a*b^3*c^2*d^2
*f*g*m + 41*a^2*b^2*c*d^3*e*h*m + 41*a^2*b^2*c*d^3*f*g*m - 16*a*b^3*c^3*d*f
*h*m^2 + 14*a^3*b*c*d^3*f*h*m^2 - 2*a*b^3*c^3*d*f*h*m^3 + 2*a^3*b*c*d^3*f*h
*m^3 - 25*a*b^3*c^2*d^2*e*h*m^2 - 25*a*b^3*c^2*d^2*f*g*m^2 + 20*a^2*b^2*c*d
^3*e*h*m^2 + 20*a^2*b^2*c*d^3*f*g*m^2 - 3*a*b^3*c^2*d^2*e*h*m^3 - 3*a*b^3*c
^2*d^2*f*g*m^3 + 3*a^2*b^2*c*d^3*e*h*m^3 + 3*a^2*b^2*c*d^3*f*g*m^3 + 15*a^2
*b^2*c^2*d^2*f*h*m - 30*a*b^3*c*d^3*e*g*m - 46*a*b^3*c^3*d*f*h*m + 36*a^3*b
*c*d^3*f*h*m))/((a*d - b*c)^4*(c + d*x)^(m + 5)*(50*m + 35*m^2 + 10*m^3 + m
^4 + 24)) - (x^2*(a + b*x)^m*(8*a^4*d^4*e*h + 8*a^4*d^4*f*g - 12*b^4*c^4*e*
h - 12*b^4*c^4*f*g - 60*b^4*c^3*d*e*g + 20*a^4*c*d^3*f*h + 14*a^4*d^4*e*h*m
 + 14*a^4*d^4*f*g*m - 19*b^4*c^4*e*h*m - 19*b^4*c^4*f*g*m + 7*a^4*d^4*e*h*m
^2 + 7*a^4*d^4*f*g*m^2 - 8*b^4*c^4*e*h*m^2 - 8*b^4*c^4*f*g*m^2 + a^4*d^4*e*
h*m^3 + a^4*d^4*f*g*m^3 - b^4*c^4*e*h*m^3 - b^4*c^4*f*g*m^3 + 48*a^2*b^2*c^
2*d^2*e*h + 48*a^2*b^2*c^2*d^2*f*g + 48*a*b^3*c^3*d*e*h + 48*a*b^3*c^3*d*f*
g - 32*a^3*b*c*d^3*e*h - 32*a^3*b*c*d^3*f*g + 2*a^3*b*d^4*e*g*m - 4*a*b^3*c
^4*f*h*m - 47*b^4*c^3*d*e*g*m + 29*a^4*c*d^3*f*h*m - 80*a^3*b*c^2*d^2*f*h +
 3*a^3*b*d^4*e*g*m^2 + a^3*b*d^4*e*g*m^3 - 5*a*b^3*c^4*f*h*m^2 - a*b^3*c^4*
f*h*m^3 - 12*b^4*c^3*d*e*g*m^2 - b^4*c^3*d*e*g*m^3 + 10*a^4*c*d^3*f*h*m^2 +
 a^4*c*d^3*f*h*m^3 + 3*a^2*b^2*c^2*d^2*e*h*m^2 + 3*a^2*b^2*c^2*d^2*f*g*m^2
+ 60*a*b^3*c^2*d^2*e*g*m - 15*a^2*b^2*c*d^3*e*g*m + 14*a*b^3*c^3*d*e*h*m^2
+ 14*a*b^3*c^3*d*f*g*m^2 - 16*a^3*b*c*d^3*e*h*m^2 - 16*a^3*b*c*d^3*f*g*m^2
+ 2*a*b^3*c^3*d*e*h*m^3 + 2*a*b^3*c^3*d*f*g*m^3 - 2*a^3*b*c*d^3*e*h*m^3 - 2
*a^3*b*c*d^3*f*g*m^3 + 41*a^2*b^2*c^3*d*f*h*m - 66*a^3*b*c^2*d^2*f*h*m + 27
*a*b^3*c^2*d^2*e*g*m^2 - 18*a^2*b^2*c*d^3*e*g*m^2 + 3*a*b^3*c^2*d^2*e*g*m^3
 - 3*a^2*b^2*c*d^3*e*g*m^3 + 15*a^2*b^2*c^2*d^2*e*h*m + 15*a^2*b^2*c^2*d^2*
f*g*m + 20*a^2*b^2*c^3*d*f*h*m^2 - 25*a^3*b*c^2*d^2*f*h*m^2 + 3*a^2*b^2*c^3
*d*f*h*m^3 - 3*a^3*b*c^2*d^2*f*h*m^3 + 36*a*b^3*c^3*d*e*h*m + 36*a*b^3*c^3*
d*f*g*m - 46*a^3*b*c*d^3*e*h*m - 46*a^3*b*c*d^3*f*g*m))/((a*d - b*c)^4*(c +
 d*x)^(m + 5)*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)) + (b*d*x^4*(a + b*x)^m*(
5*b*c - a*d*m + b*c*m)*(6*b^2*d^2*e*g + 12*a^2*d^2*f*h + 2*b^2*c^2*f*h + 7*
a^2*d^2*f*h*m + 3*b^2*c^2*f*h*m + a^2*d^2*f*h*m^2 + b^2*c^2*f*h*m^2 - 8*a*b
*d^2*e*h - 8*a*b*d^2*f*g + 2*b^2*c*d*e*h + 2*b^2*c*d*f*g - 2*a*b*d^2*e*h*m
- 2*a*b*d^2*f*g*m + 2*b^2*c*d*e*h*m + 2*b^2*c*d*f*g*m - 8*a*b*c*d*f*h - 10*
a*b*c*d*f*h*m - 2*a*b*c*d*f*h*m^2))/((a*d - b*c)^4*(c + d*x)^(m + 5)*(50*m
+ 35*m^2 + 10*m^3 + m^4 + 24))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)**m*(d*x+c)**(-5-m)*(f*x+e)*(h*x+g),x)
```

```
[Out] Timed out
```

3.36 $\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal. Leaf size=363

$$\frac{(c+dx)^{-m-2}(e+fx)^{m+1}\left(adf(cfh(m+1)+d(2fg-eh(m+3)))+b\left(c^2f^2h\left(m^2+3m+2\right)+cdf(m+1)(fg-2\right.\right.}{d^2f(m+2)(m+3)(de-cf)^2}$$

Rubi [A] time = 0.40, antiderivative size = 360, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {146, 45, 37}

$$\frac{(c+dx)^{-m-2}(e+fx)^{m+1}\left(adf(cfh(m+1)-deh(m+3)+2dfg)+b\left(c^2f^2h\left(m^2+3m+2\right)+cdf(m+1)(fg-2eh(m+3))+d^2(-e)(m+3)(fg-eh(m+2))\right)\right)}{d^2f(m+2)(m+3)(de-cf)^2}-\frac{(c+dx)^{-m-1}(e+fx)^{m+1}\left(adf(cfh(m+1)-deh(m+3)+2dfg)+b\left(c^2f^2h\left(m^2+3m+2\right)+cdf(m+1)(fg-2eh(m+3))+d^2(-e)(m+3)(fg-eh(m+2))\right)\right)}{d^2(m+1)(m+2)(m+3)(de-cf)^3}-\frac{(c+dx)^{-m-3}(e+fx)^{m+1}(adf(dg-ch)-bc(cfh(m+2)-deh(m+3)+dfg)+bdh(m+3)x(de-cf))}{d^2f(m+3)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x)*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((a*d*f*(2*d*f*g + c*f*h*(1 + m) - d*e*h*(3 + m)) + b*(c^2*f^2*h*(2 + 3*m +
 m^2) - d^2*e*(3 + m)*(f*g - e*h*(2 + m)) + c*d*f*(1 + m)*(f*g - 2*e*h*(3 +
 m))))*(c + d*x)^(-2 - m)*(e + f*x)^(1 + m))/(d^2*f*(d*e - c*f)^2*(2 + m)*(
3 + m)) - ((a*d*f*(2*d*f*g + c*f*h*(1 + m) - d*e*h*(3 + m)) + b*(c^2*f^2*h*
(2 + 3*m + m^2) - d^2*e*(3 + m)*(f*g - e*h*(2 + m)) + c*d*f*(1 + m)*(f*g -
2*e*h*(3 + m))))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d^2*(d*e - c*f)^3*(
1 + m)*(2 + m)*(3 + m)) - ((c + d*x)^(-3 - m)*(e + f*x)^(1 + m)*(a*d*f*(d*g
 - c*h) - b*c*(d*f*g + c*f*h*(2 + m) - d*e*h*(3 + m)) + b*d*(d*e - c*f)*h*(
3 + m)*x))/(d^2*f*(d*e - c*f)*(3 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx = -\frac{(c+dx)^{-3-m}(e+fx)^{1+m}(adf(dg-ch)-bc(dfg+cfh(2+m)}{d^2f(de-cf)(3+m)}$$
$$= \frac{\left(adf(2dfg+cfh(1+m)-deh(3+m))+b\left(c^2f^2h\left(2+3m+\right.\right.\right.}{}$$
$$= \frac{\left(adf(2dfg+cfh(1+m)-deh(3+m))+b\left(c^2f^2h\left(2+3m+\right.\right.\right.}{}$$

Mathematica [A] time = 0.55, size = 227, normalized size = 0.63

$$\frac{(c+dx)^{-m-3}(e+fx)^{m+1}\left(\frac{(c+dx)(cf(m+2)-d(em+e-fx))\left(adf(cfh(m+1)-deh(m+3)+2dfg)+b\left(c^2f^2h\left(m^2+3m+2\right)+cdf(m+1)(fg-2eh(m+3))+d^2e(m+3)(eh(m+2)-fg)\right)\right)}{(m+1)(m+2)(de-cf)^2}+adf(dg-ch)-b\left(c^2fh(m+2)+cd(f(g+h(m+3)x)-eh(m+3))-d^2eh(m+3)x\right)\right)}{d^2f(m+3)(cf-de)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x)*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((c + d*x)^(-3 - m)*(e + f*x)^(1 + m)*(a*d*f*(d*g - c*h) + ((a*d*f*(2*d*f*g
 + c*f*h*(1 + m) - d*e*h*(3 + m)) + b*(c^2*f^2*h*(2 + 3*m + m^2) + d^2*e*(3
 + m)*(-(f*g) + e*h*(2 + m)) + c*d*f*(1 + m)*(f*g - 2*e*h*(3 + m))))*(c + d
*x)*(c*f*(2 + m) - d*(e + e*m - f*x)))/((d*e - c*f)^2*(1 + m)*(2 + m)) - b*
(c^2*f*h*(2 + m) - d^2*e*h*(3 + m)*x + c*d*(-(e*h*(3 + m)) + f*(g + h*(3 +
m)*x)))))/(d^2*f*(-(d*e) + c*f)*(3 + m))
```

IntegrateAlgebraic [F] time = 0.12, size = 0, normalized size = 0.00

$$\int (a+bx)(c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] Defer[IntegrateAlgebraic][(a + b*x)*(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x
), x]
```

fricas [B] time = 0.71, size = 1608, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
```

```
[Out] -(((b*d^3*e^2*f - 2*b*c*d^2*e*f^2 + b*c^2*d*f^3)*h*m^2 - (3*b*d^3*e*f^2 - (
b*c*d^2 + 2*a*d^3)*f^3)*g + (6*b*d^3*e^2*f - 3*(2*b*c*d^2 + a*d^3)*e*f^2 +
(2*b*c^2*d + a*c*d^2)*f^3)*h - ((b*d^3*e*f^2 - b*c*d^2*f^3)*g - (5*b*d^3*e^
2*f - (8*b*c*d^2 + a*d^3)*e*f^2 + (3*b*c^2*d + a*c*d^2)*f^3)*h)*m)*x^4 + (a
*c*d^2*e^3 - 2*a*c^2*d*e^2*f + a*c^3*e*f^2)*g*m^2 + (((b*d^3*e^2*f - 2*b*c*
d^2*e*f^2 + b*c^2*d*f^3)*g + (b*d^3*e^3 - (b*c*d^2 - a*d^3)*e^2*f - (b*c^2*
d + 2*a*c*d^2)*e*f^2 + (b*c^3 + a*c^2*d)*f^3)*h)*m^2 - 4*(3*b*c*d^2*e*f^2 -
 (b*c^2*d + 2*a*c*d^2)*f^3)*g + 2*(3*b*d^3*e^3 + 3*b*c*d^2*e^2*f - 3*(b*c^2
*d + 2*a*c*d^2)*e*f^2 + (b*c^3 + 2*a*c^2*d)*f^3)*h + ((3*b*d^3*e^2*f - 2*(4
*b*c*d^2 + a*d^3)*e*f^2 + (5*b*c^2*d + 2*a*c*d^2)*f^3)*g + (5*b*d^3*e^3 - (
b*c*d^2 - 3*a*d^3)*e^2*f - (7*b*c^2*d + 8*a*c*d^2)*e*f^2 + (3*b*c^3 + 5*a*c
^2*d)*f^3)*h)*m)*x^3 + (((b*d^3*e^3 - (b*c*d^2 - a*d^3)*e^2*f - (b*c^2*d +
2*a*c*d^2)*e*f^2 + (b*c^3 + a*c^2*d)*f^3)*g + (a*c^3*f^3 + (b*c*d^2 + a*d^3
)*e^3 - (2*b*c^2*d + a*c*d^2)*e^2*f + (b*c^3 - a*c^2*d)*e*f^2)*h)*m^2 + 3*(
b*d^3*e^3 - 3*b*c*d^2*e^2*f - 3*b*c^2*d*e*f^2 + (b*c^3 + 4*a*c^2*d)*f^3)*g
- 3*(3*a*c*d^2*e^2*f + 3*a*c^2*d*e*f^2 - a*c^3*f^3 - (4*b*c*d^2 + a*d^3)*e^
```

```
3)*h + ((4*b*d^3*e^3 - (4*b*c*d^2 - a*d^3)*e^2*f - 4*(b*c^2*d + 2*a*c*d^2)*
e*f^2 + (4*b*c^3 + 7*a*c^2*d)*f^3)*g + (4*a*c^3*f^3 + (7*b*c*d^2 + 4*a*d^3)
*e^3 - 4*(2*b*c^2*d + a*c*d^2)*e^2*f + (b*c^3 - 4*a*c^2*d)*e*f^2)*h)*m)*x^2
 + (6*a*c^3*e*f^2 + (b*c^2*d + 2*a*c*d^2)*e^3 - 3*(b*c^3 + 2*a*c^2*d)*e^2*f
)*g - (3*a*c^3*e^2*f - (2*b*c^3 + a*c^2*d)*e^3)*h + ((5*a*c^3*e*f^2 + (b*c^
2*d + 3*a*c*d^2)*e^3 - (b*c^3 + 8*a*c^2*d)*e^2*f)*g + (a*c^2*d*e^3 - a*c^3*
e^2*f)*h)*m + (((a*c^3*f^3 + (b*c*d^2 + a*d^3)*e^3 - (2*b*c^2*d + a*c*d^2)*
e^2*f + (b*c^3 - a*c^2*d)*e*f^2)*g + (a*c*d^2*e^3 - 2*a*c^2*d*e^2*f + a*c^3
*e*f^2)*h)*m^2 + 2*(3*a*c^2*d*e*f^2 + 3*a*c^3*f^3 + (2*b*c*d^2 + a*d^3)*e^3
 - 3*(2*b*c^2*d + a*c*d^2)*e^2*f)*g - 4*(3*a*c^2*d*e^2*f - (2*b*c^2*d + a*c
*d^2)*e^3)*h + ((5*a*c^3*f^3 + (5*b*c*d^2 + 3*a*d^3)*e^3 - (8*b*c^2*d + 7*a
*c*d^2)*e^2*f + (3*b*c^3 - a*c^2*d)*e*f^2)*g + (3*a*c^3*e*f^2 + (2*b*c^2*d
+ 5*a*c*d^2)*e^3 - 2*(b*c^3 + 4*a*c^2*d)*e^2*f)*h)*m)*x)*(d*x + c)^(-m - 4)
*(f*x + e)^m/(6*d^3*e^3 - 18*c*d^2*e^2*f + 18*c^2*d*e*f^2 - 6*c^3*f^3 + (d^
3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*m^3 + 6*(d^3*e^3 - 3*c*d^2
*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*m^2 + 11*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2
*d*e*f^2 - c^3*f^3)*m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)\left(hx+g\right)(dx+c)^{-m-4}\left(fx+e\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((b*x + a)*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

maple [B] time = 0.01, size = 906, normalized size = 2.50

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x+a)*(d*x+c)^(-m-4)*(f*x+e)^m*(h*x+g),x)
```

```
[Out] -(d*x+c)^(-m-3)*(f*x+e)^(m+1)*(-b*c^2*f^2*h*m^2*x^2+2*b*c*d*e*f*h*m^2*x^2-b
*d^2*e^2*h*m^2*x^2-a*c^2*f^2*h*m^2*x+2*a*c*d*e*f*h*m^2*x-a*c*d*f^2*h*m*x^2-
a*d^2*e^2*h*m^2*x+a*d^2*e*f*h*m*x^2-b*c^2*f^2*g*m^2*x-3*b*c^2*f^2*h*m*x^2+2
*b*c*d*e*f*g*m^2*x+8*b*c*d*e*f*h*m*x^2-b*c*d*f^2*g*m*x^2-b*d^2*e^2*g*m^2*x-
5*b*d^2*e^2*h*m*x^2+b*d^2*e*f*g*m*x^2-a*c^2*f^2*g*m^2-4*a*c^2*f^2*h*m*x+2*a
*c*d*e*f*g*m^2+8*a*c*d*e*f*h*m*x-2*a*c*d*f^2*g*m*x-a*c*d*f^2*h*x^2-a*d^2*e^
2*g*m^2-4*a*d^2*e^2*h*m*x+2*a*d^2*e*f*g*m*x+3*a*d^2*e*f*h*x^2-2*a*d^2*f^2*g
*x^2+2*b*c^2*e*f*h*m*x-4*b*c^2*f^2*g*m*x-2*b*c^2*f^2*h*x^2-2*b*c*d*e^2*h*m*
x+8*b*c*d*e*f*g*m*x+6*b*c*d*e*f*h*x^2-b*c*d*f^2*g*x^2-4*b*d^2*e^2*g*m*x-6*b
*d^2*e^2*h*x^2+3*b*d^2*e*f*g*x^2+a*c^2*e*f*h*m-5*a*c^2*f^2*g*m-3*a*c^2*f^2*
h*x-a*c*d*e^2*h*m+8*a*c*d*e*f*g*m+10*a*c*d*e*f*h*x-6*a*c*d*f^2*g*x-3*a*d^2*
e^2*g*m-3*a*d^2*e^2*h*x+2*a*d^2*e*f*g*x+b*c^2*e*f*g*m+2*b*c^2*e*f*h*x-3*b*c
^2*f^2*g*x-b*c*d*e^2*g*m-6*b*c*d*e^2*h*x+10*b*c*d*e*f*g*x-3*b*d^2*e^2*g*x+3
*a*c^2*e*f*h-6*a*c^2*f^2*g-a*c*d*e^2*h+6*a*c*d*e*f*g-2*a*d^2*e^2*g-2*b*c^2*
e^2*h+3*b*c^2*e*f*g-b*c*d*e^2*g)/(c^3*f^3*m^3-3*c^2*d*e*f^2*m^3+3*c*d^2*e^2
*f*m^3-d^3*e^3*m^3+6*c^3*f^3*m^2-18*c^2*d*e*f^2*m^2+18*c*d^2*e^2*f*m^2-6*d^
3*e^3*m^2+11*c^3*f^3*m-33*c^2*d*e*f^2*m+33*c*d^2*e^2*f*m-11*d^3*e^3*m+6*c^3
*f^3-18*c^2*d*e*f^2+18*c*d^2*e^2*f-6*d^3*e^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx+a)\left(hx+g\right)(dx+c)^{-m-4}\left(fx+e\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((b*x + a)*(h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

mupad [B] time = 4.28, size = 1890, normalized size = 5.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^m*(g + h*x)*(a + b*x))/(c + d*x)^(m + 4),x)
```

```
[Out] ((e + f*x)^m*(2*b*c^3*e^3*h + 2*a*c*d^2*e^3*g + a*c^2*d*e^3*h + b*c^2*d*e^3
*g + 6*a*c^3*e*f^2*g - 3*a*c^3*e^2*f*h - 3*b*c^3*e^2*f*g - 6*a*c^2*d*e^2*f*
g + 3*a*c*d^2*e^3*g*m + a*c^2*d*e^3*h*m + b*c^2*d*e^3*g*m + 5*a*c^3*e*f^2*g
*m - a*c^3*e^2*f*h*m - b*c^3*e^2*f*g*m + a*c*d^2*e^3*g*m^2 + a*c^3*e*f^2*g*
m^2 - 2*a*c^2*d*e^2*f*g*m^2 - 8*a*c^2*d*e^2*f*g*m))/((c*f - d*e)^3*(c + d*x
)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (x*(e + f*x)^m*(6*a*c^3*f^3*g + 2*a*d
^3*e^3*g + 4*a*c*d^2*e^3*h + 4*b*c*d^2*e^3*g + 8*b*c^2*d*e^3*h + 5*a*c^3*f^
3*g*m + 3*a*d^3*e^3*g*m + a*c^3*f^3*g*m^2 + a*d^3*e^3*g*m^2 - 6*a*c*d^2*e^2
*f*g + 6*a*c^2*d*e*f^2*g - 12*a*c^2*d*e^2*f*h - 12*b*c^2*d*e^2*f*g + 5*a*c*
d^2*e^3*h*m + 5*b*c*d^2*e^3*g*m + 2*b*c^2*d*e^3*h*m + 3*a*c^3*e*f^2*h*m + 3
*b*c^3*e*f^2*g*m - 2*b*c^3*e^2*f*h*m + a*c*d^2*e^3*h*m^2 + b*c*d^2*e^3*g*m^
2 + a*c^3*e*f^2*h*m^2 + b*c^3*e*f^2*g*m^2 - a*c*d^2*e^2*f*g*m^2 - a*c^2*d*e
*f^2*g*m^2 - 2*a*c^2*d*e^2*f*h*m^2 - 2*b*c^2*d*e^2*f*g*m^2 - 7*a*c*d^2*e^2*
f*g*m - a*c^2*d*e*f^2*g*m - 8*a*c^2*d*e^2*f*h*m - 8*b*c^2*d*e^2*f*g*m))/((c
*f - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (x^4*(e + f*x)^m*
(2*a*d^3*f^3*g + a*c*d^2*f^3*h + b*c*d^2*f^3*g + 2*b*c^2*d*f^3*h - 3*a*d^3*
e*f^2*h - 3*b*d^3*e*f^2*g + 6*b*d^3*e^2*f*h - 6*b*c*d^2*e*f^2*h + a*c*d^2*f
^3*h*m + b*c*d^2*f^3*g*m + 3*b*c^2*d*f^3*h*m - a*d^3*e*f^2*h*m - b*d^3*e*f^
2*g*m + 5*b*d^3*e^2*f*h*m + b*c^2*d*f^3*h*m^2 + b*d^3*e^2*f*h*m^2 - 2*b*c*d
^2*e*f^2*h*m^2 - 8*b*c*d^2*e*f^2*h*m))/((c*f - d*e)^3*(c + d*x)^(m + 4)*(11
*m + 6*m^2 + m^3 + 6)) + (x^2*(e + f*x)^m*(3*a*c^3*f^3*h + 3*a*d^3*e^3*h +
3*b*c^3*f^3*g + 3*b*d^3*e^3*g + 12*a*c^2*d*f^3*g + 12*b*c*d^2*e^3*h + 4*a*c
^3*f^3*h*m + 4*a*d^3*e^3*h*m + 4*b*c^3*f^3*g*m + 4*b*d^3*e^3*g*m + a*c^3*f^
3*h*m^2 + a*d^3*e^3*h*m^2 + b*c^3*f^3*g*m^2 + b*d^3*e^3*g*m^2 - 9*a*c*d^2*e
^2*f*h - 9*a*c^2*d*e*f^2*h - 9*b*c*d^2*e^2*f*g - 9*b*c^2*d*e*f^2*g + 7*a*c^
2*d*f^3*g*m + 7*b*c*d^2*e^3*h*m + a*d^3*e^2*f*g*m + b*c^3*e*f^2*h*m + a*c^2
*d*f^3*g*m^2 + b*c*d^2*e^3*h*m^2 + a*d^3*e^2*f*g*m^2 + b*c^3*e*f^2*h*m^2 -
2*a*c*d^2*e*f^2*g*m^2 - a*c*d^2*e^2*f*h*m^2 - a*c^2*d*e*f^2*h*m^2 - b*c*d^2
*e^2*f*g*m^2 - b*c^2*d*e*f^2*g*m^2 - 2*b*c^2*d*e^2*f*h*m^2 - 8*a*c*d^2*e*f^
2*g*m - 4*a*c*d^2*e^2*f*h*m - 4*a*c^2*d*e*f^2*h*m - 4*b*c*d^2*e^2*f*g*m - 4
*b*c^2*d*e*f^2*g*m - 8*b*c^2*d*e^2*f*h*m))/((c*f - d*e)^3*(c + d*x)^(m + 4)
*(11*m + 6*m^2 + m^3 + 6)) + (x^3*(e + f*x)^m*(2*b*c^3*f^3*h + 6*b*d^3*e^3*
h + 8*a*c*d^2*f^3*g + 4*a*c^2*d*f^3*h + 4*b*c^2*d*f^3*g + 3*b*c^3*f^3*h*m +
 5*b*d^3*e^3*h*m + b*c^3*f^3*h*m^2 + b*d^3*e^3*h*m^2 - 12*a*c*d^2*e*f^2*h -
 12*b*c*d^2*e*f^2*g + 6*b*c*d^2*e^2*f*h - 6*b*c^2*d*e*f^2*h + 2*a*c*d^2*f^3
*g*m + 5*a*c^2*d*f^3*h*m + 5*b*c^2*d*f^3*g*m - 2*a*d^3*e*f^2*g*m + 3*a*d^3*
e^2*f*h*m + 3*b*d^3*e^2*f*g*m + a*c^2*d*f^3*h*m^2 + b*c^2*d*f^3*g*m^2 + a*d
^3*e^2*f*h*m^2 + b*d^3*e^2*f*g*m^2 - 2*a*c*d^2*e*f^2*h*m^2 - 2*b*c*d^2*e*f^
2*g*m^2 - b*c*d^2*e^2*f*h*m^2 - b*c^2*d*e*f^2*h*m^2 - 8*a*c*d^2*e*f^2*h*m -
 8*b*c*d^2*e*f^2*g*m - b*c*d^2*e^2*f*h*m - 7*b*c^2*d*e*f^2*h*m))/((c*f - d*
e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)*(d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
```

```
[Out] Timed out
```

3.37 $\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$

Optimal. Leaf size=188

$$-\frac{(dg-ch)(c+dx)^{-m-3}(e+fx)^{m+1}}{d(m+3)(de-cf)}+\frac{(c+dx)^{-m-2}(e+fx)^{m+1}(cfh(m+1)+d(2fg-eh(m+3)))}{d(m+2)(m+3)(de-cf)^2}-\frac{f(c+dx)^{-m-1}(}{d}$$

Rubi [A] time = 0.10, antiderivative size = 186, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {79, 45, 37}

$$-\frac{(dg-ch)(c+dx)^{-m-3}(e+fx)^{m+1}}{d(m+3)(de-cf)}+\frac{(c+dx)^{-m-2}(e+fx)^{m+1}(cfh(m+1)-deh(m+3)+2dfg)}{d(m+2)(m+3)(de-cf)^2}-\frac{f(c+dx)^{-m-1}(e+fx)^{m+1}(cfh(m+1)-deh(m+3)+2dfg)}{d(m+1)(m+2)(m+3)(de-cf)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] -(((d*g - c*h)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d*(d*e - c*f)*(3 + m)
)) + ((2*d*f*g + c*f*h*(1 + m) - d*e*h*(3 + m))*(c + d*x)^(-2 - m)*(e + f*x
)^(1 + m))/(d*(d*e - c*f)^2*(2 + m)*(3 + m)) - (f*(2*d*f*g + c*f*h*(1 + m)
- d*e*h*(3 + m))*(c + d*x)^(-1 - m)*(e + f*x)^(1 + m))/(d*(d*e - c*f)^3*(1
+ m)*(2 + m)*(3 + m))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*S
implify[m + n + 2])/((b*c - a*d)*(m + 1)), Int[(a + b*x)^Simplify[m + 1]*(c
 + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && I
LtQ[Simplify[m + n + 2], 0] && NeQ[m, -1] &&  !(LtQ[m, -1] && LtQ[n, -1] &&
 (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && (SumSimpler
Q[m, 1] ||  !SumSimplerQ[n, 1])
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx &= -\frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(3+m)}-\frac{(2dfg+cfh(1+m)-deh(3+m))\int}{d(de-cf)(3+}\\ &= -\frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(3+m)}+\frac{(2dfg+cfh(1+m)-deh(3+m))(}{d(de-cf)^2(2+m)(}\\ &= -\frac{(dg-ch)(c+dx)^{-3-m}(e+fx)^{1+m}}{d(de-cf)(3+m)}+\frac{(2dfg+cfh(1+m)-deh(3+m))(}{d(de-cf)^2(2+m)(}\end{aligned}$$

Mathematica [A] time = 0.13, size = 181, normalized size = 0.96

$$\frac{(dg-ch)(c+dx)^{-m-3}(e+fx)^{m+1}}{d(-m-3)(de-cf)}-\frac{\left(\frac{(c+dx)^{-m-2}(e+fx)^{m+1}}{(-m-2)(de-cf)}+\frac{f(c+dx)^{-m-1}(e+fx)^{m+1}}{(-m-2)(-m-1)(de-cf)^2}\right)(-h(cf(m+1)+de(-m-3))-2dfg)}{d(-m-3)(de-cf)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] ((d*g - c*h)*(c + d*x)^(-3 - m)*(e + f*x)^(1 + m))/(d*(d*e - c*f)*(-3 - m))
 - ((-2*d*f*g - h*(d*e*(-3 - m) + c*f*(1 + m)))*(((c + d*x)^(-2 - m)*(e + f
*x)^(1 + m))/((d*e - c*f)*(-2 - m)) + (f*(c + d*x)^(-1 - m)*(e + f*x)^(1 +
m))/((d*e - c*f)^2*(-2 - m)*(-1 - m))))/(d*(d*e - c*f)*(-3 - m))
```

IntegrateAlgebraic [F] time = 0.08, size = 0, normalized size = 0.00

$$\int (c+dx)^{-4-m}(e+fx)^m(g+hx)\,dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x),x]
```

```
[Out] Defer[IntegrateAlgebraic][(c + d*x)^(-4 - m)*(e + f*x)^m*(g + h*x), x]
```

fricas [B] time = 0.84, size = 905, normalized size = 4.81

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="fricas")
```

```
[Out] -((2*d^3*f^3*g - (d^3*e*f^2 - c*d^2*f^3)*h*m - (3*d^3*e*f^2 - c*d^2*f^3)*h)
*x^4 + (c*d^2*e^3 - 2*c^2*d*e^2*f + c^3*e*f^2)*g*m^2 + (8*c*d^2*f^3*g + (d^
3*e^2*f - 2*c*d^2*e*f^2 + c^2*d*f^3)*h*m^2 - 4*(3*c*d^2*e*f^2 - c^2*d*f^3)*
h - (2*(d^3*e*f^2 - c*d^2*f^3)*g - (3*d^3*e^2*f - 8*c*d^2*e*f^2 + 5*c^2*d*f
^3)*h)*m)*x^3 + (12*c^2*d*f^3*g + ((d^3*e^2*f - 2*c*d^2*e*f^2 + c^2*d*f^3)*
g + (d^3*e^3 - c*d^2*e^2*f - c^2*d*e*f^2 + c^3*f^3)*h)*m^2 + 3*(d^3*e^3 - 3
*c*d^2*e^2*f - 3*c^2*d*e*f^2 + c^3*f^3)*h + ((d^3*e^2*f - 8*c*d^2*e*f^2 + 7
*c^2*d*f^3)*g + 4*(d^3*e^3 - c*d^2*e^2*f - c^2*d*e*f^2 + c^3*f^3)*h)*m)*x^2
 + 2*(c*d^2*e^3 - 3*c^2*d*e^2*f + 3*c^3*e*f^2)*g + (c^2*d*e^3 - 3*c^3*e^2*f
)*h + ((3*c*d^2*e^3 - 8*c^2*d*e^2*f + 5*c^3*e*f^2)*g + (c^2*d*e^3 - c^3*e^2
*f)*h)*m + (((d^3*e^3 - c*d^2*e^2*f - c^2*d*e*f^2 + c^3*f^3)*g + (c*d^2*e^3
 - 2*c^2*d*e^2*f + c^3*e*f^2)*h)*m^2 + 2*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d
*e*f^2 + 3*c^3*f^3)*g + 4*(c*d^2*e^3 - 3*c^2*d*e^2*f)*h + ((3*d^3*e^3 - 7*c
*d^2*e^2*f - c^2*d*e*f^2 + 5*c^3*f^3)*g + (5*c*d^2*e^3 - 8*c^2*d*e^2*f + 3*
c^3*e*f^2)*h)*m)*x)*(d*x + c)^(-m - 4)*(f*x + e)^m/(6*d^3*e^3 - 18*c*d^2*e^
2*f + 18*c^2*d*e*f^2 - 6*c^3*f^3 + (d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2
 - c^3*f^3)*m^3 + 6*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*m^2
 + 11*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*m)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (hx+g)(dx+c)^{-m-4}(fx+e)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="giac")
```

```
[Out] integrate((h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

maple [B] time = 0.01, size = 509, normalized size = 2.71

$$-\frac{\left(-c^{2} f^{2} h m^{2} x+2 c d e f h m^{2} x-c d f^{2} h m x^{2}-d^{2} e^{2} h m^{2} x+d^{2} e f h m x^{2}-c^{2} f^{2} g m^{2}-4 c^{2} f^{2} h m x+2 c d e f g m^{2}+8 c d e f h m x-2 c d f^{2} g m x-c d f^{2} h x^{2}-d^{2} e^{2} g m^{2}-4 d^{2} e^{2} h m x+2 d^{2} e f g m x+3 d^{2} e f h x^{2}-2 d^{2} f^{2} g x^{2}+c^{2} e f h m-5 c^{2} f^{2} g m-3 c^{2} f^{2} h x-c d e^{2} h m+8 c d e f g m+10 c d e f h x-6 c d f^{2} g x-3 d^{2} e^{2} g m-3 d^{2} e^{2} h x+2 d^{2} e f g x+3 c^{2} e f h-6 c^{2} f^{2} g-c d e^{2} h+6 c d e f g-2 d^{2} e^{2} g\right)(d x+c)^{-m-3}(f x+e)^{m+1}}{c^{3} f^{3} m^{3}-3 c^{2} d e f^{2} m^{3}+3 c d^{2} e^{2} f m^{3}-d^{3} e^{3} m^{3}+6 c^{3} f^{3} m^{2}-18 c^{2} d e f^{2} m^{2}+18 c d^{2} e^{2} f m^{2}-6 d^{3} e^{3} m^{2}+11 c^{3} f^{3} m-33 c^{2} d e f^{2} m+33 c d^{2} e^{2} f m-11 d^{3} e^{3} m+6 c^{3} f^{3}-18 c^{2} d e f^{2}+18 c d^{2} e^{2} f-6 d^{3} e^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(-m-4)*(f*x+e)^m*(h*x+g),x)
```

```
[Out] -(d*x+c)^(-m-3)*(f*x+e)^(m+1)*(-c^2*f^2*h*m^2*x+2*c*d*e*f*h*m^2*x-c*d*f^2*h
*m*x^2-d^2*e^2*h*m^2*x+d^2*e*f*h*m*x^2-c^2*f^2*g*m^2-4*c^2*f^2*h*m*x+2*c*d*
e*f*g*m^2+8*c*d*e*f*h*m*x-2*c*d*f^2*g*m*x-c*d*f^2*h*x^2-d^2*e^2*g*m^2-4*d^2
*e^2*h*m*x+2*d^2*e*f*g*m*x+3*d^2*e*f*h*x^2-2*d^2*f^2*g*x^2+c^2*e*f*h*m-5*c^
2*f^2*g*m-3*c^2*f^2*h*x-c*d*e^2*h*m+8*c*d*e*f*g*m+10*c*d*e*f*h*x-6*c*d*f^2*
g*x-3*d^2*e^2*g*m-3*d^2*e^2*h*x+2*d^2*e*f*g*x+3*c^2*e*f*h-6*c^2*f^2*g-c*d*e
^2*h+6*c*d*e*f*g-2*d^2*e^2*g)/(c^3*f^3*m^3-3*c^2*d*e*f^2*m^3+3*c*d^2*e^2*f*
m^3-d^3*e^3*m^3+6*c^3*f^3*m^2-18*c^2*d*e*f^2*m^2+18*c*d^2*e^2*f*m^2-6*d^3*e
^3*m^2+11*c^3*f^3*m-33*c^2*d*e*f^2*m+33*c*d^2*e^2*f*m-11*d^3*e^3*m+6*c^3*f^
3-18*c^2*d*e*f^2+18*c*d^2*e^2*f-6*d^3*e^3)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (hx+g)(dx+c)^{-m-4}(fx+e)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(-4-m)*(f*x+e)^m*(h*x+g),x, algorithm="maxima")
```

```
[Out] integrate((h*x + g)*(d*x + c)^(-m - 4)*(f*x + e)^m, x)
```

mupad [B] time = 3.41, size = 869, normalized size = 4.62

$$\frac{x^{2}(e+fx)^{m}\left(hc^{3}f^{3}m^{2}+4hc^{3}f^{3}m+3hc^{3}f^{3}-hc^{2}def^{2}m^{2}-4hc^{2}def^{2}m-9hc^{2}def^{2}+gc^{2}df^{3}m^{2}+7gc^{2}df^{3}m+12gc^{2}df^{3}-hcd^{2}e^{2}fm^{2}-4hcd^{2}e^{2}fm-9hcd^{2}e^{2}f-2gcd^{2}ef^{2}m^{2}-8gcd^{2}ef^{2}m+hd^{3}e^{3}m^{2}+4hd^{3}e^{3}m+3hd^{3}e^{3}+gd^{3}e^{2}fm^{2}+gd^{3}e^{2}fm\right)}{(cf-de)^{3}(c+dx)^{m+4}(m^{3}+6m^{2}+11m+6)}+\text{[illegible]}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e + f*x)^m*(g + h*x))/(c + d*x)^(m + 4),x)
```

```
[Out] (x^2*(e + f*x)^m*(3*c^3*f^3*h + 3*d^3*e^3*h + c^3*f^3*h*m^2 + d^3*e^3*h*m^2
 + 12*c^2*d*f^3*g + 4*c^3*f^3*h*m + 4*d^3*e^3*h*m - 9*c*d^2*e^2*f*h - 9*c^2
*d*e*f^2*h + 7*c^2*d*f^3*g*m + d^3*e^2*f*g*m + c^2*d*f^3*g*m^2 + d^3*e^2*f*
g*m^2 - 8*c*d^2*e*f^2*g*m - 4*c*d^2*e^2*f*h*m - 4*c^2*d*e*f^2*h*m - 2*c*d^2
*e*f^2*g*m^2 - c*d^2*e^2*f*h*m^2 - c^2*d*e*f^2*h*m^2))/((c*f - d*e)^3*(c +
d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (x*(e + f*x)^m*(6*c^3*f^3*g + 2*d^
3*e^3*g + c^3*f^3*g*m^2 + d^3*e^3*g*m^2 + 4*c*d^2*e^3*h + 5*c^3*f^3*g*m + 3
*d^3*e^3*g*m - 6*c*d^2*e^2*f*g + 6*c^2*d*e*f^2*g - 12*c^2*d*e^2*f*h + 5*c*d
^2*e^3*h*m + 3*c^3*e*f^2*h*m + c*d^2*e^3*h*m^2 + c^3*e*f^2*h*m^2 - 7*c*d^2*
e^2*f*g*m - c^2*d*e*f^2*g*m - 8*c^2*d*e^2*f*h*m - c*d^2*e^2*f*g*m^2 - c^2*d
*e*f^2*g*m^2 - 2*c^2*d*e^2*f*h*m^2))/((c*f - d*e)^3*(c + d*x)^(m + 4)*(11*m
 + 6*m^2 + m^3 + 6)) + (c*e*(e + f*x)^m*(6*c^2*f^2*g + 2*d^2*e^2*g + c^2*f^
2*g*m^2 + d^2*e^2*g*m^2 + c*d*e^2*h - 3*c^2*e*f*h + 5*c^2*f^2*g*m + 3*d^2*e
^2*g*m - 6*c*d*e*f*g + c*d*e^2*h*m - c^2*e*f*h*m - 2*c*d*e*f*g*m^2 - 8*c*d*
e*f*g*m))/((c*f - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (d^2
*f^2*x^4*(e + f*x)^m*(c*f*h - 3*d*e*h + 2*d*f*g + c*f*h*m - d*e*h*m))/((c*f
 - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6)) + (d*f*x^3*(e + f*x)^
m*(4*c*f + c*f*m - d*e*m)*(c*f*h - 3*d*e*h + 2*d*f*g + c*f*h*m - d*e*h*m))/
((c*f - d*e)^3*(c + d*x)^(m + 4)*(11*m + 6*m^2 + m^3 + 6))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(-4-m)*(f*x+e)**m*(h*x+g),x)
```

```
[Out] Timed out
```

3.38 $\int \frac{x\left(a+bx+cx^2\right)}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=79

$$-\frac{\sqrt{1-d^2x^2}\,\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4}+\frac{b\sin^{-1}(dx)}{2d^3}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

Rubi [A] time = 0.14, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {1609, 1809, 780, 216}

$$-\frac{\sqrt{1-d^2x^2}\,\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4}+\frac{b\sin^{-1}(dx)}{2d^3}-\frac{cx^2\sqrt{1-d^2x^2}}{3d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(c*x^2*Sqrt[1 - d^2*x^2])/(3*d^2) - ((2*(2*c + 3*a*d^2) + 3*b*d^2*x)*Sqrt[
1 - d^2*x^2])/(6*d^4) + (b*ArcSin[d*x])/(2*d^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{x(a+bx+cx^2)}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{x(a+bx+cx^2)}{\sqrt{1-d^2x^2}}\,dx$$
$$= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{\int \frac{x(-2c-3ad^2-3bd^2x)}{\sqrt{1-d^2x^2}}\,dx}{3d^2}$$
$$= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{(2(2c+3ad^2)+3bd^2x)\sqrt{1-d^2x^2}}{6d^4} + \frac{b\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{cx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{(2(2c+3ad^2)+3bd^2x)\sqrt{1-d^2x^2}}{6d^4} + \frac{b\sin^{-1}(dx)}{2d^3}$$

Mathematica [A] time = 0.07, size = 57, normalized size = 0.72

$$\frac{3bd\sin^{-1}(dx) - \sqrt{1-d^2x^2}\left(3d^2(2a+bx)+2c\left(d^2x^2+2\right)\right)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(Sqrt[1 - d^2*x^2]*(3*d^2*(2*a + b*x) + 2*c*(2 + d^2*x^2))) + 3*b*d*ArcSi
n[d*x])/(6*d^4)
```

IntegrateAlgebraic [B] time = 0.17, size = 179, normalized size = 2.27

$$-\frac{\sqrt{1-dx}\left(\frac{12ad^2(1-dx)}{dx+1}+\frac{6ad^2(1-dx)^2}{(dx+1)^2}+6ad^2-\frac{3bd(1-dx)^2}{(dx+1)^2}+3bd+\frac{4c(1-dx)}{dx+1}+\frac{6c(1-dx)^2}{(dx+1)^2}+6c\right)}{3d^4\sqrt{dx+1}\left(\frac{1-dx}{dx+1}+1\right)^3} - \frac{b\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x + c*x^2))/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/3*(Sqrt[1 - d*x]*(6*c + 3*b*d + 6*a*d^2 + (6*c*(1 - d*x)^2)/(1 + d*x)^2
- (3*b*d*(1 - d*x)^2)/(1 + d*x)^2 + (6*a*d^2*(1 - d*x)^2)/(1 + d*x)^2 + (4*
c*(1 - d*x))/(1 + d*x) + (12*a*d^2*(1 - d*x))/(1 + d*x)))/(d^4*Sqrt[1 + d*x
]*(1 + (1 - d*x)/(1 + d*x))^3) - (b*ArcTan[Sqrt[1 - d*x]/Sqrt[1 + d*x]])/d^
3
```

fricas [A] time = 0.88, size = 78, normalized size = 0.99

$$-\frac{6\,bd\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)+\left(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c\right)\sqrt{dx+1}\sqrt{-dx+1}}{6\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] -1/6*(6*b*d*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)) + (2*c*d^2*x^2
 + 3*b*d^2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(-d*x + 1))/d^4
```

giac [A] time = 1.34, size = 101, normalized size = 1.28

$$-\frac{\sqrt{dx+1}\sqrt{-dx+1}\left((dx+1)\left(\frac{2\,(dx+1)c}{d^3}+\frac{3\,bd^{10}-4\,cd^9}{d^{12}}\right)+\frac{3\left(2\,ad^{11}-bd^{10}+2\,cd^9\right)}{d^{12}}\right)-\frac{6\,b\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)}{d^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*(sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*(2*(d*x + 1)*c/d^3 + (3*b*d^1
0 - 4*c*d^9)/d^12) + 3*(2*a*d^11 - b*d^10 + 2*c*d^9)/d^12) - 6*b*arcsin(1/2
*sqrt(2)*sqrt(d*x + 1))/d^2)/d
```

maple [C] time = 0.04, size = 139, normalized size = 1.76

$$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(2\sqrt{-d^2x^2+1}\,c\,d^2x^2\mathrm{csgn}(d)+3\sqrt{-d^2x^2+1}\,b\,d^2x\,\mathrm{csgn}(d)+6\sqrt{-d^2x^2+1}\,a\,d^2\mathrm{csgn}(d)-3bd\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)+4\sqrt{-d^2x^2+1}\,c\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)}{6\sqrt{-d^2x^2+1}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/6*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(2*csgn(d)*x^2*c*d^2*(-d^2*x^2+1)^(1/2)+3
*(-d^2*x^2+1)^(1/2)*csgn(d)*x*b*d^2+6*(-d^2*x^2+1)^(1/2)*csgn(d)*a*d^2+4*(-
d^2*x^2+1)^(1/2)*csgn(d)*c-3*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*b*d)*cs
gn(d)/d^4/(-d^2*x^2+1)^(1/2)
```

maxima [A] time = 0.97, size = 87, normalized size = 1.10

$$-\frac{\sqrt{-d^2x^2+1}\,cx^2}{3\,d^2}-\frac{\sqrt{-d^2x^2+1}\,bx}{2\,d^2}-\frac{\sqrt{-d^2x^2+1}\,a}{d^2}+\frac{b\arcsin\left(dx\right)}{2\,d^3}-\frac{2\,\sqrt{-d^2x^2+1}\,c}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -1/3*sqrt(-d^2*x^2 + 1)*c*x^2/d^2 - 1/2*sqrt(-d^2*x^2 + 1)*b*x/d^2 - sqrt(-
d^2*x^2 + 1)*a/d^2 + 1/2*b*arcsin(d*x)/d^3 - 2/3*sqrt(-d^2*x^2 + 1)*c/d^4
```

mupad [B] time = 7.44, size = 244, normalized size = 3.09

$$-\frac{\sqrt{1-d\,x}\,\left(\frac{a}{d^2}+\frac{a\,x}{d}\right)}{\sqrt{d\,x+1}}-\frac{2\,b\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{\frac{14\,b\,{\left(\sqrt{1-d\,x}-1\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}-\frac{14\,b\,{\left(\sqrt{1-d\,x}-1\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{2\,b\,{\left(\sqrt{1-d\,x}-1\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}-\frac{2\,b\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4}-\frac{\sqrt{1-d\,x}\,\left(\frac{2\,c}{3\,d^4}+\frac{c\,x^3}{3\,d}+\frac{c\,x^2}{3\,d^2}+\frac{2\,c\,x}{3\,d^3}\right)}{\sqrt{d\,x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x + c*x^2))/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((1 - d*x)^(1/2)*(a/d^2 + (a*x)/d))/(d*x + 1)^(1/2) - (2*b*atan(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*b*((1 - d*x)^(1/2) - 1)^3)/
((d*x + 1)^(1/2) - 1)^3 - (14*b*((1 - d*x)^(1/2) - 1)^5)/((d*x + 1)^(1/2) -
 1)^5 + (2*b*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7 - (2*b*((1 -
d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2) - 1)^2/((d*x
 + 1)^(1/2) - 1)^2 + 1)^4) - ((1 - d*x)^(1/2)*((2*c)/(3*d^4) + (c*x^3)/(3*d
) + (c*x^2)/(3*d^2) + (2*c*x)/(3*d^3)))/(d*x + 1)^(1/2)
```

sympy [C] time = 82.40, size = 313, normalized size = 3.96

$$-\frac{iaG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{aG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}-\frac{ibG_{6,6}^{6,2}\left(\begin{matrix} -\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1 \\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}+\frac{bG_{6,6}^{2,6}\left(\begin{matrix} -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}-\frac{icG_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}-\frac{cG_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix}\middle|\frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -I*a*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0),
()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - a*meijerg(((-1, -3/4, -1/2, -1/4,
```

```
0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2*I*pi)/(d**2*x
**2))/(4*pi**(3/2)*d**2) - I*b*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)),
((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**3) + b*m
eijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1,
 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*c*meijerg(((-5
/4, -3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(d*
*2*x**2))/(4*pi**(3/2)*d**4) - c*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()
), ((-7/4, -5/4), (-2, -3/2, -3/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*d**4)
```

3.39 $\int \frac{a+bx+cx^2}{\sqrt{1-dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=63

$$\frac{(2ad^2+c)\sin^{-1}(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

Rubi [A] time = 0.06, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {899, 1815, 641, 216}

$$\frac{(2ad^2+c)\sin^{-1}(dx)}{2d^3}-\frac{b\sqrt{1-d^2x^2}}{d^2}-\frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*Sqrt[1 - d^2*x^2])/d^2) - (c*x*Sqrt[1 - d^2*x^2])/(2*d^2) + ((c + 2*a*
d^2)*ArcSin[d*x])/(2*d^3)
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 899

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[(d*f + e*g*x^2)^m*(a + b*x + c*x^2)^
p, x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0] && EqQ[e
*f + d*g, 0] && (IntegerQ[m] || (GtQ[d, 0] && GtQ[f, 0]))
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && !LeQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}}\,dx = \int \frac{a+bx+cx^2}{\sqrt{1-d^2x^2}}\,dx$$
$$= -\frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\int \frac{-c-2ad^2-2bd^2x}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} - \frac{\left(-c-2ad^2\right)\int \frac{1}{\sqrt{1-d^2x^2}}\,dx}{2d^2}$$
$$= -\frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2} + \frac{\left(c+2ad^2\right)\sin^{-1}(dx)}{2d^3}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 0.71

$$\frac{\left(2ad^2+c\right)\sin^{-1}(dx) - d\sqrt{1-d^2x^2}\,(2b+cx)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-(d*(2*b + c*x)*Sqrt[1 - d^2*x^2]) + (c + 2*a*d^2)*ArcSin[d*x])/(2*d^3)
```

IntegrateAlgebraic [A] time = 0.13, size = 117, normalized size = 1.86

$$\frac{\left(-2ad^2-c\right)\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)}{d^3} - \frac{\sqrt{1-dx}\left(\frac{2bd(1-dx)}{dx+1}+2bd-\frac{c(1-dx)}{dx+1}+c\right)}{d^3\sqrt{dx+1}\left(\frac{1-dx}{dx+1}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((Sqrt[1 - d*x]*(c + 2*b*d - (c*(1 - d*x))/(1 + d*x) + (2*b*d*(1 - d*x))/(
1 + d*x)))/(d^3*Sqrt[1 + d*x]*(1 + (1 - d*x)/(1 + d*x))^2)) + ((-c - 2*a*d^
2)*ArcTan[Sqrt[1 - d*x]/Sqrt[1 + d*x]])/d^3
```

fricas [A] time = 0.94, size = 67, normalized size = 1.06

$$-\frac{(cdx+2\,bd)\sqrt{dx+1}\,\sqrt{-dx+1}\,+2\left(2\,ad^2+c\right)\arctan\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{dx}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 2*(2*a*d^2 + c)*arctan
((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/d^3
```

giac [A] time = 1.31, size = 76, normalized size = 1.21

$$-\frac{\sqrt{dx+1}\,\sqrt{-dx+1}\left(\frac{(dx+1)c}{d^2}+\frac{2\,bd^5-cd^4}{d^6}\right)-\frac{2\left(2\,ad^2+c\right)\arcsin\left(\frac{1}{2}\,\sqrt{2}\,\sqrt{dx+1}\right)}{d^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(d*x + 1)*sqrt(-d*x + 1)*((d*x + 1)*c/d^2 + (2*b*d^5 - c*d^4)/d^6
) - 2*(2*a*d^2 + c)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1))/d^2)/d
```

maple [C] time = 0.02, size = 117, normalized size = 1.86

$$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(-2a\,d^{2}\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^{2}x^{2}+1}}\right)+\sqrt{-d^{2}x^{2}+1}\,cdx\,\mathrm{csgn}(d)+2\sqrt{-d^{2}x^{2}+1}\,bd\,\mathrm{csgn}(d)-c\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^{2}x^{2}+1}}\right)\right)\mathrm{csgn}(d)}{2\sqrt{-d^{2}x^{2}+1}\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)/d^3*(csgn(d)*d*(-d^2*x^2+1)^(1/2)*x*c-2*a
rctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*a*d^2+2*csgn(d)*d*(-d^2*x^2+1)^(1/2
)*b-arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*c)/(-d^2*x^2+1)^(1/2)*csgn(d)
```

maxima [A] time = 0.97, size = 57, normalized size = 0.90

$$\frac{a\arcsin\left(dx\right)}{d}-\frac{\sqrt{-d^{2}x^{2}+1}\,cx}{2\,d^{2}}-\frac{\sqrt{-d^{2}x^{2}+1}\,b}{d^{2}}+\frac{c\arcsin\left(dx\right)}{2\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*arcsin(d*x)/d - 1/2*sqrt(-d^2*x^2 + 1)*c*x/d^2 - sqrt(-d^2*x^2 + 1)*b/d^2
 + 1/2*c*arcsin(d*x)/d^3
```

mupad [B] time = 6.99, size = 232, normalized size = 3.68

$$-\frac{\sqrt{1-d\,x}\,\left(\frac{b}{d^2}+\frac{b\,x}{d}\right)}{\sqrt{d\,x+1}}-\frac{4\,a\,\mathrm{atan}\left(\frac{d\,\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\,\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{2\,c\,\mathrm{atan}\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{\frac{14\,c\,{\left(\sqrt{1-d\,x}-1\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}-\frac{14\,c\,{\left(\sqrt{1-d\,x}-1\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{2\,c\,{\left(\sqrt{1-d\,x}-1\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}-\frac{2\,c\,\left(\sqrt{1-d\,x}-1\right)}{\sqrt{d\,x+1}-1}}{d^3\,{\left(\frac{{\left(\sqrt{1-d\,x}-1\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] - ((1 - d*x)^(1/2)*(b/d^2 + (b*x)/d))/(d*x + 1)^(1/2) - (4*a*atan((d*((1 -
d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (2*c*a
tan(((1 - d*x)^(1/2) - 1)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((1 - d*x)^(
1/2) - 1)^3)/((d*x + 1)^(1/2) - 1)^3 - (14*c*((1 - d*x)^(1/2) - 1)^5)/((d*x
 + 1)^(1/2) - 1)^5 + (2*c*((1 - d*x)^(1/2) - 1)^7)/((d*x + 1)^(1/2) - 1)^7
- (2*c*((1 - d*x)^(1/2) - 1))/((d*x + 1)^(1/2) - 1))/(d^3*(((1 - d*x)^(1/2)
 - 1)^2/((d*x + 1)^(1/2) - 1)^2 + 1)^4)
```

sympy [C] time = 49.79, size = 282, normalized size = 4.48

$$-\frac{iaG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}d}+\frac{aG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}d^{2}}-\frac{bG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}d^{2}}-\frac{icG_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1\\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{1}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}d^{3}}+\frac{cG_{6,6}^{2,6}\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -I*a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d) + a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1),
 ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi
**(3/2)*d) - I*b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/
4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - b*meijerg(((-1, -3/4,
-1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2*I
*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2) - I*c*meijerg(((-3/4, -1/4), (-1/2, -1
/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)
*d**3) + c*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-
3/2, -1, -1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

3.40 $\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=48

$$-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{b\sin^{-1}(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

Rubi [A] time = 0.18, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {1609, 1809, 844, 216, 266, 63, 208}

$$-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{b\sin^{-1}(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + (b*ArcSin[d*x])/d - a*ArcTanh[Sqrt[1 - d^2*x
^2]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx+cx^2}{x\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x\sqrt{1-d^2x^2}}\,dx\\ &= -\frac{c\sqrt{1-d^2x^2}}{d^2}-\frac{\int \frac{-ad^2-bd^2x}{x\sqrt{1-d^2x^2}}\,dx}{d^2}\\ &= -\frac{c\sqrt{1-d^2x^2}}{d^2}+a\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx+b\int \frac{1}{\sqrt{1-d^2x^2}}\,dx\\ &= -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\sin^{-1}(dx)}{d}+\frac{1}{2}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx,x,x^2\right)\\ &= -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\sin^{-1}(dx)}{d}-\frac{a\,\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx,x,\sqrt{1-d^2x^2}\right)}{d^2}\\ &= -\frac{c\sqrt{1-d^2x^2}}{d^2}+\frac{b\sin^{-1}(dx)}{d}-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)\end{aligned}$$

Mathematica [A] time = 0.06, size = 48, normalized size = 1.00

$$-a\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{b\sin^{-1}(dx)}{d}-\frac{c\sqrt{1-d^2x^2}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] -((c*Sqrt[1 - d^2*x^2])/d^2) + (b*ArcSin[d*x])/d - a*ArcTanh[Sqrt[1 - d^2*x
^2]]
```

IntegrateAlgebraic [A] time = 0.13, size = 95, normalized size = 1.98

$$-2a\tanh^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)-\frac{2b\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)}{d}-\frac{2c\sqrt{1-dx}}{d^2\sqrt{dx+1}\left(\frac{1-dx}{dx+1}+1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(x*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] (-2*c*Sqrt[1 - d*x])/(d^2*Sqrt[1 + d*x]*(1 + (1 - d*x)/(1 + d*x))) - (2*b*A
rcTan[Sqrt[1 - d*x]/Sqrt[1 + d*x]])/d - 2*a*ArcTanh[Sqrt[1 - d*x]/Sqrt[1 +
d*x]]
```

fricas [A] time = 0.94, size = 81, normalized size = 1.69

$$\frac{ad^2\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right)-2\,bd\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)-\sqrt{dx+1}\sqrt{-dx+1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas
")
```

```
[Out] (a*d^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - 2*b*d*arctan((sqrt(d*x +
 1)*sqrt(-d*x + 1) - 1)/(d*x)) - sqrt(d*x + 1)*sqrt(-d*x + 1)*c)/d^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values [70,22
]Warning, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values
[42,56]-a*ln(abs(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-
d*x+1)+2*sqrt(2))/sqrt(d*x+1)))+a*ln(abs(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*s
qrt(2))-2-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1)))-2*b*(-1/2*pi-atan(s
qrt(d*x+1)*((-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))^2-1)/(-2*sqrt(-d
*x+1)+2*sqrt(2))))/d-2*c*d^2/2/d^4*sqrt(d*x+1)*sqrt(-d*x+1)
```

maple [C] time = 0.03, size = 96, normalized size = 2.00

$$\frac{\left(-a\,d^2 \operatorname{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)\operatorname{csgn}(d)+bd\arctan\left(\frac{dx\operatorname{csgn}(d)}{\sqrt{-(dx+1)(dx-1)}}\right)-\sqrt{-d^2x^2+1}\,c\operatorname{csgn}(d)\right)\sqrt{-dx+1}\,\sqrt{dx+1}\,\operatorname{csgn}(d)}{\sqrt{-d^2x^2+1}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-csgn(d)*arctanh(1/(-d^2*x^2+1)^(1/2))*a*d^2-(-d^2*x^2+1)^(1/2)*c*csgn(d)+
arctan(csgn(d)*d*x/(-(d*x+1)*(d*x-1))^(1/2))*b*d)*(-d*x+1)^(1/2)*(d*x+1)^(1
/2)/d^2*csgn(d)/(-d^2*x^2+1)^(1/2)
```

maxima [A] time = 0.97, size = 57, normalized size = 1.19

$$-a\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{b\arcsin(dx)}{d}-\frac{\sqrt{-d^2x^2+1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima
")
```

```
[Out] -a*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + b*arcsin(d*x)/d - sqrt(-d^
2*x^2 + 1)*c/d^2
```

mupad [B] time = 3.92, size = 122, normalized size = 2.54

$$a\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right)-\frac{\sqrt{1-d\,x}\,\left(\frac{c}{d^2}+\frac{c\,x}{d}\right)}{\sqrt{d\,x+1}}-\frac{4\,b\,\mathrm{atan}\left(\frac{d\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{d^2}}\right)}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] a*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - ((1 - d*x)^(1/2)*(c/d^2 + (c*x)/d))/
(d*x + 1)^(1/2) - (4*b*atan((d*((1 - d*x)^(1/2) - 1))/(((d*x + 1)^(1/2) - 1
)*(d^2)^(1/2))))/(d^2)^(1/2)
```

sympy **[C]** time = 55.20, size = 245, normalized size = 5.10

$$\frac{iaG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2} \\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0 \end{matrix}\middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{aG_{6,6}^{2,6}\left(\begin{matrix} 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0 \end{matrix}\middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{ibG_{6,6}^{6,2}\left(\begin{matrix} \frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1 \\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} + \frac{bG_{6,6}^{2,6}\left(\begin{matrix} -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0 \end{matrix}\middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d} - \frac{icG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2} - \frac{cG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix}\middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)),
1/(d**2*x**2))/(4*pi**(3/2)) - a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((
1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3/2))
 - I*b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), (
)), 1/(d**2*x**2))/(4*pi**(3/2)*d) + b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)*d) - I*c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0,
1/4, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) - c*meijerg(((-1, -3/4
, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(-2
*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

3.41 $\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=48

$$-\frac{a\sqrt{1-d^2x^2}}{x}-b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{c\sin^{-1}(dx)}{d}$$

Rubi [A] time = 0.18, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {1609, 1807, 844, 216, 266, 63, 208}

$$-\frac{a\sqrt{1-d^2x^2}}{x}-b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)+\frac{c\sin^{-1}(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + (c*ArcSin[d*x])/d - b*ArcTanh[Sqrt[1 - d^2*x^2
]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx+cx^2}{x^2\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x^2\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} - \int \frac{-b-cx}{x\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + b\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx + c\int \frac{1}{\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} + \frac{1}{2}b\,\mathrm{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx, x, x^2\right)\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} - \frac{b\,\mathrm{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx, x, \sqrt{1-d^2x^2}\right)}{d^2}\\
&= -\frac{a\sqrt{1-d^2x^2}}{x} + \frac{c\sin^{-1}(dx)}{d} - b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.06, size = 48, normalized size = 1.00

$$-\frac{a\sqrt{1-d^2x^2}}{x} - b\tanh^{-1}\left(\sqrt{1-d^2x^2}\right) + \frac{c\sin^{-1}(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] -((a*Sqrt[1 - d^2*x^2])/x) + (c*ArcSin[d*x])/d - b*ArcTanh[Sqrt[1 - d^2*x^2
]]
```

IntegrateAlgebraic [A] time = 0.17, size = 93, normalized size = 1.94

$$\frac{2ad\sqrt{1-dx}}{\sqrt{dx+1}\left(\frac{1-dx}{dx+1}-1\right)} - 2b\tanh^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right) - \frac{2c\tan^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(x^2*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
[Out] (2*a*d*Sqrt[1 - d*x])/(Sqrt[1 + d*x]*(-1 + (1 - d*x)/(1 + d*x))) - (2*c*Arc
Tan[Sqrt[1 - d*x]/Sqrt[1 + d*x]])/d - 2*b*ArcTanh[Sqrt[1 - d*x]/Sqrt[1 + d*
x]]
```

fricas [A] time = 0.87, size = 84, normalized size = 1.75

$$\frac{bdx\log\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{x}\right) - \sqrt{dx+1}\sqrt{-dx+1}\,ad - 2\,cx\arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] (b*d*x*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - sqrt(d*x + 1)*sqrt(-d*x
+ 1)*a*d - 2*c*x*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x)))/(d*x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values [70,22
]Warning, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values
[42,56]1/d*(-2*c*(-1/2*pi-atan(sqrt(d*x+1)*((-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2
))/sqrt(d*x+1))^2-1)/(-2*sqrt(-d*x+1)+2*sqrt(2))))-b*d*ln(abs(2*sqrt(d*x+1)
/(-2*sqrt(-d*x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))
)+b*d*ln(abs(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2))-2-1/2*(-2*sqrt(-d*x+
1)+2*sqrt(2))/sqrt(d*x+1)))-4*a*d^2*(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(
2))-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))/(-(2*sqrt(d*x+1)/(-2*sqrt(
-d*x+1)+2*sqrt(2))-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))^2+4))
```

maple [C] time = 0.02, size = 97, normalized size = 2.02

$$\frac{\left(-bdx\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)\mathrm{csgn}(d)-\sqrt{-d^2x^2+1}\,ad\,\mathrm{csgn}(d)+cx\arctan\left(\frac{dx\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}}\right)\right)\sqrt{-dx+1}\,\sqrt{dx+1}\,\mathrm{csgn}(d)}{\sqrt{-d^2x^2+1}\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-csgn(d)*d*arctanh(1/(-d^2*x^2+1)^(1/2))*x*b-csgn(d)*d*(-d^2*x^2+1)^(1/2)*
a+arctan(1/(-d^2*x^2+1)^(1/2)*d*x*csgn(d))*x*c)*(-d*x+1)^(1/2)*(d*x+1)^(1/2
)*csgn(d)/(-d^2*x^2+1)^(1/2)/x/d
```

maxima [A] time = 0.97, size = 57, normalized size = 1.19

$$-b\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{c\arcsin\left(dx\right)}{d}-\frac{\sqrt{-d^2x^2+1}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -b*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) + c*arcsin(d*x)/d - sqrt(-d^
2*x^2 + 1)*a/x
```

mupad [B] time = 3.74, size = 114, normalized size = 2.38

$$b\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right)-\frac{4\,c\,\mathrm{atan}\left(\frac{d\left(\sqrt{1-d\,x}-1\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{a\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^2*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] b*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - (4*c*atan((d*((1 - d*x)^(1/2) - 1))/
(((d*x + 1)^(1/2) - 1)*(d^2)^(1/2))))/(d^2)^(1/2) - (a*(1 - d*x)^(1/2)*(d*x
 + 1)^(1/2))/x
```

sympy [C] time = 49.85, size = 221, normalized size = 4.60

$$\frac{iadG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}+\frac{adG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}+\frac{ibG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}-\frac{bG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}}-\frac{icG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}d}+\frac{cG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{-2i\pi}}{d^{2}x^{2}}\right)}{4\pi^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,))
, 1/(d**2*x**2))/(4*pi**(3/2)) + a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), (
)), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi**(3
/2)) + I*b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2),
(0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*pi*
*(3/2)) - I*c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1
, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) + c*meijerg(((-1/2, -1/4, 0, 1/4,
 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(-2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)*d)
```

3.42 $\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=71

$$-\frac{1}{2}\left(ad^2+2c\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}$$

Rubi [A] time = 0.19, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1609, 1807, 807, 266, 63, 208}

$$-\frac{1}{2}\left(ad^2+2c\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)-\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(a*Sqrt[1 - d^2*x^2])/(2*x^2) - (b*Sqrt[1 - d^2*x^2])/x - ((2*c + a*d^2)*A
rcTanh[Sqrt[1 - d^2*x^2]])/2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1609

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; F
reeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*d, 0] &
& EqQ[m, n] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
```

```
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx+cx^2}{x^3\sqrt{1-dx}\sqrt{1+dx}}\,dx &= \int \frac{a+bx+cx^2}{x^3\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{1}{2}\int \frac{-2b-\left(2c+ad^2\right)x}{x^2\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(-2c-ad^2\right)\int \frac{1}{x\sqrt{1-d^2x^2}}\,dx\\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{4}\left(-2c-ad^2\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-d^2x}}\,dx,x,x^2\right)\\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(a+\frac{2c}{d^2}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{d^2}-\frac{x^2}{d^2}}\,dx,x,\sqrt{1-d^2x^2}\right)\\
&= -\frac{a\sqrt{1-d^2x^2}}{2x^2}-\frac{b\sqrt{1-d^2x^2}}{x}-\frac{1}{2}\left(2c+ad^2\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 56, normalized size = 0.79

$$-\frac{\sqrt{1-d^2x^2}\,(a+2bx)}{2x^2}-\frac{1}{2}\left(ad^2+2c\right)\tanh^{-1}\left(\sqrt{1-d^2x^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -1/2*((a + 2*b*x)*Sqrt[1 - d^2*x^2])/x^2 - ((2*c + a*d^2)*ArcTanh[Sqrt[1 -
d^2*x^2]])/2
```

IntegrateAlgebraic [A] time = 0.26, size = 112, normalized size = 1.58

$$\left(-ad^2-2c\right)\tanh^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{dx+1}}\right)-\frac{d\sqrt{1-dx}\left(\frac{ad(1-dx)}{dx+1}+ad-\frac{2b(1-dx)}{dx+1}+2b\right)}{\sqrt{dx+1}\left(\frac{1-dx}{dx+1}-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(x^3*Sqrt[1 - d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((d*Sqrt[1 - d*x]*(2*b + a*d - (2*b*(1 - d*x))/(1 + d*x) + (a*d*(1 - d*x))
/(1 + d*x)))/(Sqrt[1 + d*x]*(-1 + (1 - d*x)/(1 + d*x))^2)) + (-2*c - a*d^2)
*ArcTanh[Sqrt[1 - d*x]/Sqrt[1 + d*x]]
```

fricas [A] time = 0.66, size = 65, normalized size = 0.92

$$\frac{\left(ad^2+2\,c\right)x^2\log\left(\frac{\sqrt{dx+1}\,\sqrt{-dx+1}-1}{x}\right)-(2\,bx+a)\sqrt{dx+1}\,\sqrt{-dx+1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fric
as")
```

```
[Out] 1/2*((a*d^2 + 2*c)*x^2*log((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/x) - (2*b*x +
 a)*sqrt(d*x + 1)*sqrt(-d*x + 1))/x^2
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac
")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values [70,22
]Warning, choosing root of [1,0,-4,0,%%%{4,[2,2]%%%}] at parameters values
[42,56]1/d*(-1/2*(a*d^3+2*c*d)*ln(abs(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt
(2))+2-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1)))+1/2*(a*d^3+2*c*d)*ln(a
bs(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2))-2-1/2*(-2*sqrt(-d*x+1)+2*sqrt(
2))/sqrt(d*x+1)))-(2*a*d^3*(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2))-1/2*(
-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))^3-4*b*d^2*(2*sqrt(d*x+1)/(-2*sqrt(-
d*x+1)+2*sqrt(2))-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))^3+8*a*d^3*(2
*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2))-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sq
rt(d*x+1))+16*b*d^2*(2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2))-1/2*(-2*sqrt
(-d*x+1)+2*sqrt(2))/sqrt(d*x+1)))/((2*sqrt(d*x+1)/(-2*sqrt(-d*x+1)+2*sqrt(2
))-1/2*(-2*sqrt(-d*x+1)+2*sqrt(2))/sqrt(d*x+1))^2-4)^2)
```

maple [C] time = 0.02, size = 108, normalized size = 1.52

$$-\frac{\sqrt{-dx+1}\,\sqrt{dx+1}\,\left(a\,d^2x^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)+2c\,x^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-d^2x^2+1}}\right)+2\sqrt{-d^2x^2+1}\,bx+\sqrt{-d^2x^2+1}\,a\right)\mathrm{csgn}(d)^2}{2\sqrt{-d^2x^2+1}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(arctanh(1/(-d^2*x^2+1)^(1/2))*
x^2*a*d^2+2*arctanh(1/(-d^2*x^2+1)^(1/2))*x^2*c+2*(-d^2*x^2+1)^(1/2)*x*b+(-
d^2*x^2+1)^(1/2)*a)/(-d^2*x^2+1)^(1/2)/x^2
```

maxima [A] time = 0.97, size = 98, normalized size = 1.38

$$-\frac{1}{2}\,ad^2\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-c\log\left(\frac{2\sqrt{-d^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\sqrt{-d^2x^2+1}\,b}{x}-\frac{\sqrt{-d^2x^2+1}\,a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxi
ma")
```

```
[Out] -1/2*a*d^2*log(2*sqrt(-d^2*x^2 + 1)/abs(x) + 2/abs(x)) - c*log(2*sqrt(-d^2*
x^2 + 1)/abs(x) + 2/abs(x)) - sqrt(-d^2*x^2 + 1)*b/x - 1/2*sqrt(-d^2*x^2 +
1)*a/x^2
```

mupad [B] time = 5.85, size = 312, normalized size = 4.39

$$c\left(\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)-\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)\right)-\frac{\frac{a\,d^2\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-\frac{a\,d^2}{2}+\frac{15\,a\,d^2\left(\sqrt{1-d\,x}-1\right)^4}{2\left(\sqrt{d\,x+1}-1\right)^4}}{\frac{16\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-\frac{32\left(\sqrt{1-d\,x}-1\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}+\frac{16\left(\sqrt{1-d\,x}-1\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}}+\frac{a\,d^2\,\ln\left(\frac{\left(\sqrt{1-d\,x}-1\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}-1\right)}{2}-\frac{a\,d^2\,\ln\left(\frac{\sqrt{1-d\,x}-1}{\sqrt{d\,x+1}-1}\right)}{2}-\frac{b\,\sqrt{1-d\,x}\,\sqrt{d\,x+1}}{x}+\frac{a\,d^2\left(\sqrt{1-d\,x}-1\right)^2}{32\left(\sqrt{d\,x+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^3*(1 - d*x)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] c*(log(((1 - d*x)^(1/2) - 1)^2/((d*x + 1)^(1/2) - 1)^2 - 1) - log(((1 - d*x
)^(1/2) - 1)/((d*x + 1)^(1/2) - 1))) - ((a*d^2*((1 - d*x)^(1/2) - 1)^2)/((d
*x + 1)^(1/2) - 1)^2 - (a*d^2)/2 + (15*a*d^2*((1 - d*x)^(1/2) - 1)^4)/(2*((
d*x + 1)^(1/2) - 1)^4))/((16*((1 - d*x)^(1/2) - 1)^2)/((d*x + 1)^(1/2) - 1)
^2 - (32*((1 - d*x)^(1/2) - 1)^4)/((d*x + 1)^(1/2) - 1)^4 + (16*((1 - d*x)^
(1/2) - 1)^6)/((d*x + 1)^(1/2) - 1)^6) + (a*d^2*log(((1 - d*x)^(1/2) - 1)^2
/((d*x + 1)^(1/2) - 1)^2 - 1))/2 - (a*d^2*log(((1 - d*x)^(1/2) - 1)/((d*x +
 1)^(1/2) - 1)))/2 - (b*(1 - d*x)^(1/2)*(d*x + 1)^(1/2))/x + (a*d^2*((1 - d
*x)^(1/2) - 1)^2)/(32*((d*x + 1)^(1/2) - 1)^2)
```

sympy [C] time = 80.29, size = 218, normalized size = 3.07

$$\frac{iad^2 G_{6,6}^{5,3}\left(\begin{matrix} \frac{7}{4}, \frac{9}{4}, 1 & 2, 2, \frac{5}{2} \\ \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2} & 0 \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{ad^2 G_{6,6}^{2,6}\left(\begin{matrix} 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1 & \\ \frac{5}{4}, \frac{7}{4} & 1, \frac{3}{2}, \frac{3}{2}, 0 \end{matrix} \middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{ibdG_{6,6}^{5,3}\left(\begin{matrix} \frac{5}{4}, \frac{7}{4}, 1 & \frac{3}{2}, \frac{3}{2}, 2 \\ 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2 & 0 \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{bdG_{6,6}^{2,6}\left(\begin{matrix} \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 1 & \\ \frac{3}{4}, \frac{5}{4} & \frac{1}{2}, 1, 1, 0 \end{matrix} \middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} + \frac{icG_{6,6}^{5,3}\left(\begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 & 1, 1, \frac{3}{2} \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} & 0 \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}} - \frac{cG_{6,6}^{2,6}\left(\begin{matrix} 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1 & \\ \frac{1}{4}, \frac{3}{4} & 0, \frac{1}{2}, \frac{1}{2}, 0 \end{matrix} \middle| \frac{e^{-2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] I*a*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0
,)), 1/(d**2*x**2))/(4*pi**(3/2)) - a*d**2*meijerg(((1, 5/4, 3/2, 7/4, 2, 1
), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(-2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)) + I*b*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7
/4, 2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) + b*d*meijerg(((1/2, 3/4, 1, 5/
4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(-2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)) + I*c*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1
, 5/4, 3/2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - c*meijerg(((0, 1/4, 1/2,
 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(-2*I*pi)/(d**2*
x**2))/(4*pi**(3/2))
```

3.43 $\int \frac{x(a+bx+cx^2)}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=87

$$\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4}+\frac{b\cosh^{-1}(dx)}{2d^3}+\frac{cx^2\sqrt{dx-1}\,\sqrt{dx+1}}{3d^2}$$

Rubi [A] time = 0.15, antiderivative size = 151, normalized size of antiderivative = 1.74, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1610, 1809, 780, 217, 206}

$$-\frac{\left(1-d^2x^2\right)\left(2\left(3ad^2+2c\right)+3bd^2x\right)}{6d^4\sqrt{dx-1}\,\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\,\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{2d^3\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(c*x^2*(1 - d^2*x^2))/(3*d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - ((2*(2*c + 3*
a*d^2) + 3*b*d^2*x)*(1 - d^2*x^2))/(6*d^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (
b*Sqrt[-1 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(2*d^3*Sqrt[-1 + d*
x]*Sqrt[1 + d*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 780

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p
+ 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
```

```
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\left(a+bx+cx^2\right)}{\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx &= \frac{\sqrt{-1+d^2x^2}\int \frac{x\left(a+bx+cx^2\right)}{\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\,\sqrt{1+dx}} \\
&= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{x\left(2c+3ad^2+3bd^2x\right)}{\sqrt{-1+d^2x^2}}\,dx}{3d^2\sqrt{-1+dx}\,\sqrt{1+dx}} \\
&= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\left(1-d^2x^2\right)}{6d^4\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int -}{2d^2\sqrt{-1+dx}\,\sqrt{}} \\
&= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\left(1-d^2x^2\right)}{6d^4\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Sul}}{2d^2\sqrt{-}} \\
&= -\frac{cx^2\left(1-d^2x^2\right)}{3d^2\sqrt{-1+dx}\,\sqrt{1+dx}} - \frac{\left(2\left(2c+3ad^2\right)+3bd^2x\right)\left(1-d^2x^2\right)}{6d^4\sqrt{-1+dx}\,\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\,\tanh^{}}{2d^3\sqrt{-1+dx}}
\end{aligned}
$$

Mathematica [A] time = 0.35, size = 149, normalized size = 1.71

$$
\frac{\sqrt{-(dx-1)^2}\sqrt{dx+1}\left(3d^2(2a+bx)+2c\left(d^2x^2+2\right)\right)+6\sqrt{dx-1}\sin^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{2}}\right)(d(2ad-b)+2c)-12\sqrt{1-dx}\tanh^{-1}\left(\sqrt{\frac{dx-1}{dx+1}}\right)(d(ad-b)+c)}{6d^4\sqrt{1-dx}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (Sqrt[-(-1 + d*x)^2]*Sqrt[1 + d*x]*(3*d^2*(2*a + b*x) + 2*c*(2 + d^2*x^2))
+ 6*(2*c + d*(-b + 2*a*d))*Sqrt[-1 + d*x]*ArcSin[Sqrt[1 - d*x]/Sqrt[2]] - 1
2*(c + d*(-b + a*d))*Sqrt[1 - d*x]*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/(6*
d^4*Sqrt[1 - d*x])
```

IntegrateAlgebraic [B] time = 0.17, size = 230, normalized size = 2.64

$$
\frac{-\frac{6ad^2(dx-1)^{5/2}}{(dx+1)^{5/2}}+\frac{12ad^2(dx-1)^{3/2}}{(dx+1)^{3/2}}-\frac{6ad^2\sqrt{dx-1}}{\sqrt{dx+1}}+\frac{3bd(dx-1)^{5/2}}{(dx+1)^{5/2}}-\frac{3bd\sqrt{dx-1}}{\sqrt{dx+1}}-\frac{6c(dx-1)^{5/2}}{(dx+1)^{5/2}}+\frac{4c(dx-1)^{3/2}}{(dx+1)^{3/2}}-\frac{6c\sqrt{dx-1}}{\sqrt{dx+1}}}{3d^4\left(\frac{dx-1}{dx+1}-1\right)^3}+\frac{b\tanh^{-1}\left(\frac{\sqrt{dx-1}}{\sqrt{dx+1}}\right)}{d^3}
$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*(a + b*x + c*x^2))/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((-6*c*(-1 + d*x)^(5/2))/(1 + d*x)^(5/2) + (3*b*d*(-1 + d*x)^(5/2))/(1 + d*
x)^(5/2) - (6*a*d^2*(-1 + d*x)^(5/2))/(1 + d*x)^(5/2) + (4*c*(-1 + d*x)^(3/
2))/(1 + d*x)^(3/2) + (12*a*d^2*(-1 + d*x)^(3/2))/(1 + d*x)^(3/2) - (6*c*Sq
rt[-1 + d*x])/Sqrt[1 + d*x] - (3*b*d*Sqrt[-1 + d*x])/Sqrt[1 + d*x] - (6*a*d
^2*Sqrt[-1 + d*x])/Sqrt[1 + d*x])/(3*d^4*(-1 + (-1 + d*x)/(1 + d*x))^3) + (
b*ArcTanh[Sqrt[-1 + d*x]/Sqrt[1 + d*x]])/d^3
```

fricas [A] time = 0.91, size = 73, normalized size = 0.84

$$
-\frac{3\,bd\log\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)-\left(2\,cd^2x^2+3\,bd^2x+6\,ad^2+4\,c\right)\sqrt{dx+1}\,\sqrt{dx-1}}{6\,d^4}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] -1/6*(3*b*d*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - (2*c*d^2*x^2 + 3*b*d^
2*x + 6*a*d^2 + 4*c)*sqrt(d*x + 1)*sqrt(d*x - 1))/d^4
```

giac [A] time = 1.30, size = 105, normalized size = 1.21

$$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\left((dx+1)\left(\frac{2\,(dx+1)c}{d^3}+\frac{3\,bd^{10}-4\,cd^9}{d^{12}}\right)+\frac{3\left(2\,ad^{11}-bd^{10}+2\,cd^9\right)}{d^{12}}\right)-\frac{6\,b\log\left(\sqrt{dx+1}-\sqrt{dx-1}\right)}{d^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(d*x + 1)*sqrt(d*x - 1)*((d*x + 1)*(2*(d*x + 1)*c/d^3 + (3*b*d^10
- 4*c*d^9)/d^12) + 3*(2*a*d^11 - b*d^10 + 2*c*d^9)/d^12) - 6*b*log(sqrt(d*x
 + 1) - sqrt(d*x - 1))/d^2)/d
```

maple [C] time = 0.03, size = 137, normalized size = 1.57

$$\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(2\sqrt{d^2x^2-1}\,c\,d^2x^2\mathrm{csgn}(d)+3\sqrt{d^2x^2-1}\,b\,d^2x\,\mathrm{csgn}(d)+6\sqrt{d^2x^2-1}\,a\,d^2\mathrm{csgn}(d)+3bd\ln\left(\left(dx+\sqrt{d^2x^2-1}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+4\sqrt{d^2x^2-1}\,c\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)}{6\sqrt{d^2x^2-1}\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] 1/6*(d*x-1)^(1/2)*(d*x+1)^(1/2)*(2*csgn(d)*x^2*c*d^2*(d^2*x^2-1)^(1/2)+3*cs
gn(d)*(d^2*x^2-1)^(1/2)*x*b*d^2+6*csgn(d)*(d^2*x^2-1)^(1/2)*a*d^2+4*csgn(d)
*(d^2*x^2-1)^(1/2)*c+3*ln((csgn(d)*(d^2*x^2-1)^(1/2)+d*x)*csgn(d))*b*d)*csg
n(d)/d^4/(d^2*x^2-1)^(1/2)
```

maxima [A] time = 0.43, size = 100, normalized size = 1.15

$$\frac{\sqrt{d^2x^2-1}\,cx^2}{3\,d^2}+\frac{\sqrt{d^2x^2-1}\,bx}{2\,d^2}+\frac{\sqrt{d^2x^2-1}\,a}{d^2}+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{2\,d^3}+\frac{2\,\sqrt{d^2x^2-1}\,c}{3\,d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] 1/3*sqrt(d^2*x^2 - 1)*c*x^2/d^2 + 1/2*sqrt(d^2*x^2 - 1)*b*x/d^2 + sqrt(d^2*
x^2 - 1)*a/d^2 + 1/2*b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^3 + 2/3*sqrt(
d^2*x^2 - 1)*c/d^4
```

mupad [B] time = 12.35, size = 318, normalized size = 3.66

$$\frac{\sqrt{d\,x-1}\,\left(\frac{2\,c}{3\,d^4}+\frac{c\,x^3}{3\,d}+\frac{c\,x^2}{3\,d^2}+\frac{2\,c\,x}{3\,d^3}\right)}{\sqrt{d\,x+1}}+\frac{2\,b\,\mathrm{atanh}\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{\frac{14\,b\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^3}{{\left(\sqrt{d\,x+1}-1\right)}^3}+\frac{14\,b\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^5}{{\left(\sqrt{d\,x+1}-1\right)}^5}+\frac{2\,b\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^7}{{\left(\sqrt{d\,x+1}-1\right)}^7}+\frac{2\,b\,\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\sqrt{d\,x+1}-1}}{d^3-\frac{4\,d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+\frac{6\,d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{d\,x+1}-1\right)}^4}-\frac{4\,d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{d\,x+1}-1\right)}^6}+\frac{d^3\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^8}{{\left(\sqrt{d\,x+1}-1\right)}^8}}+\frac{a\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x + c*x^2))/((d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (2*b*atanh(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*b*((d*
x - 1)^(1/2) - 1i)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*b*((d*x - 1)^(1/2) - 1i
)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*b*((d*x - 1)^(1/2) - 1i)^7)/((d*x + 1)^(1
/2) - 1)^7 + (2*b*((d*x - 1)^(1/2) - 1i))/((d*x + 1)^(1/2) - 1))/(d^3 - (4*
d^3*((d*x - 1)^(1/2) - 1i)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((d*x - 1)^(
```

```
1/2) - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 - (4*d^3*((d*x - 1)^(1/2) - 1i)^6)/((
d*x + 1)^(1/2) - 1)^6 + (d^3*((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1/2) - 1
)^8) + ((d*x - 1)^(1/2)*((2*c)/(3*d^4) + (c*x^3)/(3*d) + (c*x^2)/(3*d^2) +
(2*c*x)/(3*d^3)))/(d*x + 1)^(1/2) + (a*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2
```

sympy [C]   time = 78.83, size = 308, normalized size = 3.54

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix} -\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1 \\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix} -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0 \end{matrix} \middle| \frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix} -\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1 \\ -1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}-\frac{ibG_{6,6}^{2,6}\left(\begin{matrix} -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\ -\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0 \end{matrix} \middle| \frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}+\frac{cG_{6,6}^{6,2}\left(\begin{matrix} -\frac{5}{4},-\frac{3}{4} & -1,-1,-\frac{1}{2},1 \\ -\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},0 & \end{matrix} \middle| \frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}+\frac{icG_{6,6}^{2,6}\left(\begin{matrix} -2,-\frac{7}{4},-\frac{3}{2},-\frac{5}{4},-1,1 & \\ -\frac{7}{4},-\frac{5}{4} & -2,-\frac{3}{2},-\frac{3}{2},0 \end{matrix} \middle| \frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*a*meijerg(((-1, -3/4, -1/2, -1/4, 0
, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(d**2*x**
2))/(4*pi**(3/2)*d**2) + b*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1
, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**3) - I*b*mei
jerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0
)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3) + c*meijerg(((-5/4, -
3/4), (-1, -1, -1/2, 1)), ((-3/2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(d**2*x*
*2))/(4*pi**(3/2)*d**4) + I*c*meijerg(((-2, -7/4, -3/2, -5/4, -1, 1), ()),
((-7/4, -5/4), (-2, -3/2, -3/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
(3/2)*d**4)
```

3.44 $\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=52

$$\frac{(2ad^2+c)\cosh^{-1}(dx)}{2d^3}+\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,(2b+cx)}{2d^2}$$

Rubi [B] time = 0.07, antiderivative size = 135, normalized size of antiderivative = 2.60, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {901, 1815, 641, 217, 206}

$$\frac{\sqrt{d^2x^2-1}\,(2ad^2+c)\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{2d^3\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{b(1-d^2x^2)}{d^2\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{cx(1-d^2x^2)}{2d^2\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((b*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) - (c*x*(1 - d^2*x^2
))/(2*d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + ((c + 2*a*d^2)*Sqrt[-1 + d^2*x^2]
*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(2*d^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 641

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*(
a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 901

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) +
 (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[((d + e*x)^FracPart[m]*(f + g*x)^Fr
acPart[m])/(d*f + e*g*x^2)^FracPart[m], Int[(d*f + e*g*x^2)^m*(a + b*x + c*
x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && EqQ[m - n, 0]
 && EqQ[e*f + d*g, 0]
```

Rule 1815

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Expon[Pq, x],
e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(e*x^(q - 1)*(a + b*x^2)^(p + 1))/(b*
(q + 2*p + 1)), x] + Dist[1/(b*(q + 2*p + 1)), Int[(a + b*x^2)^p*ExpandToSu
m[b*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b*e*(q + 2*p + 1)*x^q, x], x
], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] &&  !LeQ[p, -1]
```

Rubi steps

$$\int \frac{a+bx+cx^2}{\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{\sqrt{-1+d^2x^2}}dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$
$$= -\frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{c+2ad^2+2bd^2x}{\sqrt{-1+d^2x^2}}dx}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}$$
$$= -\frac{b\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(c+2ad^2\right)\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+}}}{2d^2\sqrt{-1+dx}\sqrt{1+dx}}$$
$$= -\frac{b\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(c+2ad^2\right)\sqrt{-1+d^2x^2}\right)\text{Subst}(}{2d^2\sqrt{-1+dx}\sqrt{}}$$
$$= -\frac{b\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{cx\left(1-d^2x^2\right)}{2d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(c+2ad^2\right)\sqrt{-1+d^2x^2}\tanh^{-1}(}{2d^3\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [B] time = 0.22, size = 126, normalized size = 2.42

$$\frac{4\sqrt{1-dx}\tanh^{-1}\left(\sqrt{\frac{dx-1}{dx+1}}\right)(d(ad-b)+c)+d\sqrt{-(dx-1)^2}\sqrt{dx+1}(2b+cx)+2\sqrt{dx-1}(2bd-c)\sin^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{2}}\right)}{2d^3\sqrt{1-dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (d*(2*b + c*x)*Sqrt[-(-1 + d*x)^2]*Sqrt[1 + d*x] + 2*(-c + 2*b*d)*Sqrt[-1 +
 d*x]*ArcSin[Sqrt[1 - d*x]/Sqrt[2]] + 4*(c + d*(-b + a*d))*Sqrt[1 - d*x]*Ar
cTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/(2*d^3*Sqrt[1 - d*x])
```

IntegrateAlgebraic [B] time = 0.14, size = 112, normalized size = 2.15

$$\frac{\left(2ad^2+c\right)\tanh^{-1}\left(\frac{\sqrt{dx-1}}{\sqrt{dx+1}}\right)}{d^3} - \frac{\sqrt{dx-1}\left(\frac{2bd(dx-1)}{dx+1}-2bd-\frac{c(dx-1)}{dx+1}-c\right)}{d^3\sqrt{dx+1}\left(\frac{dx-1}{dx+1}-1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((Sqrt[-1 + d*x]*(-c - 2*b*d - (c*(-1 + d*x))/(1 + d*x) + (2*b*d*(-1 + d*x
))/(1 + d*x)))/(d^3*Sqrt[1 + d*x]*(-1 + (-1 + d*x)/(1 + d*x))^2)) + ((c + 2
*a*d^2)*ArcTanh[Sqrt[-1 + d*x]/Sqrt[1 + d*x]])/d^3
```

fricas [A] time = 0.92, size = 61, normalized size = 1.17

$$\frac{(cdx+2\,bd)\sqrt{dx+1}\sqrt{dx-1}-\left(2\,ad^2+c\right)\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*((c*d*x + 2*b*d)*sqrt(d*x + 1)*sqrt(d*x - 1) - (2*a*d^2 + c)*log(-d*x +
 sqrt(d*x + 1)*sqrt(d*x - 1)))/d^3
```

giac [A] time = 1.25, size = 80, normalized size = 1.54

$$\frac{\sqrt{dx+1}\,\sqrt{dx-1}\left(\frac{(dx+1)c}{d^2}+\frac{2\,bd^5-cd^4}{d^6}\right)-\frac{2\left(2\,ad^2+c\right)\log\left(\sqrt{dx+1}-\sqrt{dx-1}\right)}{d^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(sqrt(d*x + 1)*sqrt(d*x - 1)*((d*x + 1)*c/d^2 + (2*b*d^5 - c*d^4)/d^6)
- 2*(2*a*d^2 + c)*log(sqrt(d*x + 1) - sqrt(d*x - 1))/d^2)/d
```

maple [C] time = 0.02, size = 120, normalized size = 2.31

$$\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(2a\,d^2\ln\left(\left(dx+\sqrt{d^2x^2-1}\,\operatorname{csgn}(d)\right)\operatorname{csgn}(d)\right)+\sqrt{d^2x^2-1}\,cdx\operatorname{csgn}(d)+2\sqrt{d^2x^2-1}\,bd\operatorname{csgn}(d)+c\ln\left(\left(dx+\sqrt{d^2x^2-1}\,\operatorname{csgn}(d)\right)\operatorname{csgn}(d)\right)\right)\operatorname{csgn}(d)}{2\sqrt{d^2x^2-1}\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] 1/2*(d*x-1)^(1/2)*(d*x+1)^(1/2)*(csgn(d)*d*(d^2*x^2-1)^(1/2)*x*c+2*ln((d*x+
(d^2*x^2-1)^(1/2)*csgn(d))*csgn(d))*a*d^2+2*csgn(d)*d*(d^2*x^2-1)^(1/2)*b+l
n((d*x+(d^2*x^2-1)^(1/2)*csgn(d))*csgn(d))*c)*csgn(d)/d^3/(d^2*x^2-1)^(1/2)
```

maxima [B] time = 0.43, size = 90, normalized size = 1.73

$$\frac{a\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{d}+\frac{\sqrt{d^2x^2-1}\,cx}{2\,d^2}+\frac{\sqrt{d^2x^2-1}\,b}{d^2}+\frac{c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{2\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima")
```

```
[Out] a*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + 1/2*sqrt(d^2*x^2 - 1)*c*x/d^2 +
sqrt(d^2*x^2 - 1)*b/d^2 + 1/2*c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d^3
```

mupad [B] time = 12.40, size = 312, normalized size = 6.00

$$\frac{b\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2}+\frac{2\,c\,\mathrm{atanh}\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)}{d^3}-\frac{4\,a\,\mathrm{atan}\left(\frac{d\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{-d^2}}\right)}{\sqrt{-d^2}}-\frac{\frac{14\,c\left(\sqrt{d\,x-1}-\mathrm{i}\right)^3}{\left(\sqrt{d\,x+1}-1\right)^3}+\frac{14\,c\left(\sqrt{d\,x-1}-\mathrm{i}\right)^5}{\left(\sqrt{d\,x+1}-1\right)^5}+\frac{2\,c\left(\sqrt{d\,x-1}-\mathrm{i}\right)^7}{\left(\sqrt{d\,x+1}-1\right)^7}+\frac{2\,c\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\sqrt{d\,x+1}-1}}{d^3-\frac{4\,d^3\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+\frac{6\,d^3\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}-\frac{4\,d^3\left(\sqrt{d\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}+\frac{d^3\left(\sqrt{d\,x-1}-\mathrm{i}\right)^8}{\left(\sqrt{d\,x+1}-1\right)^8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (2*c*atanh(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))/d^3 - ((14*c*((d*
x - 1)^(1/2) - 1i)^3)/((d*x + 1)^(1/2) - 1)^3 + (14*c*((d*x - 1)^(1/2) - 1i
)^5)/((d*x + 1)^(1/2) - 1)^5 + (2*c*((d*x - 1)^(1/2) - 1i)^7)/((d*x + 1)^(1
/2) - 1)^7 + (2*c*((d*x - 1)^(1/2) - 1i))/((d*x + 1)^(1/2) - 1))/(d^3 - (4*
d^3*((d*x - 1)^(1/2) - 1i)^2)/((d*x + 1)^(1/2) - 1)^2 + (6*d^3*((d*x - 1)^(
1/2) - 1i)^4)/((d*x + 1)^(1/2) - 1)^4 - (4*d^3*((d*x - 1)^(1/2) - 1i)^6)/((
d*x + 1)^(1/2) - 1)^6 + (d^3*((d*x - 1)^(1/2) - 1i)^8)/((d*x + 1)^(1/2) - 1
)^8) - (4*a*atan((d*((d*x - 1)^(1/2) - 1i))/(((d*x + 1)^(1/2) - 1)*(-d^2)^(
1/2))))/(-d^2)^(1/2) + (b*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2
```

sympy [C] time = 48.29, size = 277, normalized size = 5.33

$$\frac{aG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\-\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\-\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{cG_{6,6}^{6,2}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{4} & -\frac{1}{2},-\frac{1}{2},0,1\\-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}-\frac{icG_{6,6}^{2,6}\left(\begin{matrix}-\frac{3}{2},-\frac{5}{4},-1,-\frac{3}{4},-\frac{1}{2},1 & \\-\frac{5}{4},-\frac{3}{4} & -\frac{3}{2},-1,-1,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 1
/(d**2*x**2))/(4*pi**(3/2)*d) - I*a*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1),
()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
(3/2)*d) + b*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1
/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*b*meijerg(((-1, -3/4, -1
/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi
)/(d**2*x**2))/(4*pi**(3/2)*d**2) + c*meijerg(((-3/4, -1/4), (-1/2, -1/2, 0
, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**3
) - I*c*meijerg(((-3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2
, -1, -1, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**3)
```

3.45 $\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\,\sqrt{1+dx}}\,dx$

Optimal. Leaf size=55

$$a\tan^{-1}\left(\sqrt{dx-1}\,\sqrt{dx+1}\right)+\frac{b\cosh^{-1}(dx)}{d}+\frac{c\sqrt{dx-1}\,\sqrt{dx+1}}{d^2}$$

Rubi [B] time = 0.18, antiderivative size = 135, normalized size of antiderivative = 2.45, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1610, 1809, 844, 217, 206, 266, 63, 205}

$$\frac{a\sqrt{d^2x^2-1}\,\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{\sqrt{dx-1}\,\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\,\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d\sqrt{dx-1}\,\sqrt{dx+1}}-\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{dx-1}\,\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((c*(1 - d^2*x^2))/(d^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) + (a*Sqrt[-1 + d^2*
x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (b*Sqrt[-
1 + d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(d*Sqrt[-1 + d*x]*Sqrt[1 +
d*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1809

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, Simp[(f*(c*x)^(m + q -
1)*(a + b*x^2)^(p + 1))/(b*c^(q - 1)*(m + q + 2*p + 1)), x] + Dist[1/(b*(m
+ q + 2*p + 1)), Int[(c*x)^m*(a + b*x^2)^p*ExpandToSum[b*(m + q + 2*p + 1)*
Pq - b*f*(m + q + 2*p + 1)*x^q - a*f*(m + q - 1)*x^(q - 2), x], x], x] /; G
tQ[q, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[
Pq, x] && ( !IGtQ[m, 0] || IGtQ[p + 1/2, -1])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{ad^2+bd^2x}{x\sqrt{-1+d^2x^2}}\,dx}{d^2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+}}}{\sqrt{-1+dx}\sqrt{1+d}}$$

$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\mathrm{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx, x, x^2\right)}{2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+\right.}{}$$

$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(a\sqrt{-1+d^2x^2}\right)\mathrm{Subs}}{d^2\sqrt{-1}}$$

$$= -\frac{c\left(1-d^2x^2\right)}{d^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{a\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\tanh^{-}}{d\sqrt{-1+dx}\sqrt{}}$$

Mathematica [B] time = 0.42, size = 128, normalized size = 2.33

$$\frac{\frac{ad^2\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)+cd^2x^2-2c\sqrt{1-d^2x^2}\sin^{-1}\left(\frac{\sqrt{1-dx}}{\sqrt{2}}\right)-c}{\sqrt{dx-1}\sqrt{dx+1}}-2(c-bd)\tanh^{-1}\left(\sqrt{\frac{dx-1}{dx+1}}\right)}{d^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((-c + c*d^2*x^2 - 2*c*Sqrt[1 - d^2*x^2]*ArcSin[Sqrt[1 - d*x]/Sqrt[2]] + a*
d^2*Sqrt[-1 + d^2*x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 +
 d*x]) - 2*(c - b*d)*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d^2
```

IntegrateAlgebraic [A] time = 0.15, size = 91, normalized size = 1.65

$$2a\tan^{-1}\left(\frac{\sqrt{dx-1}}{\sqrt{dx+1}}\right)+\frac{2b\tanh^{-1}\left(\frac{\sqrt{dx-1}}{\sqrt{dx+1}}\right)}{d}-\frac{2c\sqrt{dx-1}}{d^2\sqrt{dx+1}\left(\frac{dx-1}{dx+1}-1\right)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (-2*c*Sqrt[-1 + d*x])/(d^2*Sqrt[1 + d*x]*(-1 + (-1 + d*x)/(1 + d*x))) + 2*a
*ArcTan[Sqrt[-1 + d*x]/Sqrt[1 + d*x]] + (2*b*ArcTanh[Sqrt[-1 + d*x]/Sqrt[1
+ d*x]])/d
```

fricas [A] time = 0.74, size = 73, normalized size = 1.33

$$\frac{2\,ad^2\arctan\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)-bd\log\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)+\sqrt{dx+1}\,\sqrt{dx-1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="fricas"
)
```

```
[Out] (2*a*d^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) - b*d*log(-d*x + sqrt(d
*x + 1)*sqrt(d*x - 1)) + sqrt(d*x + 1)*sqrt(d*x - 1)*c)/d^2
```

giac [A] time = 1.29, size = 71, normalized size = 1.29

$$-2\,a\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)-\frac{b\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)}{d}+\frac{\sqrt{dx+1}\,\sqrt{dx-1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac")
```

```
[Out] -2*a*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - b*log((sqrt(d*x + 1) -
 sqrt(d*x - 1))^2)/d + sqrt(d*x + 1)*sqrt(d*x - 1)*c/d^2
```

maple [C] time = 0.02, size = 95, normalized size = 1.73

$$\frac{\left(-a\,d^2\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)\mathrm{csgn}(d)+bd\ln\left(\left(dx+\sqrt{(dx+1)(dx-1)}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)+\sqrt{d^2x^2-1}\,c\,\mathrm{csgn}(d)\right)\sqrt{dx-1}\,\sqrt{dx+1}\,\mathrm{csgn}(d)}{\sqrt{d^2x^2-1}\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-csgn(d)*arctan(1/(d^2*x^2-1)^(1/2))*a*d^2+(d^2*x^2-1)^(1/2)*c*csgn(d)+ln(
(csgn(d)*((d*x+1)*(d*x-1))^(1/2)+d*x)*csgn(d))*b*d)*(d*x-1)^(1/2)*(d*x+1)^(
1/2)/d^2*csgn(d)/(d^2*x^2-1)^(1/2)
```

maxima [A] time = 0.96, size = 56, normalized size = 1.02

$$-a\arcsin\left(\frac{1}{d|x|}\right)+\frac{b\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{d}+\frac{\sqrt{d^2x^2-1}\,c}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxima"
)
```

```
[Out] -a*arcsin(1/(d*abs(x))) + b*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + sqrt(d
^2*x^2 - 1)*c/d^2
```

mupad [B] time = 3.97, size = 118, normalized size = 2.15

$$\frac{c\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{d^2}-\frac{4\,b\,\mathrm{atan}\left(\frac{d\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{-d^2}}\right)}{\sqrt{-d^2}}-a\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (c*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/d^2 - (4*b*atan((d*((d*x - 1)^(1/2) - 1
i))/(((d*x + 1)^(1/2) - 1)*(-d^2)^(1/2))))/(-d^2)^(1/2) - a*(log(((d*x - 1)
^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - log(((d*x - 1)^(1/2) - 1i)/((
d*x + 1)^(1/2) - 1)))*1i
```

sympy [C] time = 47.40, size = 240, normalized size = 4.36

$$-\frac{aG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iaG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{bG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}+\frac{cG_{6,6}^{6,2}\left(\begin{matrix}-\frac{1}{4},\frac{1}{4} & 0,0,\frac{1}{2},1\\ -\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}+\frac{icG_{6,6}^{2,6}\left(\begin{matrix}-1,-\frac{3}{4},-\frac{1}{2},-\frac{1}{4},0,1 & \\ -\frac{3}{4},-\frac{1}{4} & -1,-\frac{1}{2},-\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1
/(d**2*x**2))/(4*pi**(3/2)) + I*a*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), (
(1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2))
 + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ())
, 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*meijerg(((-1/2, -1/4, 0, 1/4, 1/2, 1
), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*p
i**(3/2)*d) + c*meijerg(((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4
, 1/2, 0), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*c*meijerg(((-1, -3/4,
 -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I
*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2)
```

3.46 $\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=55

$$\frac{a\sqrt{dx-1}\sqrt{dx+1}}{x}+b\tan^{-1}\left(\sqrt{dx-1}\sqrt{dx+1}\right)+\frac{c\cosh^{-1}(dx)}{d}$$

Rubi [B] time = 0.18, antiderivative size = 135, normalized size of antiderivative = 2.45, number of steps used = 8, number of rules used = 8, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1610, 1807, 844, 217, 206, 266, 63, 205}

$$-\frac{a\left(1-d^2x^2\right)}{x\sqrt{dx-1}\sqrt{dx+1}}+\frac{b\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{\sqrt{dx-1}\sqrt{dx+1}}+\frac{c\sqrt{d^2x^2-1}\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-1}}\right)}{d\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((a*(1 - d^2*x^2))/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x])) + (b*Sqrt[-1 + d^2*x^
2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (c*Sqrt[-1
+ d^2*x^2]*ArcTanh[(d*x)/Sqrt[-1 + d^2*x^2]])/(d*Sqrt[-1 + d*x]*Sqrt[1 + d*
x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
```

```
e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] && !IntegerQ[m]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^2\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^2\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{b+cx}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\int \frac{1}{x\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(c\sqrt{-1+d^2x^2}\right)\int \frac{1}{\sqrt{-1+}}}{\sqrt{-1+dx}\sqrt{1+d}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Subst}\left(\int \frac{1}{x\sqrt{-1+d^2x}}\,dx, x, x^2\right)}{2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(c\sqrt{-1+}\right.}{}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{c\sqrt{-1+d^2x^2}\tanh^{-1}\left(\frac{dx}{\sqrt{-1+d^2x^2}}\right)}{d\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{-1+d^2x^2}\right)\text{Subst}}{d^2\sqrt{-1}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{b\sqrt{-1+d^2x^2}\tan^{-1}\left(\sqrt{-1+d^2x^2}\right)}{\sqrt{-1+dx}\sqrt{1+dx}} + \frac{c\sqrt{-1+d^2x^2}\tanh^{-}}{d\sqrt{-1+dx}\sqrt{}}$$

Mathematica [A] time = 0.19, size = 89, normalized size = 1.62

$$\frac{a\left(d^2x^2-1\right)+bx\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{x\sqrt{dx-1}\sqrt{dx+1}}+\frac{2c\tanh^{-1}\left(\sqrt{\frac{dx-1}{dx+1}}\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (a*(-1 + d^2*x^2) + b*x*Sqrt[-1 + d^2*x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(x*S
qrt[-1 + d*x]*Sqrt[1 + d*x]) + (2*c*ArcTanh[Sqrt[(-1 + d*x)/(1 + d*x)]])/d
```

IntegrateAlgebraic [A] time = 0.12, size = 89, normalized size = 1.62

$$\frac{2ad\sqrt{dx-1}}{\sqrt{dx+1}\left(\frac{dx-1}{dx+1}+1\right)}+2b\tan^{-1}\left(\frac{\sqrt{dx-1}}{\sqrt{dx+1}}\right)+\frac{2c\tanh^{-1}\left(\frac{\sqrt{dx-1}}{\sqrt{dx+1}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (2*a*d*Sqrt[-1 + d*x])/(Sqrt[1 + d*x]*(1 + (-1 + d*x)/(1 + d*x))) + 2*b*Arc
Tan[Sqrt[-1 + d*x]/Sqrt[1 + d*x]] + (2*c*ArcTanh[Sqrt[-1 + d*x]/Sqrt[1 + d*
x]])/d
```

fricas [A] time = 1.03, size = 82, normalized size = 1.49

$$\frac{ad^2x+2\,bdx\arctan\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)+\sqrt{dx+1}\sqrt{dx-1}\,ad-cx\log\left(-dx+\sqrt{dx+1}\sqrt{dx-1}\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] (a*d^2*x + 2*b*d*x*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + sqrt(d*x +
1)*sqrt(d*x - 1)*a*d - c*x*log(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)))/(d*x)
```

giac [A] time = 1.35, size = 83, normalized size = 1.51

$$-\frac{2\,bd\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)-\frac{8\,ad^2}{\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4}+c\log\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -(2*b*d*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) - 8*a*d^2/((sqrt(d*x
+ 1) - sqrt(d*x - 1))^4 + 4) + c*log((sqrt(d*x + 1) - sqrt(d*x - 1))^2))/d
```

maple [C] time = 0.02, size = 96, normalized size = 1.75

$$\frac{\left(-bdx\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)\mathrm{csgn}(d)+\sqrt{d^2x^2-1}\,ad\,\mathrm{csgn}(d)+cx\ln\left(\left(dx+\sqrt{d^2x^2-1}\,\mathrm{csgn}(d)\right)\mathrm{csgn}(d)\right)\right)\sqrt{dx-1}\,\sqrt{dx+1}\,\mathrm{csgn}(d)}{\sqrt{d^2x^2-1}\,dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] (-csgn(d)*d*arctan(1/(d^2*x^2-1)^(1/2))*x*b+csgn(d)*d*(d^2*x^2-1)^(1/2)*a+l
n((d*x+(d^2*x^2-1)^(1/2)*csgn(d))*csgn(d))*x*c)*(d*x-1)^(1/2)*(d*x+1)^(1/2)
*csgn(d)/(d^2*x^2-1)^(1/2)/x/d
```

maxima [A] time = 0.97, size = 56, normalized size = 1.02

$$-b\arcsin\left(\frac{1}{d|x|}\right)+\frac{c\log\left(2\,d^2x+2\,\sqrt{d^2x^2-1}\,d\right)}{d}+\frac{\sqrt{d^2x^2-1}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^2/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -b*arcsin(1/(d*abs(x))) + c*log(2*d^2*x + 2*sqrt(d^2*x^2 - 1)*d)/d + sqrt(d
^2*x^2 - 1)*a/x
```

mupad [B] time = 3.86, size = 118, normalized size = 2.15

$$\frac{a\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x}-\frac{4\,c\,\mathrm{atan}\left(\frac{d\left(\sqrt{d\,x-1}-\mathrm{i}\right)}{\left(\sqrt{d\,x+1}-1\right)\sqrt{-d^2}}\right)}{\sqrt{-d^2}}-b\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^2*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] (a*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/x - (4*c*atan((d*((d*x - 1)^(1/2) - 1i)
)/(((d*x + 1)^(1/2) - 1)*(-d^2)^(1/2))))/(-d^2)^(1/2) - b*(log(((d*x - 1)^(
1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - log(((d*x - 1)^(1/2) - 1i)/((d*
x + 1)^(1/2) - 1)))*1i
```

sympy [C] time = 45.93, size = 216, normalized size = 3.93

$$-\frac{adG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{iadG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{ibG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{cG_{6,6}^{6,2}\left(\begin{matrix}\frac{1}{4},\frac{3}{4} & \frac{1}{2},\frac{1}{2},1,1\\ 0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,0 & \end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}-\frac{icG_{6,6}^{2,6}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4},0,\frac{1}{4},\frac{1}{2},1 & \\ -\frac{1}{4},\frac{1}{4} & -\frac{1}{2},0,0,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**2/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)),
 1/(d**2*x**2))/(4*pi**(3/2)) - I*a*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1),
()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3
/2)) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0
,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1),
()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**
(3/2)) + c*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0
), ()), 1/(d**2*x**2))/(4*pi**(3/2)*d) - I*c*meijerg(((-1/2, -1/4, 0, 1/4,
1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(d**2*x**2)
)/(4*pi**(3/2)*d)
```

3.47 $\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=83

$$\frac{1}{2}\left(ad^2+2c\right)\tan^{-1}\left(\sqrt{dx-1}\sqrt{dx+1}\right)+\frac{a\sqrt{dx-1}\sqrt{dx+1}}{2x^2}+\frac{b\sqrt{dx-1}\sqrt{dx+1}}{x}$$

Rubi [A] time = 0.19, antiderivative size = 129, normalized size of antiderivative = 1.55, number of steps used = 6, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1610, 1807, 807, 266, 63, 205}

$$\frac{\sqrt{d^2x^2-1}\left(ad^2+2c\right)\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{2\sqrt{dx-1}\sqrt{dx+1}}-\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{dx-1}\sqrt{dx+1}}-\frac{b\left(1-d^2x^2\right)}{x\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(a*(1 - d^2*x^2))/(2*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - (b*(1 - d^2*x^2))
/(x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + ((2*c + a*d^2)*Sqrt[-1 + d^2*x^2]*ArcTa
n[Sqrt[-1 + d^2*x^2]])/(2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^3\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^3\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{2b+\left(2c+ad^2\right)x}{x^2\sqrt{-1+d^2x^2}}\,dx}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\right)\int \frac{}{x\sqrt{-}}}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\right)\text{Subst}}{4\sqrt{-1+dx}\sqrt{1}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\right)\text{Subst}}{2d^2\sqrt{-1+d}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(2c+ad^2\right)\sqrt{-1+d^2x^2}\tan^{-1}\left(\right.}{2\sqrt{-1+dx}\sqrt{1+dx}}$$

Mathematica [A] time = 0.14, size = 82, normalized size = 0.99

$$\frac{\left(d^2x^2-1\right)(a+2bx)+x^2\sqrt{d^2x^2-1}\left(ad^2+2c\right)\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{2x^2\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((a + 2*b*x)*(-1 + d^2*x^2) + (2*c + a*d^2)*x^2*Sqrt[-1 + d^2*x^2]*ArcTan[S
qrt[-1 + d^2*x^2]])/(2*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

IntegrateAlgebraic [A] time = 0.12, size = 107, normalized size = 1.29

$$\left(ad^2+2c\right)\tan^{-1}\left(\frac{\sqrt{dx-1}}{\sqrt{dx+1}}\right)-\frac{d\sqrt{dx-1}\left(\frac{ad(dx-1)}{dx+1}-ad-\frac{2b(dx-1)}{dx+1}-2b\right)}{\sqrt{dx+1}\left(\frac{dx-1}{dx+1}+1\right)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -((d*Sqrt[-1 + d*x]*(-2*b - a*d - (2*b*(-1 + d*x))/(1 + d*x) + (a*d*(-1 + d
*x))/(1 + d*x)))/(Sqrt[1 + d*x]*(1 + (-1 + d*x)/(1 + d*x))^2)) + (2*c + a*d
^2)*ArcTan[Sqrt[-1 + d*x]/Sqrt[1 + d*x]]
```

fricas [A] time = 0.88, size = 69, normalized size = 0.83

$$\frac{2\,bdx^2+2\,\left(ad^2+2\,c\right)x^2\arctan\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)+\left(2\,bx+a\right)\sqrt{dx+1}\,\sqrt{dx-1}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/2*(2*b*d*x^2 + 2*(a*d^2 + 2*c)*x^2*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x -
 1)) + (2*b*x + a)*sqrt(d*x + 1)*sqrt(d*x - 1))/x^2
```

giac [B] time = 1.35, size = 145, normalized size = 1.75

$$-\frac{\left(ad^3+2\,cd\right)\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)+\frac{2\left(ad^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^6-4\,bd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4-4\,ad^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2-16\,bd^2\right)}{\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -((a*d^3 + 2*c*d)*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(a*d^3*
(sqrt(d*x + 1) - sqrt(d*x - 1))^6 - 4*b*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1))
^4 - 4*a*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 16*b*d^2)/((sqrt(d*x + 1)
- sqrt(d*x - 1))^4 + 4)^2)/d
```

maple [C] time = 0.02, size = 103, normalized size = 1.24

$$-\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(a\,d^2x^2\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)+2c\,x^2\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)-2\sqrt{d^2x^2-1}\;bx-\sqrt{d^2x^2-1}\;a\right)\mathrm{csgn}(d)^2}{2\sqrt{d^2x^2-1}\;x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/2*(d*x-1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(arctan(1/(d^2*x^2-1)^(1/2))*x^2
*a*d^2+2*arctan(1/(d^2*x^2-1)^(1/2))*x^2*c-2*(d^2*x^2-1)^(1/2)*x*b-(d^2*x^2
-1)^(1/2)*a)/(d^2*x^2-1)^(1/2)/x^2
```

maxima [A] time = 0.97, size = 61, normalized size = 0.73

$$-\frac{1}{2}\,ad^2\arcsin\left(\frac{1}{d|x|}\right)-c\arcsin\left(\frac{1}{d|x|}\right)+\frac{\sqrt{d^2x^2-1}\,b}{x}+\frac{\sqrt{d^2x^2-1}\,a}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^3/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*a*d^2*arcsin(1/(d*abs(x))) - c*arcsin(1/(d*abs(x))) + sqrt(d^2*x^2 - 1
)*b/x + 1/2*sqrt(d^2*x^2 - 1)*a/x^2
```

mupad [B] time = 9.89, size = 316, normalized size = 3.81

$$\frac{\frac{a\,d^2\,1\mathrm{i}}{32}+\frac{a\,d^2\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2 1\mathrm{i}}{16\left(\sqrt{d\,x+1}-1\right)^2}-\frac{a\,d^2\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4 15\mathrm{i}}{32\left(\sqrt{d\,x+1}-1\right)^4}}{\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+\frac{2\left(\sqrt{d\,x-1}-\mathrm{i}\right)^4}{\left(\sqrt{d\,x+1}-1\right)^4}+\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^6}{\left(\sqrt{d\,x+1}-1\right)^6}}-c\left(\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)-\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\right)1\mathrm{i}-\frac{a\,d^2\,\ln\left(\frac{\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2}{\left(\sqrt{d\,x+1}-1\right)^2}+1\right)1\mathrm{i}}{2}+\frac{a\,d^2\,\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)1\mathrm{i}}{2}+\frac{b\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x}+\frac{a\,d^2\left(\sqrt{d\,x-1}-\mathrm{i}\right)^2 1\mathrm{i}}{32\left(\sqrt{d\,x+1}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^3*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((a*d^2*1i)/32 + (a*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(16*((d*x + 1)^(1/2) -
 1)^2) - (a*d^2*((d*x - 1)^(1/2) - 1i)^4*15i)/(32*((d*x + 1)^(1/2) - 1)^4))
/(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + (2*((d*x - 1)^(1/2) -
1i)^4)/((d*x + 1)^(1/2) - 1)^4 + ((d*x - 1)^(1/2) - 1i)^6/((d*x + 1)^(1/2)
- 1)^6) - c*(log(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1) - lo
g(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1)))*1i - (a*d^2*log(((d*x - 1)
^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*d^2*log(((d*x - 1)^(
1/2) - 1i)/((d*x + 1)^(1/2) - 1))*1i)/2 + (b*(d*x - 1)^(1/2)*(d*x + 1)^(1/2
))/x + (a*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(32*((d*x + 1)^(1/2) - 1)^2)
```

sympy [C] time = 74.80, size = 212, normalized size = 2.55

$$-\frac{ad^2G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{iad^2G_{6,6}^{2,6}\left(\begin{matrix}1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bdG_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{ibdG_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{cG_{6,6}^{5,3}\left(\begin{matrix}\frac{3}{4},\frac{5}{4},1 & 1,1,\frac{3}{2}\\ \frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{icG_{6,6}^{2,6}\left(\begin{matrix}0,\frac{1}{4},\frac{1}{2},\frac{3}{4},1,1 & \\ \frac{1}{4},\frac{3}{4} & 0,\frac{1}{2},\frac{1}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**3/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,
)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*a*d**2*meijerg(((1, 5/4, 3/2, 7/4, 2,
1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(4*
pi**(3/2)) - b*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4
, 2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - I*b*d*meijerg(((1/2, 3/4, 1, 5/
4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(d**2*x**2
))/(4*pi**(3/2)) - c*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5
/4, 3/2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*c*meijerg(((0, 1/4, 1/2,
3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(d**2*x*
*2))/(4*pi**(3/2))
```

3.48 $\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx$

Optimal. Leaf size=116

$$\frac{\sqrt{dx-1}\sqrt{dx+1}\left(2ad^2+3c\right)}{3x}+\frac{a\sqrt{dx-1}\sqrt{dx+1}}{3x^3}+\frac{1}{2}bd^2\tan^{-1}\left(\sqrt{dx-1}\sqrt{dx+1}\right)+\frac{b\sqrt{dx-1}\sqrt{dx+1}}{2x^2}$$

Rubi [A] time = 0.22, antiderivative size = 171, normalized size of antiderivative = 1.47, number of steps used = 7, number of rules used = 7, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.219, Rules used = {1610, 1807, 835, 807, 266, 63, 205}

$$-\frac{\left(1-d^2x^2\right)\left(2ad^2+3c\right)}{3x\sqrt{dx-1}\sqrt{dx+1}}-\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{dx-1}\sqrt{dx+1}}-\frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{dx-1}\sqrt{dx+1}}+\frac{bd^2\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{2\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] -(a*(1 - d^2*x^2))/(3*x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - (b*(1 - d^2*x^2))
/(2*x^2*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) - ((3*c + 2*a*d^2)*(1 - d^2*x^2))/(3*
x*Sqrt[-1 + d*x]*Sqrt[1 + d*x]) + (b*d^2*Sqrt[-1 + d^2*x^2]*ArcTan[Sqrt[-1
+ d^2*x^2]])/(2*Sqrt[-1 + d*x]*Sqrt[1 + d*x])
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
```

```
p])
```

Rule 1610

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.
)*(x_))^(p_.), x_Symbol] :> Dist[((a + b*x)^FracPart[m]*(c + d*x)^FracPart[
m])/(a*c + b*d*x^2)^FracPart[m], Int[Px*(a*c + b*d*x^2)^m*(e + f*x)^p, x],
x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px, x] && EqQ[b*c + a*
d, 0] && EqQ[m, n] &&  !IntegerQ[m]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rubi steps

$$\int \frac{a+bx+cx^2}{x^4\sqrt{-1+dx}\sqrt{1+dx}}\,dx = \frac{\sqrt{-1+d^2x^2}\int \frac{a+bx+cx^2}{x^4\sqrt{-1+d^2x^2}}\,dx}{\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{3b+\left(3c+2ad^2\right)x}{x^3\sqrt{-1+d^2x^2}}\,dx}{3\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\sqrt{-1+d^2x^2}\int \frac{2\left(3c+2ad^2\right)+3bd^2x}{x^2\sqrt{-1+d^2x^2}}}{6\sqrt{-1+dx}\sqrt{1+dx}}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(bd\right.}{}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(bd\right.}{}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{\left(b\sqrt{\right.}}{}$$

$$= -\frac{a\left(1-d^2x^2\right)}{3x^3\sqrt{-1+dx}\sqrt{1+dx}} - \frac{b\left(1-d^2x^2\right)}{2x^2\sqrt{-1+dx}\sqrt{1+dx}} - \frac{\left(3c+2ad^2\right)\left(1-d^2x^2\right)}{3x\sqrt{-1+dx}\sqrt{1+dx}} + \frac{bd^2}{}$$

Mathematica [A] time = 0.13, size = 94, normalized size = 0.81

$$\frac{\left(d^2x^2-1\right)\left(a\left(4d^2x^2+2\right)+3x(b+2cx)\right)+3bd^2x^3\sqrt{d^2x^2-1}\tan^{-1}\left(\sqrt{d^2x^2-1}\right)}{6x^3\sqrt{dx-1}\sqrt{dx+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] ((-1 + d^2*x^2)*(3*x*(b + 2*c*x) + a*(2 + 4*d^2*x^2)) + 3*b*d^2*x^3*Sqrt[-1
 + d^2*x^2]*ArcTan[Sqrt[-1 + d^2*x^2]])/(6*x^3*Sqrt[-1 + d*x]*Sqrt[1 + d*x]
)
```

IntegrateAlgebraic [A] time = 0.16, size = 168, normalized size = 1.45

$$\frac{d\sqrt{dx-1}\left(\frac{4ad^2(dx-1)}{dx+1}+\frac{6ad^2(dx-1)^2}{(dx+1)^2}+6ad^2-\frac{3bd(dx-1)^2}{(dx+1)^2}+3bd+\frac{12c(dx-1)}{dx+1}+\frac{6c(dx-1)^2}{(dx+1)^2}+6c\right)}{3\sqrt{dx+1}\left(\frac{dx-1}{dx+1}+1\right)^3}+bd^2\tan^{-1}\left(\frac{\sqrt{dx-1}}{\sqrt{dx+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)/(x^4*Sqrt[-1 + d*x]*Sqrt[1 + d*x]),x]
```

```
[Out] (d*Sqrt[-1 + d*x]*(6*c + 3*b*d + 6*a*d^2 + (6*c*(-1 + d*x)^2)/(1 + d*x)^2 -
 (3*b*d*(-1 + d*x)^2)/(1 + d*x)^2 + (6*a*d^2*(-1 + d*x)^2)/(1 + d*x)^2 + (1
2*c*(-1 + d*x))/(1 + d*x) + (4*a*d^2*(-1 + d*x))/(1 + d*x)))/(3*Sqrt[1 + d*
x]*(1 + (-1 + d*x)/(1 + d*x))^3) + b*d^2*ArcTan[Sqrt[-1 + d*x]/Sqrt[1 + d*x
]]
```

fricas [A] time = 0.95, size = 90, normalized size = 0.78

$$\frac{6\,bd^2x^3\arctan\left(-dx+\sqrt{dx+1}\,\sqrt{dx-1}\right)+2\left(2\,ad^3+3\,cd\right)x^3+\left(2\left(2\,ad^2+3\,c\right)x^2+3\,bx+2\,a\right)\sqrt{dx+1}\,\sqrt{dx-1}}{6\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="frica
s")
```

```
[Out] 1/6*(6*b*d^2*x^3*arctan(-d*x + sqrt(d*x + 1)*sqrt(d*x - 1)) + 2*(2*a*d^3 +
3*c*d)*x^3 + (2*(2*a*d^2 + 3*c)*x^2 + 3*b*x + 2*a)*sqrt(d*x + 1)*sqrt(d*x -
 1))/x^3
```

giac [B] time = 1.35, size = 197, normalized size = 1.70

$$-\frac{3\,bd^3\arctan\left(\frac{1}{2}\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2\right)+\frac{2\left(3\,bd^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^{10}-12\,cd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^8-96\,ad^4\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4-96\,cd^2\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4-48\,bd^3\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^2-128\,ad^4-192\,cd^2\right)}{\left(\left(\sqrt{dx+1}-\sqrt{dx-1}\right)^4+4\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="giac"
)
```

```
[Out] -1/3*(3*b*d^3*arctan(1/2*(sqrt(d*x + 1) - sqrt(d*x - 1))^2) + 2*(3*b*d^3*(s
qrt(d*x + 1) - sqrt(d*x - 1))^10 - 12*c*d^2*(sqrt(d*x + 1) - sqrt(d*x - 1))
^8 - 96*a*d^4*(sqrt(d*x + 1) - sqrt(d*x - 1))^4 - 96*c*d^2*(sqrt(d*x + 1) -
 sqrt(d*x - 1))^4 - 48*b*d^3*(sqrt(d*x + 1) - sqrt(d*x - 1))^2 - 128*a*d^4
- 192*c*d^2)/((sqrt(d*x + 1) - sqrt(d*x - 1))^4 + 4)^3)/d
```

maple [C] time = 0.02, size = 123, normalized size = 1.06

$$-\frac{\sqrt{dx-1}\,\sqrt{dx+1}\,\left(3b\,d^2x^3\arctan\left(\frac{1}{\sqrt{d^2x^2-1}}\right)-4\sqrt{d^2x^2-1}\,a\,d^2x^2-6\sqrt{d^2x^2-1}\,c\,x^2-3\sqrt{d^2x^2-1}\,bx-2\sqrt{d^2x^2-1}\,a\right)\mathrm{csgn}(d)^2}{6\sqrt{d^2x^2-1}\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x)
```

```
[Out] -1/6*(d*x-1)^(1/2)*(d*x+1)^(1/2)*csgn(d)^2*(3*arctan(1/(d^2*x^2-1)^(1/2))*x
^3*b*d^2-4*(d^2*x^2-1)^(1/2)*x^2*a*d^2-6*(d^2*x^2-1)^(1/2)*x^2*c-3*(d^2*x^2
-1)^(1/2)*b*x-2*(d^2*x^2-1)^(1/2)*a)/(d^2*x^2-1)^(1/2)/x^3
```

maxima [A] time = 0.98, size = 86, normalized size = 0.74

$$-\frac{1}{2}\,bd^2\arcsin\left(\frac{1}{d|x|}\right)+\frac{2\,\sqrt{d^2x^2-1}\,ad^2}{3\,x}+\frac{\sqrt{d^2x^2-1}\,c}{x}+\frac{\sqrt{d^2x^2-1}\,b}{2\,x^2}+\frac{\sqrt{d^2x^2-1}\,a}{3\,x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/x^4/(d*x-1)^(1/2)/(d*x+1)^(1/2),x, algorithm="maxim
a")
```

```
[Out] -1/2*b*d^2*arcsin(1/(d*abs(x))) + 2/3*sqrt(d^2*x^2 - 1)*a*d^2/x + sqrt(d^2*
x^2 - 1)*c/x + 1/2*sqrt(d^2*x^2 - 1)*b/x^2 + 1/3*sqrt(d^2*x^2 - 1)*a/x^3
```

mupad [B] time = 9.44, size = 304, normalized size = 2.62

$$\frac{\frac{b\,d^2\,1\mathrm{i}}{32}+\frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{16\,{\left(\sqrt{d\,x+1}-1\right)}^2}-\frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4\,15\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^4}}{\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+\frac{2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^4}{{\left(\sqrt{d\,x+1}-1\right)}^4}+\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^6}{{\left(\sqrt{d\,x+1}-1\right)}^6}}-\frac{b\,d^2\,\ln\left(\frac{{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2}{{\left(\sqrt{d\,x+1}-1\right)}^2}+1\right)\,1\mathrm{i}}{2}+\frac{b\,d^2\,\ln\left(\frac{\sqrt{d\,x-1}-\mathrm{i}}{\sqrt{d\,x+1}-1}\right)\,1\mathrm{i}}{2}+\frac{c\,\sqrt{d\,x-1}\,\sqrt{d\,x+1}}{x}+\frac{\sqrt{d\,x-1}\,\left(\frac{2\,a\,d^3\,x^3}{3}+\frac{2\,a\,d^2\,x^2}{3}+\frac{a\,d\,x}{3}+\frac{a}{3}\right)}{x^3\,\sqrt{d\,x+1}}+\frac{b\,d^2\,{\left(\sqrt{d\,x-1}-\mathrm{i}\right)}^2\,1\mathrm{i}}{32\,{\left(\sqrt{d\,x+1}-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/(x^4*(d*x - 1)^(1/2)*(d*x + 1)^(1/2)),x)
```

```
[Out] ((b*d^2*1i)/32 + (b*d^2*((d*x - 1)^(1/2) - 1i)^2*1i)/(16*((d*x + 1)^(1/2) -
 1)^2) - (b*d^2*((d*x - 1)^(1/2) - 1i)^4*15i)/(32*((d*x + 1)^(1/2) - 1)^4))
/(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + (2*((d*x - 1)^(1/2) -
1i)^4)/((d*x + 1)^(1/2) - 1)^4 + ((d*x - 1)^(1/2) - 1i)^6/((d*x + 1)^(1/2)
- 1)^6) - (b*d^2*log(((d*x - 1)^(1/2) - 1i)^2/((d*x + 1)^(1/2) - 1)^2 + 1)*
1i)/2 + (b*d^2*log(((d*x - 1)^(1/2) - 1i)/((d*x + 1)^(1/2) - 1))*1i)/2 + (c
*(d*x - 1)^(1/2)*(d*x + 1)^(1/2))/x + ((d*x - 1)^(1/2)*(a/3 + (2*a*d^2*x^2)
/3 + (2*a*d^3*x^3)/3 + (a*d*x)/3))/(x^3*(d*x + 1)^(1/2)) + (b*d^2*((d*x - 1
)^(1/2) - 1i)^2*1i)/(32*((d*x + 1)^(1/2) - 1)^2)
```

sympy [C] time = 129.78, size = 219, normalized size = 1.89

$$-\frac{ad^3 G_{6,6}^{5,3}\left(\begin{matrix}\frac{9}{4},\frac{11}{4},1 & \frac{5}{2},\frac{5}{2},3\\ 2,\frac{9}{4},\frac{5}{2},\frac{11}{4},3 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{iad^3 G_{6,6}^{2,6}\left(\begin{matrix}\frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2},1 & \\ \frac{7}{4},\frac{9}{4} & \frac{3}{2},2,2,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{bd^2 G_{6,6}^{5,3}\left(\begin{matrix}\frac{7}{4},\frac{9}{4},1 & 2,2,\frac{5}{2}\\ \frac{3}{2},\frac{7}{4},2,\frac{9}{4},\frac{5}{2} & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}+\frac{ibd^2 G_{6,6}^{2,6}\left(\begin{matrix}1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2,1 & \\ \frac{5}{4},\frac{7}{4} & 1,\frac{3}{2},\frac{3}{2},0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{cd G_{6,6}^{5,3}\left(\begin{matrix}\frac{5}{4},\frac{7}{4},1 & \frac{3}{2},\frac{3}{2},2\\ 1,\frac{5}{4},\frac{3}{2},\frac{7}{4},2 & 0\end{matrix}\middle|\frac{1}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}-\frac{icd G_{6,6}^{2,6}\left(\begin{matrix}\frac{1}{2},\frac{3}{4},1,\frac{5}{4},\frac{3}{2},1 & \\ \frac{3}{4},\frac{5}{4} & \frac{1}{2},1,1,0\end{matrix}\middle|\frac{e^{2i\pi}}{d^2x^2}\right)}{4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/x**4/(d*x-1)**(1/2)/(d*x+1)**(1/2),x)
```

```
[Out] -a*d**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3), (
0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - I*a*d**3*meijerg(((3/2, 7/4, 2, 9/4, 5
/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), exp_polar(2*I*pi)/(d**2*x**2))/(
4*pi**(3/2)) - b*d**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2,
9/4, 5/2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) + I*b*d**2*meijerg(((1, 5/4,
 3/2, 7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(2*I*pi)/(d
**2*x**2))/(4*pi**(3/2)) - c*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1,
 5/4, 3/2, 7/4, 2), (0,)), 1/(d**2*x**2))/(4*pi**(3/2)) - I*c*d*meijerg(((1
/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*
pi)/(d**2*x**2))/(4*pi**(3/2))
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
```

```
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
```

```
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
```

```
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
```

```
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
```

```
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```